Towards Machine Understanding: Some Considerations Regarding Mathematical Semiosis

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Abstract: *The discussion on the possibility of machines to achieve comprehension, understanding and true meaning grounded in the real world is a very controversial debate within Artificial Intelligence and Cognitive Science. One of the biggest problems is the requirement to involve "reality" in this discussion, bringing forth a lot of unsolved questions regarding the nature of what would be such thing we use to call "reality". In this work, we present an attempt of escaping this problem, by re-defining the meaning process (semiosis, according to Peirce), in an entirely mathematical framework. We are calling this "transposition" of the Peircean theory to a purely abstract mathematical model as "Mathematical Semiosis". By doing this, we aim at growing a more understandable theory for explaining what is to comprehend, to understand and to mean, in a strictly mathematical sense, avoiding complications related to the connection of signs to a real world. The main application of such a theory would be in order to develop machines with these capabilities. In such a regard, what we are calling here "Mathematical Semiosis" would be a kind of purely mathematical abstraction for what is "Semiosis" in the real world.*

1. Introduction

The creation of agents and/or (multi) agent systems able to fully understand and comprehend its environment and communicate with humans, answering questions and giving information about it is one of the big dreams of artificial intelligence. This problem proved to be much more complicated than could be imagined at once, and to the extension this challenge became more acknowledgeable, gradually this dream gave rise to more modest claims, paving the road for what is currently the technology of Intelligent Systems. One of the problems related to achieving the aforementioned dream is the lack of good models (or too many inadequate models) for what is the meaning of terms like "meaning", "understanding" and "comprehension". Current models of such terms usually misconsider the relation between representations and reality. Some recent efforts, based on semiotic considerations, are trying to address this issue in a more proper manner. But the requirement to involve "reality" in this discussion, brings forth a lot of unsolved questions regarding the nature of what would be such thing we call "reality". We have to ourselves that a possible solution to this problem would be to consider the model of meaning given by the theory of semiotics given by the philosopher Charles Sanders Peirce. Since 1997, we have been working with Peirce's theory of sign, trying to bring his model into the theory of intelligent systems. But to work with the theory of Peirce is not an easy work. Why ? First, because Peirce himself didn't left an organized account of his workings. Peirce left a legacy of thousands of manuscripts, which are still being organized by experts in the Peirce Edition Project. Most of what is known is due to the work of interpreters and commentators of Peirce. So, his work, despite being very relevant for our purposes, sometimes is very hard to be correctly understood. Secondly, due to a very different philosophical perspective, in which Peirce situate his work, conceiving a reality that is not just deterministic as in standard physicalism, but a blend of random, deterministic and teleological sub-components. The issue of reality is central when we are defining "meaning", because one of the main failures in AI was a bad approach whenever connecting representations to reality. The notion of "reality" in contemporary philosophy is a very controversial one, with many different positions (sometimes opposed one another). To dig into this controversy may not be fruitful for our purposes here. So, the main motivation for this work, is to escape this problem by redefining the Peircean concept of "meaning process" (Semiosis, according to Peirce), in an entirely mathematical framework, creating what we are calling here a mere "Mathematical Semiosis". We may understand this as a kind of "transposition" of the Peircean model of semiosis to a purely abstract mathematical model, that we aim to be more interesting for the engineering point of view. Our goal is to bring a more understandable theory for explaining what is to 'signify', to understand and to mean, in a strictly mathematical sense. It is mainly a theoretical work, but with possible great impact in the practical construction of artificial systems able to fully "understand" its surrounding environment.

2. Meaning according to Peirce

We will start our argumentation by presenting what is meaning according to Peirce. Peirce identify meaning as a special kind of process which he calls "semiosis". The definition of "semiosis" appears in many different parts of his work. For example:

"... by 'semiosis' I mean [...] an action, or influence, which is, or involves, a cooperation of three subjects, such as a sign, its object, and its interpretant, this tri-relative influence not being in any way resolvable into actions between pairs".

Peirce conceives a 'Sign' or 'Representamen' as a 'First' which stands in such a genuine triadic relation to a

'Second', called its 'Object', so as to be capable of 'determining a Third', called its 'Interpretant', to assume the same triadic relation to its Object in which it stands itself to the same Object. To cite him:

"My definition of a sign is: A Sign is a Cognizable that, on the one hand, is so determined (i.e., specialized, bestimmt) by something other than itself, called its Object, while, on the other hand, it so determines some actual or potential Mind, the determination whereof I term the Interpretant created by the Sign, that that Interpreting Mind is therein determined mediately by the Object" (CP 8.177. Emphasis in the original).

Another important concept is the notion of Peirce's logical-phenomenological categories. For Peirce, reality can be decomposed into components that should fit into just three different categories, which he calls firstness, secondness and thirdness.

The category of firstness comprises what is such as it is, without reference to anything else. Firstness is the category of mere potentiality, freedom, immediacy, undifferentiated quality, randomness, independence, novelty, creativity and originality.

The category of secondness comprises what is such as it is, in relation with something else. Secondness is the category of action and reaction, opposition, polarity, differentiation, existence.

