RECONSTRUÇÃO 3D - IA867 - 2s 2018 (Leo Pini) 1/8

• Livro texto : Cap 7.4 livro Trucco - Verri

7.4.1 Reconstrução por Triangulação

7.4.2 Reconstrução só dependente de fator de escala

7.4.3 Reconstrução dependente de transformação projetiva

The aim of this section is to show that you can compute a 3-D reconstruction even in the absence of *any* information on the intrinsic and extrinsic parameters. The price to pay is that *the reconstruction is unique only up to an unknown projective transformation of the world*. The Further Readings point you to methods for determining this transformation.

Assumptions and Problem Statement

Assuming that only n point correspondences are given, with $n \geq 8$ (and therefore the location of the epipoles, $\mathbf{e}$ and $\mathbf{e}'$), compute the location of the 3-D points from their projections, $\mathbf{p}_l$ and $\mathbf{p}_r$.

☞ It is worth noticing that, if no estimates of the intrinsic and extrinsic parameters are available and nonlinear deformations can be neglected, the accuracy of the reconstruction is only affected by that of the algorithms computing the disparities, not by calibration.

Algorithm UNCAL_STEREO

The input is formed by n pairs of corresponding points, $\mathbf{p}_i$ and $\mathbf{p}'_i$, with $i =, \ldots, n$ and $n \geq 5$, images of n points, $\mathbf{P}_1, \ldots, \mathbf{P}_n$. We assume that, of the first five $\mathbf{P}_i$ ($\mathbf{P}_1, \mathbf{P}_2, \ldots \mathbf{P}_5$), no three are collinear and no four are coplanar.

We assume to have estimated the location of the epipoles, $\mathbf{e}$ and $\mathbf{e}'$, using EPIPOLES_LOCATION. Let $\mathbf{P}_1, \ldots, \mathbf{P}_5$ be the standard projective basis of P^3. We assume the same notation used throughout the section.

1. Determine the planar projective transformations T and T' that map the $\mathbf{p}_i$ and $\mathbf{p}'_i$ ($i = 1, \ldots, 4$) into the standard projective basis of P^2 on each image plane. Apply T to the $\mathbf{p}_i$ and the epipole $\mathbf{e}$, and T' to the $\mathbf{p}'_i$ and the epipole $\mathbf{e}'$. Let (α, β, γ) and $(\alpha', \beta', \gamma')$ be the new coordinates of $\mathbf{p}_5$ and $\mathbf{p}'_5$.
2. Determine x and x' from (7.42) and (7.43).
3. Determine $\mathbf{O}$ and $\mathbf{O}'$ from (7.37) and (7.38).
4. Given a pair of corresponding points $\mathbf{p}$ and $\mathbf{p}'$, reconstruct the location of the point $\mathbf{P}$ in the standard projective basis of P^3 using (7.44) with λ and μ nontrivial solution of (7.45).

The output is formed by the coordinates of $\mathbf{P}_1, \ldots, \mathbf{P}_n$ in the standard projective basis.

PASSO 1 do ALGORITMO

We let $\mathbf{P}_1, \ldots, \mathbf{P}_n$ be the points in P^3 to be recovered from their left and right images, $\mathbf{p}_1, \ldots, \mathbf{p}_n$ and $\mathbf{p}'_i, \ldots, \mathbf{p}'_n$, and assume that, of the first five $\mathbf{P}_i$ ($\mathbf{P}_1, \mathbf{P}_2, \ldots \mathbf{P}_5$), no three are collinear and no four are coplanar.

We first show that, if we choose $\mathbf{P}_1, \mathbf{P}_2, \ldots, \mathbf{P}_5$ as the standard projective basis of P^3 (see Appendix, section A.4), *each projection matrix can be determined up to a projective factor that depends on the location of the epipoles*. Since a *spatial projective transformation* is fixed if the destiny of five points is known, we can, without losing generality, set up a projective transformation that sends $\mathbf{P}_1, \mathbf{P}_2, \ldots \mathbf{P}_5$ into the standard projective basis of P^3, $\mathbf{P}_1 = [1, 0, 0, 0]^\top$, $\mathbf{P}_2 = [0, 1, 0, 0]^\top$, $\mathbf{P}_3 = [0, 0, 1, 0]^\top$, $\mathbf{P}_4 = [0, 0, 0, 1]^\top$, and $\mathbf{P}_5 = [1, 1, 1, 1]^\top$.

