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A novel numerical optimization algorithm inspired from weed colonization

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ABSTRACT

This paper introduces a novel numerical stochastic optimization algorithm inspired from colonizing weeds. Weeds are plants whose vigorous, invasive habits of growth pose a serious threat to desirable, cultivated plants making them a threat for agriculture. Weeds have shown to be very robust and adaptive to change in environment. Thus, capturing their properties would lead to a powerful optimization algorithm. It is tried to mimic robustness, adaptation and randomness of colonizing weeds in a simple but effective optimizing algorithm designated as Invasive Weed Optimization (IWO). The feasibility, the efficiency and the effectiveness of IWO are tested in details through a set of benchmark multi-dimensional functions, of which global and local minima are known. The reported results are compared with other recent evolutionary-based algorithms: genetic algorithms, memetic algorithms, particle swarm optimization, and shuffled frog leaping. The results are also compared with different versions of simulated annealing — a generic probabilistic meta-algorithm for the global optimization problem — which are simplex simulated annealing, and direct search simulated annealing. Additionally, IWO is employed for finding a solution for an engineering problem, which is optimization and tuning of a robust controller. The experimental results suggest that results from IWO are better than results from other methods. In conclusion, the performance of IWO has a reasonable performance for all the test functions.

1. Introduction

Engineering design problems and applications always involve optimization problems that must be solved efficiently and effectively. To solve a problem, an engineer must sketch a proper view from the problem in her hand. So, the design is the struggle of the designer for finding a solution which best suits the sketched view. In support of this need, there have been various optimization techniques proposed by scientists. In practice, many engineering problems do not have explicit presentation of control variables and/or do not have continuity, which are necessary for applying gradient-based optimization techniques. In order to overcome this difficulty, scientists proposed direct optimization methods that only use objective function and constrain values to steer towards the solution. Since derivative information is not used, the direct search methods are typically slow, requiring many function evaluations for convergence. For the same reason, they can also be applied to different problems without applying major changes in the algorithm.

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Recently, in the literature, there has been a considerable attention paid for employing algorithms inspired from natural processes and/or events in order to solve optimization problems. For example, genetic algorithms (GAs) which was first introduced by Holland (1975) are now a standard optimization tool in engineering. There are also other numerical direct search optimization methods, e.g. simulated annealing (SA); tabu search (TS) (Pham and Karaboga, 2000); Ant colony optimization (ACO) (Dorigo et al., 1996); and particle swarm optimization (PSO) (Kennedy and Eberhart, 1995).

Recently, many studies were carried out with inspirations from ecological phenomena for developing optimization techniques. For instance, a novel evolutionary algorithm inspired by the nature of spatial interactions in ecological systems is introduced in (Kirley, 2002), where the author have examined the response of the evolving population to the process of fragmentation and disturbance cased by natural events (like fire, floods or climate changes). Another ecology-inspired EA is introduced in (Yuchi and Kim, 2005). In the mentioned research, in each generation, according to the feasibility of the individuals, the whole population is divided into two groups: feasible group and infeasible group. Evaluation and ranking of these two groups are performed in parallel and separately. The best individuals from feasible and infeasible groups are selected together as parents. The number of feasible parents has a sigmoid-type relation with that of feasible individuals, which is inspired by the natural ecological population growth in a confined space.

This work is motivated by a common phenomenon in agriculture that is colonization of invasive weeds. According to the common definition, a weed is any plant growing where it is not wanted. Any tree, vine, shrub, or herb may qualify as a weed, depending on the situation; generally, however, the term is reserved for those plants whose *vigorous, invasive habits of growth pose a serious threat to desirable, cultivated plants*. Weeds have shown very robust and adaptive nature which turns them to undesirable plants in agriculture. That's why many Journals (e.g. Weed Biology and Management Journal, Weed Research Journal, and Weed Science Journal) are being published world-wide focusing on the study of weed taxonomy, ecology and physiology, weed management and control methodologies, etc. In this paper it is tried to introduce a simple numerical general-purpose optimization algorithm that is inspired by weed colonization designated as Invasive Weed Optimization (IWO). The algorithm is simple but has shown to be effective in converging to optimal solution by employing basic properties, e.g. seeding, growth and competition, in a weed colony. Simulation studies are conducted to evaluate convergence and performance of the proposed algorithm.

2. Weed ecology[1]

In this section our goal is to provide the reader some sense of weed biology, ecology, and colonization.

[1] Selected topics from (Dekker, 2005). Used with permission.

2.1. What weed is, and why it is important

A plant is called weed if, in any specified geographical area, its populations grow entirely or predominantly in situations markedly disturbed by man (without, of course, being deliberately cultivated plants) (Baker and Stebbins, 1965). The most interesting feature of weeds that is now become a common belief in agronomy it that "The Weeds Always Win". The harder people try, the better they get:

- After many thousands of years of tillage and hand-weeding we still have weeds.
- After 50 years of herbicides we still have weeds in the same fields.
- In every field, in every year, we always miss some weeds.
- New weed species appear frequently, spreading across the country.
- No weed species have disappeared from production fields.
- Humans have recently created an entirely new category of very nasty weeds: herbicide resistant weeds.

These properties indicates that the weeds are of the most robust and troublous plants in agriculture. It is also a confirmation of the fact that weeds adapt with environment and change their behavior and gets better (fitter). Weed biology and ecology is the story of their success. The behavior of weeds in occupying a territory and colonizing is made in the following steps:

1) Our cropping systems create opportunity spaces by leaving unused resources in local fields that are frequently disturbed (e.g. tillage, herbicides).
2) Weeds invade these opportunity spaces by means of dispersal, followed by colonization, followed by enduring occupation of the field.
3) Weed biodiversity provides diverse plants with traits well adapted to seize and exploit these opportunity spaces, which become locally adapted and improved over time by means of natural selection and adaptation.
4) Weedy traits are expressed at optimum times in the life history of the plant as the agricultural season unfolds such that they maximize their fitness in a plant community.
5) Agricultural plant communities assemble and interact with each other based these traits.

2.2. Weed reproduction

Weeds may reproduce with or without using sex cells, depending on the type of the plant. Sexual reproduction is made by means of *seeds* or *spores*. In sexual reproduction a plant is born, and begins its life history, when the egg is fertilized by pollen and forms a seed in a parent plant. Then it is distributed by wind, water, animals, etc. (spatial dispersal) until it can find an *opportunity space* for growth (independent ramet). Viable seeds germinate and grow when conditions are good (juvenile). They vegetate to adult plants while in interaction with other neighbor plants (vegetative plant). They turn to flowering plants and produce seeds at the final stage of their life (seed productive growth). In their colonization neighboring plants interfere with each other's activities according to their age, size and distance apart. Such density stress affects the *birth rates* and *death rates* of plant parts. As plants in a population develop, the biomass produced becomes limited by the rate of availability of resources so that yield per unit area becomes independent of density — the carrying capacity of the environment. The stress of density increases the risk of mortality to whole plants as well as their parts and the rate of death becomes a function of the growth rate of the survivors (Harper, 1977). *Thus birth, growth, and reproduction of plants are influenced by density, population, and fitness of the plants' colony*. There are mainly three components of fitness in the community, where these different components are in conflict with each other, and any estimate of fitness must consider all of them:

- reproduction,
- struggle for existence with competitors,
- avoidance of predators.

