Estimation of Non-Homogeneous Potts-Strauss MRF Model Parameters on Higher-Order Neighborhood Systems by Maximum Pseudo-Likelihood

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Abstract

This paper addresses the problem of maximum pseudo-likelihood estimation of the non-homogeneous Potts-Strauss image model parameters using higher-order non-causal neighborhood systems in a computationally efficient way. The motivation is the development of a new methodology for contextual classification that uses combination of sub-optimal MRF algorithms for multispectral image classification, which requires accurate parameters estimation. The results show that the method is consistent with real image data and in the presence of random noise.

1. Introduction

In most MRF applications, model parameters are still chosen by trial and error [1]. The objective of this paper is to propose a method for parameter estimation of non-homogeneous isotropic Potts-Strauss MRF model through maximum pseudo-likelihood approach non restricted to first-order neighborhood systems, and also make multispectral contextual classification fully operational without human intervention.

2. An approach for parameter estimation using higher-order neighborhood systems

For a general neighborhood system, the local conditional density function (LCDF) for a pairwise interaction inhomogeneous Potts-Strauss model is:

$$p\left(x_{ij}=m \mid x_{n_{ij}},\beta_k\right)=\frac{\exp\left\{\beta_k U_{ij}(m)\right\}}{\sum_{l=1}^{M}\exp\left\{\beta_k U_{ij}(l)\right\}} \qquad (1)$$

where $U_{ij}(m)$ is the number of neighbors equal to the central element, β_k is the spatial dependency parameter of region/class k and M is the total number of regions/classes. In this work, we adopt Besag's maximum pseudo-likelihood approach (MPL). Taking the derivatives of the log-PL functions and setting the results to zero leads to:

$$\Psi(\beta_k)=\sum_{i,j\in S_k}U_{ij}\left(x_{ij}\right)-\sum_{i,j\in S_k}\left[\frac{\sum_{l=1}^{M}U_{ij}(l)\exp\left\{\beta_k U_{ij}(l)\right\}}{\sum_{l=1}^{M}\exp\left\{\beta_k U_{ij}(l)\right\}}\right]=0 \qquad (2)$$

We derive the proposed estimator by extending a method that expands the derivative of the log-PL function based on the number of occurrences of each possible configuration pattern along the image [2]. The proposed estimation method will be part of the MRF parameter estimation module from the contextual classification system illustrated in Figure 1.

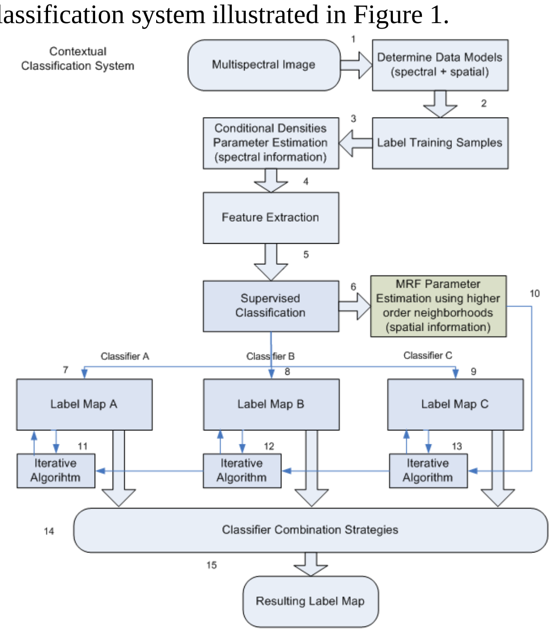

Figure 1. Contextual classification system using combination of sub-optimal MRF iterative algorithms

2.1. Mapping the possible configurations

In first-order neighborhood system, the enumeration of all possible interactions is straightforward, since there are only five different cases, as shows Figure 2, from all different labels to identical labels. These configurations can be represented by vectors (Equation 3), indicating the number of occurrences of each label around the central element, similar to a histogram. Note that in the Potts-Strauss model, location information is irrelevant (isotropic model). By denoting this vector representation, we can generate all possible configurations that define the interaction of a central pixel with its neighbors through the solutions of the following set of *N-2* equations:

$$\sum_{i=1}^{M} x_i = N, \quad M = 2,3,...,N-1 \tag{3}$$

with $0 < x_i < N$ and $x_i \in \{0,1,...,N\}$. The number of possible configurations (λ) grows exponentially as the number of neighbors increases (ie., 5, 22, 637,…).