For the corresponding image points $\mathbf{p}_i$ in the left camera, we can write

$$M\mathbf{P}_i = \rho_i \mathbf{p}_i, \tag{7.32}$$

where M is the projection matrix and $\rho_i \neq 0$. Similarly, since a *planar projective transformation* is fixed if the destiny of four points is known, we can also set up a projective transformation that sends the first four $\mathbf{p}_i$ into the standard projective basis of P^2, that is, $\mathbf{p}_1 = (1, 0, 0)^\top$, $\mathbf{p}_2 = (0, 1, 0)^\top$, $\mathbf{p}_3 = (0, 0, 1)^\top$, and $\mathbf{p}_4 = (1, 1, 1)^\top$.

In the following it

is assumed that the coordinates of the fifth point, $\mathbf{p}_5$, of the epipole $\mathbf{e}$, and of any other image point, $\mathbf{p}_i$, are obtained applying this transformation to their *old* coordinates.

The purpose of all this is to simplify the expression of the projection matrix: substituting $\mathbf{P}_1, \ldots, \mathbf{P}_4$ and $\mathbf{p}_1, \ldots, \mathbf{p}_4$ into (7.32), we see that the matrix M can be rewritten as

$$M = \begin{bmatrix} \rho_1 & 0 & 0 & \rho_4 \\ 0 & \rho_2 & 0 & \rho_4 \\ 0 & 0 & \rho_3 & \rho_4 \end{bmatrix}. \tag{7.33}$$

$$M = \begin{bmatrix} \rho_1 & 0 & 0 & \rho_4 \\ 0 & \rho_2 & 0 & \rho_4 \\ 0 & 0 & \rho_3 & \rho_4 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \rho_5 \, p_5$$

$$\begin{bmatrix} \rho_1 + \rho_4 \\ \rho_2 + \rho_4 \\ \rho_3 + \rho_4 \end{bmatrix} = \rho_5 \begin{bmatrix} p_5^x \\ p_5^y \\ p_5^z \end{bmatrix} = \rho_5 \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}$$

$$\rho_1 + \rho_4 = \rho_5 \alpha \Rightarrow \rho_1 = \alpha \rho_5 - \rho_4$$

$$\rho_2 + \rho_4 = \rho_5 \beta \Rightarrow \rho_2 = \beta \rho_5 - \rho_4$$

$$\rho_3 + \rho_4 = \rho_5 \gamma \Rightarrow \rho_3 = \gamma \rho_5 - \rho_4$$

$$\Rightarrow M = \begin{bmatrix} \alpha\rho_5 - \rho_4 & 0 & 0 & \rho_4 \\ 0 & \beta\rho_5 - \rho_4 & 0 & \rho_4 \\ 0 & 0 & \gamma\rho_5 - \rho_4 & \rho_4 \end{bmatrix}$$

$$\div \rho_4 \Rightarrow M = \begin{bmatrix} \alpha x - 1 & 0 & 0 & 1 \\ 0 & \beta x - 1 & 0 & 1 \\ 0 & 0 & \gamma x - 1 & 1 \end{bmatrix} \quad \text{c/ } x = \frac{\rho_5}{\rho_4}$$

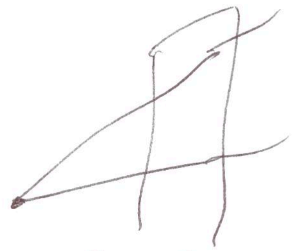
<u>atenção</u>: α, β, γ são conhecidos, porque podem ser obtidos aplicando a mesma transformação aplicada em P_i e p_i $(i = 1, 4)$

$\Rightarrow M$ é conhecida a menos do fator $\underline{x}$ } câmera da esquerda

Passo 2 do Algoritmo

Obtenção de $\underline{\underline{x}} = \rho_5 / \rho_4$

- relacionar M e O (centro de projeção)
  - M modela uma projeção de perspectiva a partir de O