Any colony tries to improve its fitness to achieve a longer life.

2.3. Forces of selection acting on plant community dynamics

The study of population biology ought to display those forces that are important at the level of the life of the individual and what sort of variation is important in determining survivorship and reproduction. The forces of selection have been described as (I) directional selection (II) stabilizing selection, and (III) disruptive selection. These broad categories of selective force are in a sense statistical rather than biological category; the biological categories need to take into account the nature as well as the direction of selection. A number of generalized biological categories can be recognized:

- r and K selection,
- ecological combining ability,
- selection by activity of predators and pathogens,
- the evolutionary consequences of disturbances,
- selection in a patchy environment.

Where we are only interested in r and K selection:

2.3.1. r-selection: "live fast, reproduce quick, die young."

Selection for the qualities needed to succeed in *unstable* and *unpredictable* environments, where ability to reproduce rapidly and opportunistically is at a premium, and where there is little value in adaptations to succeed in competition. A variety of qualities are thought to be favored by r-selection, including high fecundity, small size, and adaptations for long-distance dispersal. Weeds, and their animal equivalents, are examples, in contrast with K-selection. It is customary to emphasize that r-selection and K-selection are the extremes of a continuum, most real cases lying somewhere between. Ecologist enjoys a curious love/hate relationship with the r/K concept, often pretending to disapprove of it while finding it indispensable (Dawkins, 1999).

2.3.2. K-selection: "live slow, reproduce slow, die old."

Selection for the qualities needed to succeed in stable, predictable environments where there is likely to be heavy competition for limited resources between individuals well-equipped to compete, at population sizes close to the maximum that the habitat can bear. A variety of qualities are thought to be favored by K-selection, including large size, long life, and small numbers of intensively cared-for offspring, in contrast with r-selection (Dawkins, 1999).

3. Simulating weed colonizing behavior

To simulate colonizing behavior of weeds some basic properties of the process is considered:

1) a finite number of seeds are being dispread over the search area (initializing a population),
2) every seed grows to a flowering plant and produces seeds depending on its fitness (reproduction),
3) the produced seeds are being randomly dispread over the search area and grow to new plants (spatial dispersal),
4) this process continues until maximum number of plants is reached; now only the plants with lower fitness can survive and produce seeds, others are being eliminated (competitive exclusion). The process continue until maximum iterations is reached and hopefully the plant with best fitness it the closest to the optimal solution.

The process is addressed in details as follows:

3.1. Initialize a population

A population of initial solutions is being dispread over the d dimensional problem space with random positions.

3.2. Reproduction

A member of the population of plants is allowed to produce seeds depending on its own and the colony's lowest and highest fitness: the number of seeds each plant produce increases linearly from minimum possible seed production to its maximum. In other words, a plant will produce seeds based on its fitness, the colony's lowest fitness and highest fitness to make sure the increase is linear. Fig. 1 illustrates the procedure.

This step adds a significant property to the search algorithm. Often when evolutionary algorithms are adopted to solve optimization problems, intuitively, feasible individuals could be thought to be the ones with better fitness values than infeasible individuals (here "better" means to have more chance to survive and reproduce); thus, the infeasible individuals are not allowed to be reproduced. However, this kind of view ignores one important thing that evolutionary algorithm is a probabilistic and recurrent method (Yuchi and Kim, 2005). It is possible that some of the infeasible individuals carry more useful information than feasible individuals during evolution process. Moreover, quite often the system can reach the optimal point more easily if it is possible to "cross" an infeasible region (especially in non-convex feasible search space) (Yuchi and Kim, 2005). Thus, the above reproduction technique is proposed to give a chance to infeasible individuals to survive and reproduce similar to the mechanism happens in the nature.

3.3. Spatial dispersal

Randomness and adaptation in the algorithm is provided in this part. The generated seeds are being randomly distributed over the d dimensional search space by normally distributed random numbers with mean equal to zero; but varying variance. This means that seeds will be randomly distributed such that they abode near to the parent plant. However, standard deviation (SD), σ , of the random function will be reduced from a previously defined initial value, σ_{initial}, to a final value, σ_{final}, in every step (generation). In simulations, a nonlinear alteration has shown satisfactory performance, which is given in Eq. (1)

$$\sigma_{\text{iter}} = \frac{(\text{iter}_{\max}-\text{iter})^n}{(\text{iter}_{\max})^n}(\sigma_{\text{initial}}-\sigma_{\text{final}}) + \sigma_{\text{final}} \tag{1}$$

where $\text{iter}_{\max}$ is the maximum number of iterations, σ_{iter} is the SD at the present time step and n is the nonlinear modulation index.

This alteration ensures that the probability of dropping a seed in a distant area decreases nonlinearly at each time step which results in grouping fitter plants and elimination of inappropriate plants, representing transformation from r-selection to K-selection mechanism.

3.4. Competitive exclusion

If a plant leaves no offspring then it would go extinct, otherwise they would take over the world. Thus, there is a need of some kind of competition between plants for limiting maximum number of plants in a colony. After passing some iterations, the number of plants in a colony will reach its maximum by fast reproduction, however, it is expected that the fitter plants have been reproduced more than undesirable plants. By reaching the maximum number of plants in the colony, $p_{\max}$, a mechanism for eliminating the plants with poor fitness in the generation activates. The elimination mechanism works as follows: when the maximum number of weeds in a colony is reached, each weed is allowed to produce seeds according to the mechanism mentioned in the section 3.2. The produced seeds are then allowed to spread over the search area according to the section 3.3. When all seeds have found their position in the search area, they are ranked together with their parents' (as a colony of weeds). Next, weeds with lower fitness are eliminated to reach the maximum allowable population in a colony. In this way, plants and offspring are ranked together and the ones with better fitness survive and are allowed to replicate. As mentioned in step (2), this mechanism give a chance to plants with lower fitness to reproduce, and if their offspring has a good fitness in the colony then they can survive. The population control mechanism also is applied to their offspring to the end of a given run, realizing competitive exclusion.

4. Simulation studies

In this section several simulation studies are carried out to demonstrate merits of the proposed optimization algorithm. In the first step, the capability of the algorithm in finding global minimum of three benchmark functions, which are frequently employed in the literature, is demonstrated. These functions are 'Sphere', 'Griewank' and 'Rastrigin'; to show that the algorithm converges to the global solution, the results are compared to a standard GA.