And, the category of thirdness comprises what is such as it is or becomes, insofar as it is capable of bringing a second entity into relation with a first. Thirdness is the category of mediation, law, habit, semiosis, representation.

In this sense, according to Peirce, reality could be segmented into components that should fit one of these three categories.

3. Related Approaches and Background

This is not the first time someone tries to define semiosis in terms of a mathematical model. Other approaches were conducted e.g. by Robert Marty (Université de Perpignan – FRANCE) [1-4], by Joseph Goguen (University of California at San Diego – USA)[5-7], by Robert Burch (Texas A&M University- USA) [8,9] and also by ourselves [10,11], but with different purposes.

Our mathematical model of semiosis is actually based on some key notions:

- Peircean Semiotics
- Uexkull's notion of Umwelt
- Cellular Automata
- Rosen's Anticipatory Systems Theory

3.1 Umwelt

According to Uexkull, the concept of Umwelt may be defined as the phenomenal aspect of the parts of the environment of a subject (an animal/organism), that is, the parts that it selects with its species-specific sense organs according to its organization and its biological needs [12-14]. Or, in other words, the part of reality which is cognizable by an agent. Or, to put it simple, what we may call a sensorial reality.

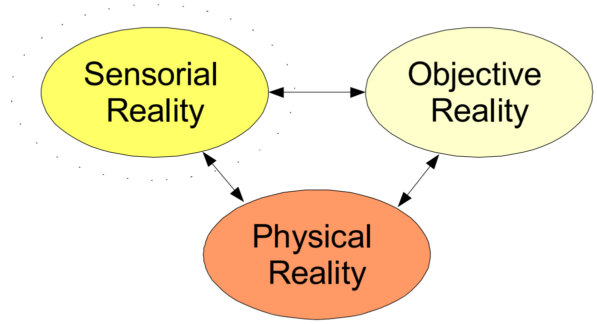


Figure 1 – The Structure of Reality

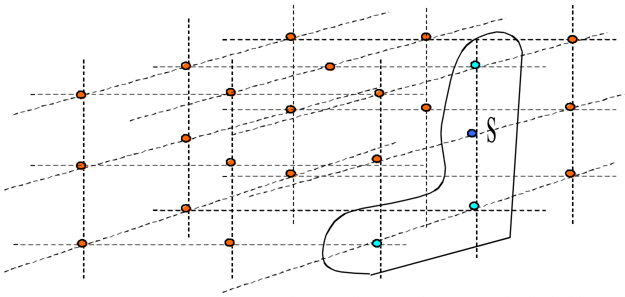
Even though Uexkull's definition of Umwelt is directed to biological organisms, we may use his concepts and terminology applied to artificial devices like a robot, as discussed by Emmeche [15].

The notion of a sensorial reality is very important here. It assumes that what is cognizable by our senses is not the full-complete reality, but just a part of it. There may be parts of reality that are not detectable by our senses. So, we may have to create a distinction between a "physical" reality and a "sensorial" reality. Another related concept that must be discussed regarding this point is the notion of an objective reality, or a reality made of objects. This is the reality we refer most of the times while we use language. We see a world of "things", and our language is adapted to convey information about these "things", even though some times these things didn't really exist, like e.g. seeing horses or faces in clouds in sky. We assume that this objective reality is just our mind representation for the sensorial reality, and so is distinct from it and from the physical reality either. As representations must refer to some reality, we need to make it clear to which reality we are referring to. This makes the problem of meaning either more difficult.

In order to escape from this problem, we will be defining a "mathematical reality" and using it as the ground were mathematical semiosis will take place. But in order to be useful in the future, we will make this mathematical reality up to some sense similar to one kind of reality. As we can not be sure on the extension of physical reality, and we know that the objective reality is just a creation of our mind, we chose to make our mathematical reality similar to the sensorial reality (Umwelt).

3.2 Cellular Automata

Cellular Automata [16] comprises a discrete model studied in computability theory, mathematics, and theoretical biology, consisting of a (potentially) infinite, regular grid of *cells*, each one in a finite number of states. The grid may have any finite number of dimensions. Time is also discrete, and the state of a cell at time *t* is a function of the states of a finite number of cells (called its *neighborhood*) at time *t-1*. These neighbors are a selection of cells relative to the specified cell (which may include the own cell), and usually do not change. Every cell has the same rule for updating,



$$S(t+1)=f(neigh(S)(t))$$

Figure 2 – A 3D Cellular Automaton

based on the values of states in this neighborhood. Each time the rules are applied to the whole grid a new *generation* is created. An example of a three-dimensional cellular automaton is given in figure 2. In figure 2, we have a three-dimensional grid of states, and the values of these states are correlated in time. In the figure, the value of state S in instant t+1 is a function f of the values of states in its neighborhood neigh(S) in an instant t.

The concept of cellular automata is useful to us because we will use it in order to define our mathematical reality. Our mathematical reality will be defined as a kind of cellular automata. But in order to fully define it, we will need first another important concept which is the concept of an anticipatory system.