- assim $MO = 0$ (o centro de projeção não é projetado)

$$\begin{bmatrix} \alpha x - 1 & 0 & 0 & 1 \\ 0 & \beta x - 1 & 0 & 1 \\ 0 & 0 & \gamma x - 1 & 1 \end{bmatrix} \begin{bmatrix} O_x \\ O_y \\ O_z \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$(\alpha x - 1) O_x + 1 = 0 \Rightarrow O_x = \frac{-1}{\alpha x - 1} \Rightarrow O_x = \frac{1}{1 - \alpha_x}$$

$$\Rightarrow O = \left[\frac{1}{1-\alpha_x}, \frac{1}{1-\beta x}, \frac{1}{1-\gamma x}, 1 \right]$$

De maneira análoga obtemos:

$$M' = \begin{bmatrix} \alpha' x' - 1 & 0 & 0 & 1 \\ 0 & \beta' x' - 1 & 0 & 1 \\ 0 & 0 & \gamma' x' - 1 & 1 \end{bmatrix}$$

$$O' = \left[\frac{1}{1-\alpha' x'}, \frac{1}{1-\beta' x'}, \frac{1}{\gamma' x' - 1}, 1 \right]$$

Lembremos:

- Assim como obtivemos p_5 e p'_5 com as mesmas transformações de $P_i \to p_i$ (1 a 4), obtemos as posições dos epipolos.

- Então:

μ e μ'

$$e \begin{cases} M O' = \sigma \bar{e} \\ M' O = \sigma' \bar{e}' \end{cases} \quad \text{com } \sigma \neq e \ \sigma' \neq 0$$

Consideremos

$$\left.\begin{aligned} & MO' = \sigma \bar{e} \\ & M = \begin{bmatrix} & & \\ & & \end{bmatrix} \\ & O' = \begin{bmatrix} & & \end{bmatrix} \end{aligned}\right\} \Rightarrow \begin{bmatrix} \alpha x - 1 & 0 & 0 & 1 \\ 0 & \beta x - 1 & 0 & 1 \\ 0 & 0 & \gamma x - 1 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{1-\alpha' x'} \\ \frac{1}{1-\beta' x'} \\ \frac{1}{1-\gamma' x'} \\ 1 \end{bmatrix} = \sigma \bar{e}$$

$$\begin{bmatrix} \frac{\alpha x - 1}{1-\alpha' x'} + 1 \\ \frac{\beta x - 1}{1-\beta' x'} + 1 \\ \frac{\gamma x - 1}{1-\gamma' x'} + 1 \end{bmatrix} = \begin{bmatrix} \sigma e_x \\ \sigma e_y \\ \sigma e_z \end{bmatrix}$$

$$\frac{\alpha x \cancel{-1+1} - \alpha' x'}{} = \frac{\alpha x - \alpha' x'}{1-\alpha' x'} = \Gamma e_x$$

$$\frac{\beta x - 1 + 1 - \beta' x'}{} = \frac{\beta x - \beta' x'}{1-\beta' x'} = \Gamma e_y$$

$$= \frac{\gamma x - \gamma' x'}{1-\gamma' x'} = \Gamma e_z$$

ou ainda

$$\boxed{\alpha x - \alpha' x' + \alpha' e_x \Gamma x' = \Gamma e_x} \qquad \alpha x - \alpha' x' = \Gamma e_x - \Gamma e_x \alpha' x'$$

$$\boxed{\alpha x - \alpha' x' + \Gamma e_x \alpha' x' = \Gamma e_x}$$

assim 7.41 OK

$$\begin{bmatrix} \alpha & -\alpha' & \alpha' e_x \\ \beta & -\beta' & \beta' e_y \\ \gamma & -\gamma' & \gamma' e_z \end{bmatrix} \begin{bmatrix} x \\ x' \\ \Gamma x' \end{bmatrix} = \begin{pmatrix} \Gamma e_x \\ \Gamma e_y \\ \Gamma e_z \end{pmatrix}$$

Γ é desconhecida

$\Rightarrow$ desconhecemos x, x', Γ $\Rightarrow$ vamos considerar $\underline{x, x', \Gamma x'}$ $\Downarrow$ Linear