As the second step, IWO algorithm is applied for finding optimal solution of a high dimension Rastrigin function with the dimension of $d=30$. Results are reported and discussed to give a better insight in the effects of tuning parameters of the IWO algorithm in finding global optima of continuous functions.

Next, with the purpose of comparison with commonly used numerical optimization algorithms, i.e. Genetic algorithms (GAs), Memetic algorithms (MAs), Particle swarm optimization (PSO) and Shuffled frog leaping (SFL), a set of studies are conducted for optimizing very high order Griewank function ($d=10$, 20, 50, 100) and EF10 function.

Finally, the performance of IWO is compared with different versions of simulated annealing; specifically simplex simulated annealing and direct search simulated annealing. The simulations are performed for optimization of Easom function and Griewank function.

4.1. Convergence of the invasive weed optimization algorithm

Three studies are conducted to demonstrate ability of the IWO algorithm in locating global minima of continues functions. Employed benchmark examples are 'Sphere', 'Griewank' and 'Rastrigin' functions, which have properties reported in Table 1.

4.1.1. Sphere Function

Numerical values for a given run executed for minimizing the Sphere function with dimension of $d=2$, in provided in Table 2. Process of colonizing of weeds around the point with the best fitness is shown in Fig. 2. It can be observed that the plants grow towards the optimal point from the initialization area. In their progress towards the optimal point, plants with higher (worse) fitness are being excluded, and only weeds with lower (better) fitness are allowed to be reproduced, which leads in colonization about the optimal point. The final value of the fitness function for Sphere function is found to be *fitness* $(x_0)=2.4362\text{e}{-8}$, for the point: $x_0=[-0.1413\text{e}{-3},-0.0662\text{e}{-3}]$. It is known that the optimal value of the function is zero for the point [0, 0] in x–y plane.

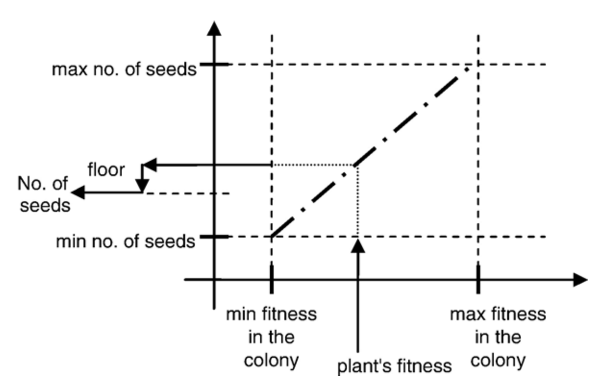
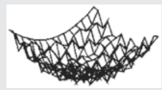
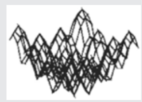


Fig. 1 – Seed production procedure in a colony of weeds.

Table 1 – Optimization test functions

Name	Formula	Sketch in 2D
Sphere	$f(x) = \sum_{i=1}^{n} x_i^2$	
Griewank	$f(x) = \frac{1}{4000}\sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n} \cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$	
Rastrigin	$f(x) = \sum_{i=1}^{n} (x_i^2 - 10\cos(2\pi x_i) + 10)$	

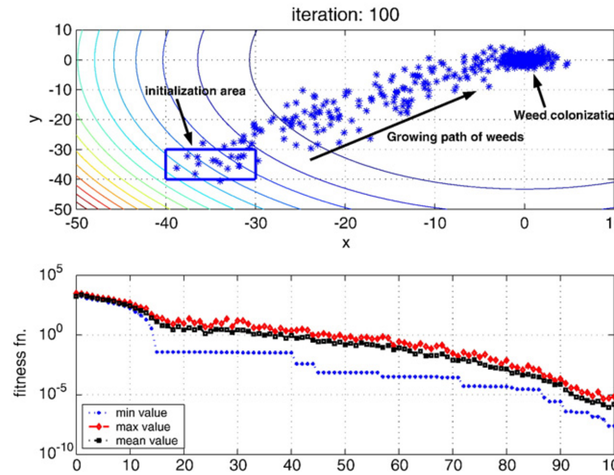


Fig. 2 – Convergence of IWO to the optimal value of the Sphere function.

4.1.2. *Griewank function*

Finding minima of the Griewank function is a challenging problem, which is the main reason for being a favorite benchmark for optimization algorithms. A schematic of Griewank function with dimension of $d=2$ is illustrated in Fig. 3. It is observed from Fig. 3 that the function has only one global minima at [0, 0] in x–y plane but numerous local minima.

Setup of IWO algorithm for minimization of this function is specified in Table 3. Fig. 4 illustrates the process of obtaining optimal solution of the problem. To demonstrate merits of proposed algorithm, same simulation is performed using GA toolbox provided in MATLAB®, where the initial conditions and number of maximum agents were identical in both simulations. As depicted in Fig. 4, the proposed algorithm outperformed GA in finding the minima of the Griewank function.

4.1.3. *Rastrigin function*

In order to demonstrate IWO abilities in minimization of different functions, another challenging optimization problem that is minimization of Rastrigin function is addressed in this part. Fig. 5 illustrates schematic of Rastrigin function with the dimension of $d=2$. Fig. 5 clearly shows that the Rastrigin function has numerous local minima — the "valleys" in the plot — same as the Griewank function. However, the function has just one global minimum, which occurs at the point [0, 0] in the x–y plane, as indicated by the vertical line in the plot, where the value of the function is zero. At any local minimum other than global minima, the value of Rastrigin function is greater than zero. The farther the local minimum is from the origin, the larger the value of the function is at that point. Low or high dimension Rastrigin function is regularly used in the literature to test evolutionary algorithms, because of its numerous local minima which make it difficult for standard, gradient-based methods to find the global optimum (Elbeltagi et al., 2005; Chatterjee and Siarry, 2006). The contour plot in Fig. 6 of Rastrigin function shows the alternating maxima and minima.

Setup of IWO algorithm for minimization of this function is given in Table 3, which is the same as Griewank function. Minimization procedure of the function is depicted in Fig. 7. For a comparison, result of a GA run using MATLAB® GA Toolbox is also added to illustrate performance of IWO algorithm.

