A Statistical Approach for Image Interpolation

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Abstract

Interpolation is an image processing operation for improve the resolution of a digital image. In this work, following the orthogonality principle, and under the assumption that the actual image is a locally stationary random process, we propose an alternative scheme for image interpolation. In our approach, the computational complexity is similar to the first-order spline algorithm. The algorithm was compared with classical B-spline methods and also with a statistical interpolator previously proposed in the literature. According to the normalized mean square error criteria, the proposed method produced accurate results.

1. Introduction

Interpolation is an important image processing operation which can be applied from resolution enhancement on desktop printers to algorithms for high resolution image reconstruction [1, 6]. A review of the interpolation problem can be find in references [1] and [4].

Interpolation procedures that are able to give accurate results are often necessary in most of applications. In addition to the quality of the resulted image, the effort required to compute the interpolated image is an important concern. At the lowest level of computational complexity, we have pixel replication, also known as nearest neighbor interpolation, which is widely used in many applications [1]. However, when continuity of derivatives is a concern, pixel replication cannot produce acceptable results. Then, one can linearly interpolate between adjacent samples. The extension of this idea to two-dimensional images is called bilinear interpolation. This strategy is an example of B-spline interpolation, which can be generalized to arbitrary order K. In this sense, a spline is a polynomial between each pair of given points, and the order of the interpolation is the number of points (minus one) used in an interpolation scheme. On the other hand, increasing the order does not necessarily increase the accuracy of the resulted image. It is well known that spline interpolation does not yield images that are sufficiently sharp. Moreover, increasing the order imply a higher computational complexity.

Hence, in the last years, there has been a great deal of interest in techniques for improving the quality of interpolated images whiling preserving edges. Most of these algorithms are based on some type of image models and a few of them explicitly estimate high-resolution edge information from the low-resolution image and use this information to control the interpolation [1].

In this work, following the orthogonality principle in a linear mean-square estimation framework, and under the assumption that the actual image is a stationary random process, we propose an alternative scheme for image interpolation. The algorithm is compared with classical B-spline methods and also with the statistical procedure proposed by Leung [3]. The Leung's work describes a Bayesian interpolator that models the sampling process and also consider the autocovariances of both the signal and the noise in the interpolation scheme. According to the normalized mean square error (NMSE) criteria, our approach is able to give superior results when compared with the B-spline procedures and also with the Leung's algorithm. Furthermore, the computational complexity of the proposed method is similar to the bilinear interpolation procedure.

2. The Proposed Method

First, consider a continuous one-dimensional signal s. Given $s(t)$ and $s(t+t_0)$, for $t, t_0 \in R$, and under the assumption that s is a stationary random process, we want to estimate $s(t^*)$, $t < t^* < t + t_0$, assuming that

$$s(t^*) = as(t) + bs(t+t_0), \tag{1}$$

for $a, b \in R$. Following the orthogonality principle [5], the error must be orthogonal to the data, that is,

$$\begin{aligned} E\{[s(t^*) - as(t) - bs(t+t_0)]s(t)\} &= 0 \\ E\{[s(t^*) - as(t) - bs(t+t_0)]s(t+t_0)\} &= 0. \end{aligned} \tag{2}$$

Thus, it can be shown that

$$\begin{aligned} R(t^*) &= aR(t) + bR(t+t_0) \\ R(t+t_0-t^*) &= aR(t+t_0) + bR(t), \end{aligned} \tag{3}$$

where $R(t)$ is the autocorrelation function of $s(t)$. Hence, if the actual autocorrelation of the signal is known, the values of a and b can be found by solving the linear system in equation (3). For the proposes of this work, we assume that

$$R(t) = \sigma^2 \eta^{|t|} + m^2, \tag{4}$$

where m and σ^2 are the mean and variance of the signal, and $0 < \eta < 1$ is the correlation coefficient. Now, consider a discrete low resolution image $f[x, y]$, with $0 \leq x < M$ and $0 \leq y < N$. In order to estimate a higher resolution image $g[u, v]$, with $0 \leq u < U$, $0 \leq v < V$, $U > M$, and $V > N$, we first interpolate the desired pixels on the rows (through the procedure described above), following by the columns interpolation, or vice-versa. We note that in order to estimate a new pixel in a row (or column), we only need two adjacent pixels on the same row (or column). In this sense, the computational complexity of the proposed method is similar to the first-order spline procedure.

3. Results

The algorithm was evaluated in a simulation framework. Figure 1(a) shows a piece of the image that was considered the actual image. It was decimated by 4 in both directions and also contaminated with additive Gaussian noise. Table 1 presents the NMSE values for the interpolation results considering different levels for the signal to noise ratio. Figure 1(b) to figure 1(f) present the visual results for the spline methods and also for the statistical algorithms.

	Noiseless	30 dB	45 dB	60 dB
1	14.7995	14.7880	14.7969	14.7991
2	14.9631	14.9572	14.9606	14.9626
3	14.8954	14.8835	14.8927	14.8949
4	14.9436	14.9361	14.9411	14.9431
5	252.4131	252.1938	252.3699	252.4053
6	4042.3933	4038.7631	4041.7031	4042.2690

Table 1. NMSE: (1) proposed algorithm; (2) Leung's algorithm; (3) bilinear interpolation; (4) pixel replication; (5) cubic convolution; (6) cubic spline.

In this simulation, we assume that $\sigma^2 = 1$, $\eta = 0.95$, and m is the mean value of adjacent pixels. We note that in our experiments, the results do not change significantly for values of η greater than 0.75. From the numerical results, we conclude that the proposed algorithm performs better when compared with B-spline interpolation methods and also with the Leung's algorithm. We also note that the computational complexity of the Leung's algorithm is higher than the method proposed in this work.

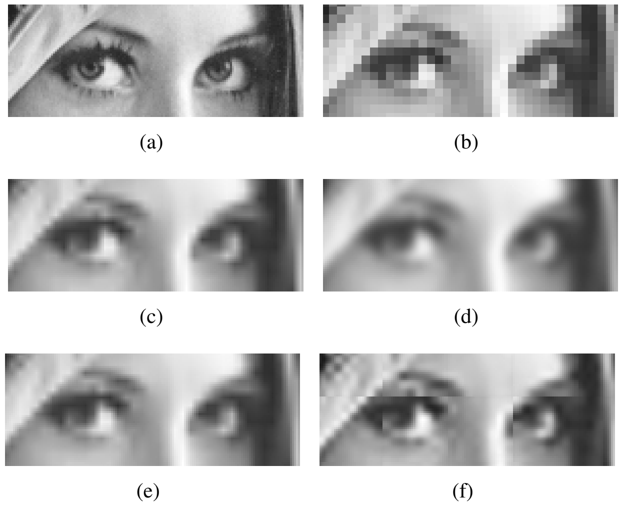

Figure 1. (a) original image; (b) pixel replication (zero-order); (c) bilinear interpolation (first-order); (d) cubic convolution (second-order); (e) proposed procedure; (f) Leung's algorithm.

4. Concluding Remarks

In this paper, we propose an efficient algorithm for image interpolation based on a statistical framework. In future works, we intent to compare our algorithm with the method described in reference [2]. This work was supported by FAPESP, Brazil, grant numbers 04/01632-1 and 2002/07153-2.

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