$\Rightarrow$ Resolvendo este sistema de eq p/ $\Gamma x'$

temos: $\Gamma x' = \Gamma \dfrac{e^T \cdot (\vec{p}_5 \times \vec{p}_5')}{v^T \cdot (\vec{p}_5 \times \vec{p}_5')}$; $\vec{v} = (\alpha' e_x, \beta' e_y, \gamma' e_z)$

$\vec{e}, \vec{p}_5, \vec{p}_5'$ conhecidos

(Eq. 7.42)

Lembrando que

Prod. escalar:

$$\vec{a} = a_1 \vec{i} + a_2 \vec{j} + a_3 \vec{k}$$

$$\vec{b} = b_1 \vec{i} + b_2 \vec{j} + b_3 \vec{k}$$

$$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 \quad \text{(escalar!)}$$

Prod. vetorial:

$$\vec{a} \times \vec{b} = (a_2 b_3)\vec{i} + (a_3 b_1)\vec{j} + (a_1 b_2)\vec{k} -$$

$$- (a_3 b_2)\vec{i} - (a_1 b_3)\vec{j} - (a_2 b_1)\vec{k} -$$

com $i \times j = k$; $j \times k = i$; $k \times i = j$; $j \times i = -k$;

$k \times j = -i$; $i \times k = -j$

cancelamos ∇ (ambos lados da eq 7.42):

$\Rightarrow$ $\boxed{x' = }$

idem para $\boxed{x = }$

Passo 3 do Algoritmo

Com x' e x $\Rightarrow$ $\boxed{O = \quad}$

$\boxed{O' = \quad}$

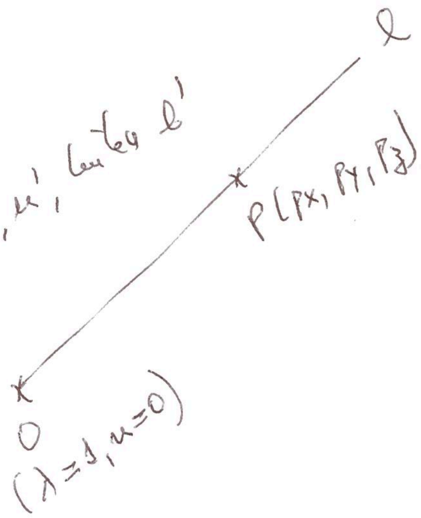
Passo 4 do Algoritmo

Dado P e P' quaisquer P é obtido a partir da eq. paramétrica da reta:

$$\text{linha-}l = \lambda \bar{O} + \mu \left[O_x P_x, O_y P_y, O_z P_z, 0\right]^T$$ ← em p/ $\lambda = 1$, $\mu = 0$

$$M \cdot \begin{pmatrix} O_x P_x \\ O_y P_y \\ O_z P_z \\ 0 \end{pmatrix} = \begin{pmatrix} P_x \\ P_y \\ P_z \end{pmatrix}$$

valores λ', μ', linha l'



$$\text{linha-}l' = \lambda' \bar{O}' + \mu' \left[O'_x P'_x, O'_y P'_y, O'_z P'_z, 0\right]^T$$

$\Rightarrow$ P está na interseção de l' e l

$\Rightarrow$ Resolver a eq. $\begin{bmatrix} & & & \end{bmatrix} \begin{bmatrix} \lambda \\ \mu \\ \lambda' \\ \mu' \end{bmatrix} = 0$ (*) $\Rightarrow$ obtemos $\boxed{\begin{matrix} \lambda = \\ \mu = \\ \lambda' = \\ \mu' = \end{matrix}}$

equação

$$\lambda O_x + \mu O_x P_x - O'_x \lambda' - O'_x P'_x \mu' = 0$$

ou seja $\lambda O_x + \mu O_x P_x = \lambda' O'_x + \mu' O'_x P'_x$ (encontro em x das 2 retas)

Voltando à (*) c/ $\lambda, \mu, \lambda', \mu'$ $\Rightarrow$ $\left.\begin{matrix} P_x \\ P_y \\ P_z \end{matrix}\right\}$ da eq. da linha l ou l'

$P(x,y,z) = \lambda \bar{O} + \mu [\quad]$