4.2. *Effects of tuning parameters on the convergence of the IWO algorithm*

Armed with successful tests of IWO algorithm in finding global optimal solution of different challenging functions, in this section, further studies are conducted for understanding the effects of tuning parameters of IWO algorithm on its

Table 2 – IWO parameter values for sphere function minimization

Symbol	Quantity	Value
N_0	Number of initial population	10
it_{max}	Maximum number of iterations	100
dim	Problem dimension	2
p_{max}	Maximum number of plant population	15
s_{max}	Maximum number of seeds	5
s_{min}	Minimum number of seeds	0
n	Nonlinear modulation index	3
$\sigma_{initial}$	Initial value of standard deviation	3
σ_{final}	Final value of standard deviation	0.001
x_{ini}	Initial search area	$-40<x_{ini}<-30$

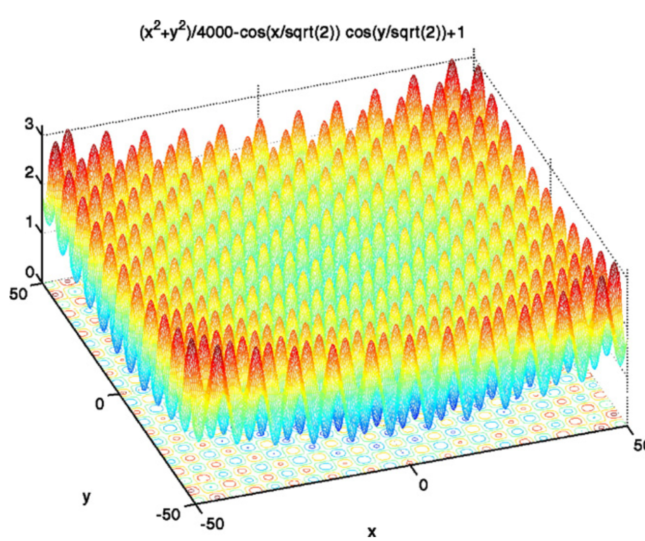


Fig. 3 – The Griewank function.

Table 3 – IWO parameter values for Griewank and Rastrigin function minimization

Symbol	Quantity	Value
N_0	Number of initial population	10
it_{max}	Maximum number of iterations	500
dim	Problem dimension	2
p_{max}	Maximum number of plant population	30
s_{max}	Maximum number of seeds	5
s_{min}	Minimum number of seeds	0
n	Nonlinear modulation index	3
$\sigma_{initial}$	Initial value of standard deviation	10
σ_{final}	Final value of standard deviation	0.1
x_{ini}	Initial search area	$-20<x_{ini}<20$

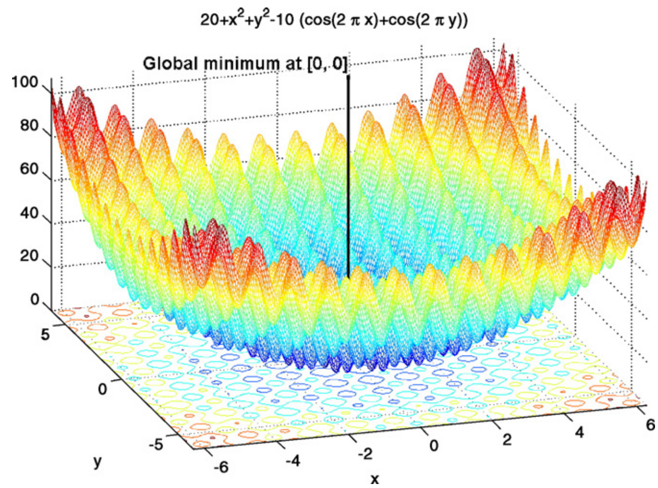


Fig. 5 – The Rastrigin function.

convergence. To this end, the Rastrigin function with the dimension of $d=30$ is considered as a benchmark. It is known that the global optimum of the considered function is 0.0. The benchmark is more complex than the ones in previous section due to larger dimension size. The performance of IWO algorithm is studied with different population size of weeds, different algorithm iterations and different nonlinear modulation index n, in each experiment with 100 trial runs in each experiment. The parameters of the algorithm are specified in Table 4. During the optimization process the weeds are allowed to 'grow' outside the region specified by x_{ini}. The performance of the proposed algorithm is specified in two criteria: (1) the percentage of success, as represented by the number of trials required for the object function to reach its known target values, which is equal or lower than 0.05 in this experiment; (2) the average value of the solution obtained in all trails. In all experiments, the solution stopped when maximum allowable iteration was reached. Table 5 presents the performance obtained over all trail runs for the Rastrigin function.

Considering the performance of the proposed algorithm for all runs, one can observe that increasing the number of iterations leads to a lower mean value for solution; however, basically, it does not increase the number of successful convergences to the desired global optima. The percentage of success is increased by decreasing population of weeds in a colony. Hence, increasing the number of agents (weeds) does not essentially lead to a satisfactory result (lower mean value and/or successful convergence). The effect of the nonlinear modulation index is also studied in this example. It is shown that nonlinear modulation index has magnificent contribution on the performance of IWO algorithm, where much better results are obtained when n is set equal to 3. The effect of the nonlinear modulation index is considered in next subsection of the paper too.

4.3. *Comparing IWO with GAs, MAs, PSO, and SFL*

In this section, the performance of IWO algorithm is compared with four evolutionary algorithms (EAs): Genetic algorithms (GAs) (Holland, 1975), Memetic algorithms (MAs) (Moscato, 1989), Particle swarm optimization (PSO) (Kennedy and Eberhart, 1995) and Shuffled frog leaping (SFL) (Eusuff and Lansey, 2003). The results are compared with the ones reported in (Elbeltagi et al., 2005). It should be noted that the aim of reporting the results in (Elbeltagi et al., 2005) is

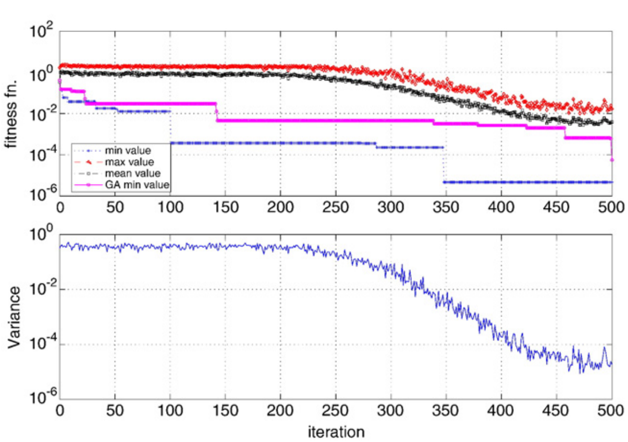


Fig. 4 – Upper diagram: optimizing process of the Griewank function by IWO algorithm vs. standard genetic algorithm. Lower diagram: associated variance of each generation.

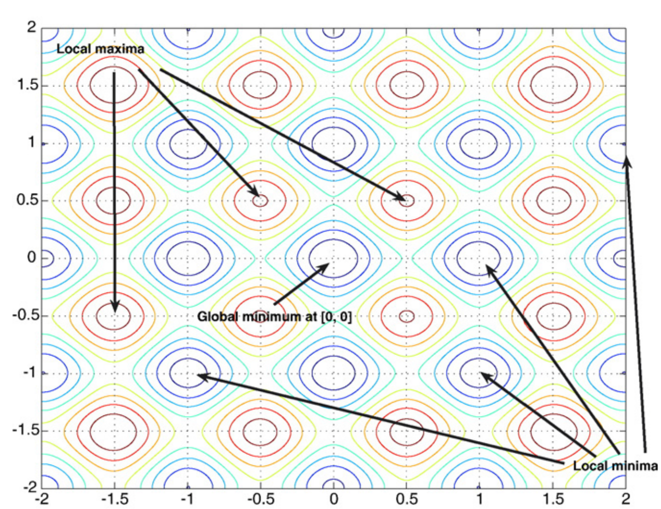


Fig. 6 – Contour plot of Rastrigin function.