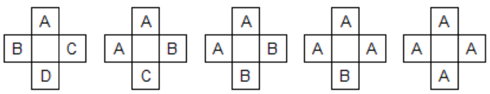

Figure 2. Possible interactions between a central pixel and its neighbors for a first-order neighborhood system.

$$\begin{aligned} &\vec{v}_0 = (1,1,1,1); \quad \vec{v}_1 = (2,1,1,0); \\ &\vec{v}_2 = (2,2,0,0); \quad \vec{v}_3 = (3,1,0,0); \\ &\vec{v}_4 = (4,0,0,0); \end{aligned} \tag{4}$$

2.2. Expanding the derivative log-PL function

Given the complete set of possible interactions, now it is possible to expand the second term of (2). We can regard the numerator as an inner product of two vectors $\vec{U}_{ij}$ and $\vec{w}_{ij}$, where $\vec{U}_{ij}$ represents the configuration vector of the current pixel and $\vec{w}_{ij}$ is the vector $w_{ij}[n] = \exp\{\beta_k U_{ij}[n]\}$. Similarly, the denominator is the inner product of $\vec{w}_{ij}$ with $\vec{r} = [1,1,...,1]$. Thus, the derivative of log-PL functions are expanded in a summation of λ terms, each one associated with a possible configuration. However, as it involves a sum for all image pixels, we define constants K_i, $i = 1,...,\lambda$ representing the number of occurrences of each possible neighborhood configuration pattern along the field. Thus, the solution can be obtained in an efficient way by finding the zero of the resultant equation through a numerical algorithm, preferably one that does not require the computation (or even the existence) of derivatives. In the experiments we implemented Brent's method [3], which uses a combination of bisection, secant, and inverse quadratic interpolation methods.

3. Experiments and results

In the experiments we considered a sample of a multispectral NMR fruit image, formed by T1, T2 and P_D bands, provided by Embrapa Agricultural Instrumentation. These images have been used on the development of a non-invasive fruit quality assessment system. We chose a mango transversal section P_D band image with dimensions of 256 x 256 pixels, 255 gray levels and 3 classes: fruit, seed and background. To test the method against different types of noise, we degraded the image with a signal dependent Poisson noise and a zero mean independent Gaussian noise. The image partition in regions was generated by a maximum likelihood classifier.

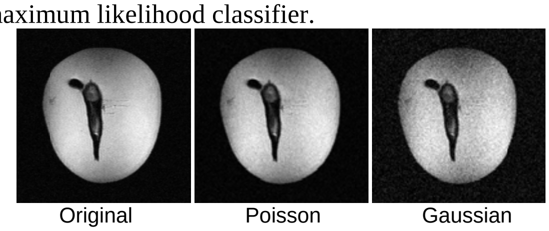

Figure 3. NMR P_D band mango images

Table 1. MPL parameter values for first, second and third-order neighborhood systems

	Original	Poisson	Gaussian
First-Order Neighborhood	$\hat{\beta}_F = 1.9289$	$\hat{\beta}_F = 1.4441$	$\hat{\beta}_F = 0.9723$
	$\hat{\beta}_S = 2.1480$	$\hat{\beta}_S = 1.5350$	$\hat{\beta}_S = 1.1890$
	$\hat{\beta}_B = 2.2045$	$\hat{\beta}_B = 2.1696$	$\hat{\beta}_B = 2.1661$
Second-Order Neighborhood	$\hat{\beta}_F = 1.5898$	$\hat{\beta}_F = 1.2711$	$\hat{\beta}_F = 0.9628$
	$\hat{\beta}_S = 1.6615$	$\hat{\beta}_S = 1.3399$	$\hat{\beta}_S = 1.1125$
	$\hat{\beta}_B = 1.6876$	$\hat{\beta}_B = 1.6784$	$\hat{\beta}_B = 1.3035$
Third-Order Neighborhood	$\hat{\beta}_F = 1.3958$	$\hat{\beta}_F = 1.1808$	$\hat{\beta}_F = 0.8992$
	$\hat{\beta}_S = 1.4490$	$\hat{\beta}_S = 1.2492$	$\hat{\beta}_S = 1.0237$
	$\hat{\beta}_B = 1.4104$	$\hat{\beta}_B = 1.4114$	$\hat{\beta}_B = 0.9273$

4. Conclusions

The proposed method allows the statistical modeling of less restrictive contextual systems in a large number of MRF applications. Furthermore, the numerical methods adopted are not based on intensive global optimization algorithms, leading to an efficient estimation procedure. Also, the final solution is robust to variations on the initial conditions.

5. Acknowledgements

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6. References

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