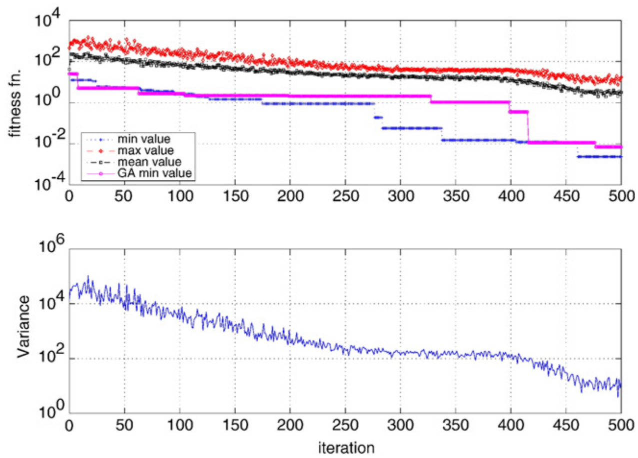


Fig. 7 – Upper diagram: optimizing process of the Rastrigin function by IWO algorithm vs. standard genetic algorithm. Lower diagram: associated variance of each generation.

comparison of abovementioned optimization algorithms; therefore, it was tried to find the best possible parameters values for the algorithms in order to have a reasonable comparison and providing guidelines for determining the best operators for each algorithm.

4.3.1. *Griewank function*

To have a sensible comparison between IWO and the mentioned optimization algorithms, in addition to studying the influence of the parameter values on the performance of IWO, the simulations are performed for a high-dimension Griewank function (d=10, 20, 50, and 100). The performance of IWO for finding the global optima of the benchmark is examined with different population size of the weed colony, different algorithm iterations and different nonlinear modulation index n. Twenty trial runs are performed for each problem. Same as the previous example during the optimization process the weeds are allowed to 'grow' outside the region specified by x_{ini}. The performance of the proposed algorithm is specified in two criteria, same as the previous example: (1) the percentage of success; and (2) mean value of solutions. The parameters of the algorithm are specified in Table 6. Note that in all experiments, the solution stopped when maximum allowable iteration was reached.

The results found from solving the test problem using the four EAs vis-à-vis IWO are summarized in Table 7. As it can be observed from Table 7, a well-tuned IWO performs very well in finding global optima of the benchmark problem. The interesting thing is that the GA performs more poorly than all other four algorithms, which made the authors of Elbeltagi et al., (2005) to verify their results with a commercial GA package named Evolver (Elbeltagi et al., 2005; Evolver, 1998). Thus, outperforming of IWO over a GA, reported in Section 4.1 is not really very surprising. Table 7 shows that increasing the number of agents (weeds) in a colony does no essentially increase the percentage of success, the fact that was observed in the previous section as well. Moreover, in contrast with GA, increase in dimensionality of the problem does not decrease performance of IWO. It is also shown that IWO, in some cases, has surpassed PSO and SFL. Hence, it can be concluded that the proposed algorithm can be considered as one of efficient EAs.

It is observed that the value of the nonlinear modulation index, n, has considerable contribution on the performance of IWO. The nonlinear modulation index has a key control on IWO convergence. It makes the weed colony to change their behavior in time and softly switch from a high value of standard deviation to a lower one. Thus, the algorithm starts with a high initial standard deviation which should allow it to explore new search areas aggressively and then decreases it gradually according to (1) to find a finer local optimum solution in later

Table 4 – IWO Numerical parameter values for high-dimension Rastrigin function optimization

Symbol	Quantity	Value
N_0	Number of initial population	10
it_{max}	Maximum number of iterations	100 or 500
dim	Problem dimension	30
p_{max}	Maximum number of plant population	20, 40, or 60
s_{max}	Maximum number of seeds	3
s_{min}	Minimum number of seeds	0
n	Nonlinear modulation index	1, 2, or 3
$\sigma_{initial}$	Initial value of standard deviation	10
σ_{final}	Final value of standard deviation	0.02
x_{ini}	Initial search area	$-100<x_{ini}<100$

Table 5 – Simulation results of high-dimensional Rastrigin function optimization

Max. population of weeds	Nonlinear modulation Index (n)	Max. no. of iterations	Comparison criteria	
			% Success	Mean solution
20			14	90.4242
40	3	500	12	69.3683
60			5	62.2004
20			27	2574.2
40	1	100	14	2427.1
60			9	2368.3
20			93	494.23
40	3	100	72	1538.7
60			67	1617.7
	1		11	230.74
20	2	500	26	92.957
	3		20	85.76

Table 6 – IWO parameter values for high-dimension Griewank function optimization for comparison with GAs, MAs, PSO AND SFL

Symbol	Quantity	Value
N_0	Number of initial population	5
it_{max}	Maximum number of iterations	30, 120, 200, 210
dim	Problem dimension	30
p_{max}	Maximum number of plant population	10, 20, 50, and 100
s_{max}	Maximum number of seeds	3
s_{min}	Minimum number of seeds	0
n	Nonlinear modulation index	1, 2, or 3
$\sigma_{initial}$	Initial value of standard deviation	300
σ_{final}	Final value of standard deviation	0.05
x_{ini}	Initial search area	$-512<x_{ini}<511$

Table 7 – Results of the Griewank function optimization for comparison with GAs, MAs, PSO AND SFL

Comparison criteria	Algorithm	Max. population of weeds	The nonlinear modulation index (n)	Max. no. of iterations	Problem dimension 10	20	50	100
% Success		10			70	80	95	100
		20	3	200	65	95	100	100
		30			75	85	100	95
			1		10	10	40	50
	IWO	20	2	200	95	25	80	85
			3		65	95	100	100
				30	35	35	45	35
		20	3	120	95	75	100	95
				210	80	90	100	100
	GAs (Evolver)	–	–	–	50	30	10	0
	MAs	–	–	–	90	100	100	100
	PSO	–	–	–	30	80	100	100
	SFL	–	–	–	50	70	90	100
Mean solution		10			0.0459	0.2108	2.4932	0
		20	3	200	0.0373	0.0494	0	0
		30			0.0215	0.1432	0	25.7871
			1		1.0132	3.4364	64.239	395.33
	IWO	20	2	200	0.019302	0.88883	7.8221	75.727
			3		0.0373	0.0494	0	0
				30	4.0067	65.3527	363.31	1085.9
		20	3	120	0.018437	0.4245	0	49.34
				210	0.016336	0.1066	0	0
	GAs (Evolver)	–	–	–	0.06	0.097	0.161	0.432
	MAs	–	–	–	0.014	0.013	0.011	0.009
	PSO	–	–	–	0.093	0.081	0.011	0.011
	SFL	–	–	–	0.08	0.063	0.049	0.019

iterations. As it can be concluded from simulations, this phenomenon is accomplished better when n is set to 3.

The termination criterion that is used for IWO in simulations is attaining to the maximum number of allowable iterations; however, in other four EAs different termination criteria was applied (Elbeltagi et al., 2005). For instance, in PSO algorithm, upon experimentation, the suitable numbers of particles and generations were found to be 40 and 10,000, respectively (Elbeltagi et al., 2005). Hence, IWO is able to find the optimal solution in fewer iterations vis-à-vis PSO in this example, which is an advantage of the proposed algorithm.

4.3.2. *EF10 Function*

The F10 function is a non-linear, non-separable and involves two variables, x and y (Elbeltagi et al., 2005)

$$f10(x,y) = (x^2+y^2)^{0.25}\left\{\sin^2\left[(x^2+y^2)^{0.1}\right]+1\right\} \tag{2}$$

An extended EF10 function is created to scale the original F10 function (2) to any number of variables (Elbeltagi et al., 2005):

$$EF10(x) = \sum_{j=1}^{N}\sum_{i=1}^{N} f10(x_i, x_j) \tag{3}$$

Similar to Griewank function, the global optimum solution of the F10 function is known to be zero when all variables are equal to zero.

The simulations are performed for a high-dimension F10 function with the dimension of d=10, 20, and 50. The performance of IWO for finding the global optima of the benchmark is examined with different population size of the weed colony and different ratios of maximum number of allowable seed to population (seed/pop ratio). Same as the previous example, twenty trial runs are performed for each problem and during the optimization process the weeds are allowed to 'grow' outside the region specified by x_{ini}. In addition the performance of the proposed algorithm is specified in two criteria, same as the previous example: (1) the percentage of success; and (2) mean value of solutions. The parameters of the algorithm are specified in Table 8. Note that in all experiments, the solution stopped when maximum allowable iteration was reached.

Simulation results, as reported in Table 9, show that the best performance is obtained when the maximum population of weeds is set to 10 or 20. Additionally, the number of

Table 8 – IWO Parameter values for EF10 function optimization

Symbol	Quantity	Value
N_0	Number of initial population	10
it_{max}	Maximum number of iterations	800
dim	Problem dimension	10
p_{max}	Maximum number of plant population	10, 20, or 30
s_{max}	Maximum number of seeds	15, 10, 6, 5, 4, 3, 2, or 1
s_{min}	Minimum number of seeds	0
n	Nonlinear modulation index	3
$\sigma_{initial}$	Initial value of standard deviation	75
σ_{final}	Final value of standard deviation	1e− 6
x_{ini}	Initial search area	$-100<x_{ini}<100$

Table 9 – Simulation results of the EF10 function optimization

Comparison criteria	Algorithm	Max. population of weeds	Seed/pop ratio	Problem dimension 10	20	50
% Success			1/2	0	0	–
		10	1/5	100	100	100
			1/10	100	100	100
			1/2	0	0	–
	IWO	20	1/5	10	0	–
			1/10	100	100	–
			1/2	0	0	–
		30	1/5	0	0	–
			1/10	25	45	–
	GAs (Evolver)	–	–	20	0	0
	MAs	–	–	100	70	0
	PSO	–	–	100	80	60
	SFL	–	–	80	20	0
Mean solution			1/2	13.29	86.199	–
		10	1/5	0	0	0
			1/10	0	0	0
			1/2	4.5622	59.452	–
	IWO	20	1/5	2.0437	34.14	–
			1/10	0	0	–
			1/2	3.2214	43.652	–
		30	1/5	0.68206	19.303	–
			1/10	0.60547	10.655	–
	GAs (Evolver)	–	–	0.455	1.128	5.951
	MAs	–	–	0.014	0.068	0.552
	PSO	–	–	0.009	0.075	2.895
	SFL	–	–	0.058	2.252	6.469

allowable seeds for a weed has to be set equal to one or two in order to achieve the best performance.

4.4. *Comparison of IWO with SDS, SSA, and DSSA*

Simulated annealing (SA) is a generic probabilistic meta-algorithm for the global optimization problem, namely locating a good approximation to the global optimum of a given function in a large search space. It was independently invented by Kirkpatrick et al. (1983) and Cerny (1985). The name and inspiration come from annealing in metallurgy, a technique involving heating and controlled cooling of a material to increase the size of its crystals and reduce their defects. The heat causes the atoms to become unstuck from their initial positions (a local minimum of the internal energy) and wander randomly through states of higher energy; the slow cooling gives them more chances of finding configurations with lower internal energy than the initial one (Simulated annealing — Wikipedia). By analogy with this physical process, each step of the SA algorithm replaces the current solution by a random "nearby" solution, chosen with a probability that depends on the difference between the corresponding function values and on a global parameter T (called the temperature), that is gradually decreased during the process. The dependency is such that the current solution changes almost randomly when T is large, but increasingly "downhill" as T goes to zero. The allowance for "uphill" moves saves the method from becoming stuck at local minima—which are the bane of greedier methods.

Because of some similarities between the proposed algorithm and SA method, a comparison between IWO and SA based algorithms is addressed in this section. The comparison is based on the simulation results reported in (Hedar and Fukushima, 2002) where a combination of SA and a direct search (SDS) method is introduced for solving nonlinear unconstrained global optimization problems. In the mentioned research paper, the authors have first suggested a Simple Direct Search (SDS) method, which comes from some ideas of other well-known direct search methods. Then, the idea of SDS is hybridized with the standard SA to design a new method, called Simplex Simulated Annealing (SSA) method, which is expected to have some ability to look for a global minimum. To obtain faster convergence, the authors first accelerated the cooling schedule in SSA, and in the final stage, they applied Kelley's modification of the Nelder–Mead method on the best solutions found by the accelerated SSA method to improve the final results. They referred to the modified method as 'Direct Search Simulated Annealing' (DSSA) method.

Table 10 – IWO parameter values for Easom function optimization

Symbol	Quantity	Value
N_0	Number of initial population	5
it_{max}	Maximum number of iterations	200
dim	Problem dimension	2
p_{max}	Maximum number of plant population	10
s_{max}	Maximum number of seeds	2
s_{min}	Minimum number of seeds	0
n	Nonlinear modulation index	3
$\sigma_{initial}$	Initial value of standard deviation	7.5
σ_{final}	Final value of standard deviation	1e– 3
x_{ini}	Initial search area	$-10<x_{ini}<10$

Table 11 – Simulation results for Easom function optimization

Function	Rate of success				Average number of function evaluation				Average error			
	SDS	SSA	DSSA	IWO	SDS	SSA	DSSA	IWO	SDS	SSA	DSSA	IWO
Easom	12	68	93	100	–	4318	1442	1609	–	4e–3	3e–9	<1e–9
Griewank	36	82	90	100	–	12208	1830	1996	–	0.1	5e–9	<1e–9

Two examples are considered to compare the performance of IWO with SDS, SSA, and DSSA, which are Easom function and Griewank function. The performance of IWO, SDS, SSA and DSSA methods is evaluated based on (1) the percentage of successful trials, (2) the average number of function evaluation, and (3) the average error are related to only successful trials over 100 runs with different starting points. The SDS, SSA and DSSA algorithms are terminated when the function values at all the vertices become closer than 1e−6 in SDS, SSA, and 1e−8 in DSSA. Nevertheless, like earlier examples, assigning maximum number of iteration for IWO algorithm is used to terminate the search. More details on tuning of initial parameters and control parameters of SDS, SSA, and DSSA can be found elsewhere (Hedar and Fukushima, 2002).

4.4.1. *Easom function*

Easom function is a nonlinear function with a global minimum lying in a very narrow hole and outside this narrow hole the graph the function is almost flat. Easom function is defined as follows (Hedar and Fukushima, 2002):

$$\mathrm{ES}(x,y) = -\cos(x)\cos(y)\exp\left[-(x-\pi)^2-(y-\pi)^2\right] \tag{4}$$

The global minimum of the function is located in x, $y=\pi$, where the function value is equal to −1. Parameter values of IWO algorithm for optimization of Easom function is specified in Table 10. The parameter values are selected from experiences obtained from previous sections, for instance, the nonlinear modulation index is selected equal to 3, the maximum population of weeds in the colony is set equal to 10, and the maximum allowable number of seeds produced by a weed is set to 2 (see Table 10 for more details). Note that the final value of standard deviation mainly depends on the resolution requested for the final answer. In this example the resolution of 1e−2 is satisfactory. In addition, like earlier examples, assigning maximum number of iteration for the algorithm is used to terminate the search.

Table 12 – IWO Parameter values for optimization of Griewank function with the aim of comparison with SDS, SSA, AND DSSA

Symbol	Quantity	Value
N_0	Number of initial population	5
it_{max}	Maximum number of iterations	200
dim	Problem dimension	6
p_{max}	Maximum number of plant population	10
s_{max}	Maximum number of seeds	2
s_{min}	Minimum number of seeds	0
n	Nonlinear modulation index	3
$\sigma_{initial}$	Initial value of standard deviation	0.75
σ_{final}	Final value of standard deviation	1e− 4
x_{ini}	Initial search area	$-1<x_{ini}<1$

In Table 11 the final results obtained from simulations for optimization of Easom function using IWO are reported and is compared with the performance of SDS, SSA, and DSSA. From Table 11 it can be seen that the rate of success for IWO is much better than SDS and SSA; IWO outperforms DSSA as well. Note that for IWO when the fitness function is reached the value of 1e−9 or less, then the search is considered successful.

4.4.2. *Griewank function*

Griewank function with dimension of $d=6$ is also considered to compare the performance of IWO with SDS, SSA, and DSSA algorithms. Table 12 presents the initial parameter values of IWO. The results found from solving the test problem using the three mentioned algorithms vis-à-vis IWO are summarized in Table 11, which clearly shows that the behavior of IWO is the best of the four methods in all terms.

5. A practical example

An interesting application of optimization problems appears in dynamic and control systems theory. A system is considered an *optimum control system* when the system parameters are adjusted so that an index — a quantitative measure of the performance of the system — reaches an extreme value. For example, consider a dynamic model of a flexible structure is given by Eq. (5):

$$G(s) = \frac{(1+k\omega_n^2)s^2 + 2\zeta\omega_n s + \omega_n^2}{s^2(s^2+2\zeta\omega_n s+\omega_n^2)} \tag{5}$$

where ω_n is natural frequency of the flexible mode and ζ is the corresponding damping ratio. Generally, it is difficult to know the structural damping precisely, while the natural frequency can be predicted more accurately using well-established modeling techniques (Dorf and Bishop, 1995). Assuming the nominal values of $\omega_n=2$ rad/s, and $k=0.1$ it is desired to design a robust second order compensator to balance the uncertainty ζ which may vary between zero and 0.1. The controlled closed-

Table 13 – IWO Parameter values for controller optimization

Symbol	Quantity	Value
N_0	Number of initial population	5
it_{max}	Maximum number of iterations	150
dim	Problem dimension	5
p_{max}	Maximum number of plant population	10
s_{max}	Maximum number of seeds	2
s_{min}	Minimum number of seeds	0
n	Nonlinear modulation index	3
$\sigma_{initial}$	Initial value of standard deviation	0.5
σ_{final}	Final value of standard deviation	0.01
x_{ini}	Initial search area	$-0.1<x_{ini}<0.1$

loop system is supposed to follow the time response of a second order system specified in Eq. (6):

$$G_r(s) = \frac{(1+k\omega_n^2)s^2 + 2\zeta\omega_n s + \omega_n^2}{s^2(s^2 + 2\zeta\omega_n s + \omega_n^2)} \quad (6)$$

5.1. Controller optimization

A transfer function of a second order compensator is:

$$G_c(s) = \frac{K(s+z_1)(s+z_2)}{(s+p_1)(s+p_2)} \quad (7)$$

where $s=-z_i$ and $s=-p_i$ are called zeros and poles of the controller respectively, while K is called gain of the system. The design problem becomes, then, the selection of zi, pi and K in order to obtain a suitable closed-loop performance. Thus, the optimization problem is to minimize the difference between time responses of closed-loop system Eq. (8):

$$G_{CL}(s) = \frac{G_c(s)G(s)}{1+G_c(s)G(s)} \quad (8)$$

and the reference model $G_r(s)$ to a step command, which can be specified as follows:

$$J(K, z_1, z_2, p_1, p_2) = \int_{t_0}^{t_{max}} |y_{CL}(t) - y_r(t)| dt \quad (9)$$

where $y_{CL}(t)$ and $y_r(t)$ are time response of the closed-loop system and the reference model Eq. (5) respectively; $t_0=0$ and $t_{max}=200$ stand for starting time and final time in seconds. In this paper, IWO is employed to solve the abovementioned optimization problem.

Numerical values for parameters of IWO algorithm are specified in Table 13. Five trails are made to find best parameters of the controller, which is found to be:

$$G_c(s) = \frac{0.42723(s+0.5483)(s+0.0639)}{(s+3.847)(s+1.2)} \quad (10)$$

Note that although the first generation in the IWO algorithm was initialized in the area between −0.1 and 0.1, final values of the some of search parameters are outside this area, which demonstrates the ability of the proposed algorithm in searching and locating optimal solutions out of the initial search area.

Time responses of the closed-loop systems with different values for ζ, and the reference model to a step command are depicted in Fig. 8. As shown in Fig. 8, the closed-loop system is very robust to uncertainties in damping ratio, which makes the design very favorable.

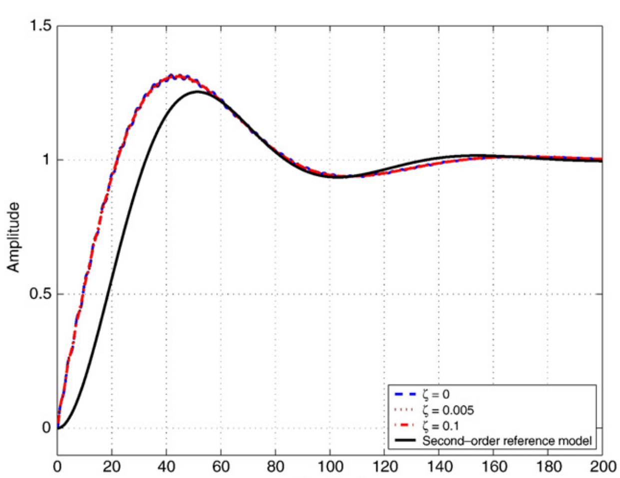


Fig. 8 – Step response of the closed-loop system with the designed controller versus the reference model's time response.

6. Conclusion

Invasive Weed Optimization (IWO) is a numerical stochastic search algorithm mimicking natural behavior of weed colonizing in opportunity spaces for function optimization. Adapting with their environments, invasive weed ride opportunity spaces left behind by improper tillage; followed by enduring occupation of the field. They reproduce rapidly by making seeds and raise their population. Their behavior changes with time as the colony become dense leaving lesser opportunity of life for the ones with lesser fitness.

Numerous numerical simulations are performed to demonstrate effectiveness of IWO algorithm. As the first step, convergence of IWO is studied for finding global minima of three benchmark functions, which are 'Sphere', 'Griewank', and 'Rastrigin' functions. It is shown that the proposed algorithm can outperform a standard genetic algorithm. Next, the effect of tuning parameters on performance of the proposed algorithm is studied. As the third step, the feasibility and efficiency of IWO for optimization of two examples are compared to four recent evolutionary algorithms — genetic algorithms (GAs), memetic algorithms (MAs), particle swarm optimization (PSO), and shuffled frog leaping (SFL). In order to give a better idea on performance of IWO, its merits for optimization of two functions — Easom function and Griewank function — is demonstrated and compared to simulated annealing based search algorithms — simplex simulated annealing and direct search simulated annealing. In the final step, the proposed algorithm is employed for solving an engineering problem that is optimal-robust tuning of a second order compensator for controlling a flexible structure with uncertain damping ratio.

It is shown in simulations that the proposed algorithm can capture properties of colonizing weeds fairly well and is capable in finding desired minima very fast in comparison with other stochastic search algorithms. As an optimization algorithm, it has the additional desirable properties of capability to deal with complex and non-differentiable objective functions and escapes from local optima. The experimental studies suggest that results from IWO are as good as (in some cases are better than) results from other methods. In conclusion, the performance of IWO is comparable with other evolutionary algorithms and IWO results are satisfactory for all test functions.

From reported simulations, it is observed that increasing the number of plant population in a colony decreases the mean solution, but does not essentially increase the percentage of success. A colony with population of 10 to 20 weeds has shown satisfactory performance. The maximum number of allowable seeds for plants also plays an important rule in the performance of the algorithm. It is shown, when the maximum and the minimum number of seeds are set to 2 and zero respectively, the behavior of colony is very satisfactory. In addition, a suitable value for the nonlinear modulation index, n, is found equal to 3 in simulations.

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REFERENCES

Baker, H.G., Stebbins, G.L. (Eds.), 1965. The Genetics of Colonizing Species. Academic Press, New York.

Cerny, V., 1985. A thermodynamical approach to the travelling salesman problem: an efficient simulation algorithm. Journal of Optimization Theory and Applications 45, 41–51.

Chatterjee, A., Siarry, P., 2006. Nonlinear inertia variation for dynamic adaptation in particle swarm optimization, Computers and Operations Research, vol. 33, pp. 859–871.

Dawkins, R., 1999. The extended phenotype: the long reach of the gene. Ch. 9 Selfish DNA. Oxford University Press, Oxford, pp. 156–164 (First edition, 1982).

Dekker, Jack, 2005. Course works of Agronomy 517: a course on the biology and evolutionary ecology of invasive weeds. http://www.agron.iastate.edu/~weeds/AG517/AG517HP.html.

Dorf, R.C., Bishop, R.H., 1995. Modern Control Systems, 7th edition. Addison-Wesley Pub. Co., Reading, Massachusetts, Menlo Park, CA, USA.

Dorigo, M., Maniezzo, V., Colorni, A., 1996. The ant system: optimization by a colony of cooperating agents. IEEE Transactions on Systems, Man and Cybernetics. Part B. Cybernetics 26 (1), 1–13.

Elbeltagi, E., Hegazy, T., Grierson, D., 2005. Comparison among five evolutionary-based optimization algorithms. Advanced Engineering Informatics 19, 43–53.

Eusuff, M.M., Lansey, K.E., 2003. Optimization of water distribution network design using the shuffled frog leaping algorithm. Journal of Water Resources planning and management 129 (3), 210–225.

Evolver, 1998. Evolver Version 4.0.2. Palisade Corporation.

Harper, J.L., 1977. Population biology of plants. Academic Press, NY, pp. 195–235. Ch. 6.

Hedar, A., Fukushima, M., 2002. Hybrid simulated annealing and direct search method for nonlinear unconstrained global optimization. Optimization Methods and Software 17, 891–912.

Holland, J.H., 1975. Adoption in Natural and Artificial Systems. University of Michigan, Ann Arbor.

Kennedy, J., Eberhart, R., 1995. Particle swarm optimization. Proceedings of the IEEE International Conference on Neural Networks, Piscataway, NJ, pp. 1942–1948.

Kirkpatrick, S., Gelatt, C.D., Vecchi, M.P., 1983. Optimization by simulated annealing. Science 220 (4598), 671–680.

Kirley, M., 2002. A cellular genetic algorithm with disturbances: optimisation using dynamic spatial interactions. Journal of Heuristics 8, 321–342.

Moscato, P., 1989. On evolution, search, optimization, genetic algorithms and martial arts: towards memetic algorithms. Technical Report Caltech Concurrent Computation Program, Report, vol. 826. California Institute of Technology, Pasadena, CA.

Pham, D.T., Karaboga, D., 2000. Intelligent Optimization Techniques. Springer.

Simulated annealing — Wikipedia, 2006. http://en.wikipedia.org/wiki/Simulated_annealing.

Yuchi, M., Kim, J.-H., 2005. Ecology-inspired evolutionary algorithm using feasibility-based grouping for constrained optimization. Proceedings of the IEEE Congress on Evolutionary Computation, Edinburgh, UK, pp. 1455–1461 (Sep.).