3.3 Anticipatory Systems

An anticipatory system is a system whose current state is determined by a future state, or, according to Robert Rosen [17]:

"A system containing a predictive model of itself and/or its environment, which allows it to change state at an instant in accordance with the model's predictions pertaining to a latter instant."

These predictions can be goals, plans or simply estimations of future states.

Acording to Mihai Nadin [18,19]:

"Anticipation is a recursive process described through the functioning of a mechanism whose past, present, and future states allow it to evolve from an initial to a final state that is implicitly embedded in the mechanism".

Anticipatory systems are very different from the standard kind of systems we are used to find in engineering and systems sciences, and have many interesting properties that make them more than pure mechanical deterministic systems. Rosen argues that their behavior is what make living systems different from non-living systems. Living systems would be anticipatory systems. Anticipatory systems may provide also the kind of teleological behavior that is particularly related to the property we use to call "intelligence" in human beings. In our point of view, this teleological behavior situate anticipatory systems as a natural candidate to instantiate the Peircean notion of "thirdness".

Peirce argues that all that can be known must fit into three different categories: firstnesses, secondnesses and thirdnesses. This is equivalent to say that if reality can be decomposed, all the components should be classified as firstnesses, secondnesses and thirdnesses. This is in the kernel of Peircean philosophy. But how to understand Peirce's claims if we assume very simple systems as our reality ?

A simple system like

$$S(t+1)=Random() \quad (1)$$

will be a system of pure firstness. Supposing that $t+1$ is equivalent to the present time, we have a system where the present is completely random.

A system like

$$S(t+1)=f(S(t)) \quad (2)$$

will be a purely deterministic, or mechanical system. This is a system where there is only secondness. It is a system where the present is completely determined by the past.

A system like

$$S(t+1)=f(S(t))+Random() \quad (3)$$

will be a system where there is firstness and secondness.

But what will be a system with thirdness ?

A candidate for a system with thirdness will be something like:

$$S(t+1)=f(S(t+\tau)) \quad (4)$$

or, in other words, a system where the present depends on the future. But, if time evolves from the past to present, this seems to be impossible. How can this possible ? To understand that, we need to make a change in the equation:

$$S(t+1)=f(E(S(t+\tau))(t)) \quad (5)$$

In this case, $E(.)$ is the estimation in time *t* of a future state $S(t+\tau)$. This system is perfectly feasible. But, $f(E(S(t+\tau))(t))$ can be rewritten to $f_2(S(t))$, and then it simply reduces again to a deterministic system. So, this is clearly not the answer. How to still have a true anticipatory system ? The solution is to provide an open system instead of a closed one:

$$S(t)=f(S(t))+g(U(t)) \quad (6)$$

where $U(t)=S(t+\tau)$ is an external input.

But, if $S(t)$ is our reality, then to assume the existence of $U(t)$ is equivalent to accept a dualist position (in cognitive science). Or, if we accept that the $S(t)$ is just our sensorial reality, and that there is something else in physical reality that is not in sensorial reality, we may argue that this something else is responsible for the anticipatory component of the system.

With this brief introductory background, we are now prepared to our definitions.

4. A Mathematical Model of Semiosis

The main contribution to this work is to propose a general framework for the discussion of means for applying Peirce's theory of semiotics in the construction of intelligent systems. The reader should be warned that it is not our intention here to propose that these models are scientific models of reality (i.e. models that fully explain this strange thing we call reality), mainly because we are exactly trying to escape this problem. Our purpose here is to create a model of reality that clearly envision a technological destination. What we define in this session is a general framework (a mathematical one) which we propose to be an instance of the Peircean model, and which may allow a better comprehension of the Peircean model of representation in purely mathematical sense. Nevertheless, our intention with this is to allow a better appreciation by the engineering community of the potentialities of the Peircean model of representation in order to build agents able to perform understanding and comprehension.

In our general framework, we define a Mathematical Universe U which is composed of a Mathematical Reality R and a set of agents $A = \{A_i\}$.

$$U = (R,A) \tag{7}$$

The Mathematical Reality R is defined as

$$R: P \times T \to V \tag{8}$$

where R is a function which returns a value $v \in V$ to each place $p \in P$ (p is a coordinate in an n-dimensional grid) and time $t \in T$.

The definition of R is quite open, and serves our purpose of generality for the framework. To allow us a better understanding of a potential use of R, let us compare it to a cellular automaton. In order for the function R to be a cellular automaton, this function should be written as:

$$R(p,t+1) = f(R(q_1,t), \dots, R(q_m,t)) \tag{9}$$

where

$q_i \in Neigh(p)$ and $Neigh(p)$ is a set of places which comprises the neighborhood of place p. But this will make R a purely deterministic system. In our case, we want our mathematical reality to be composed of firstness, secondness and thirdness components, and so our definition for R will be a little bit different. In our case our mathematical reality will be defined as:

$$R(p,t+1) = R_1 + R_2 + R_3 \tag{10}$$

where

$$R_1 = Random()$$

$$R_2 = f(R(q_1,t), \dots, R(q_m,t)), q_i \in Neigh(p)$$

$$R_3 = \sum_i Act(A_i, p, t)$$

$Act(A_i, p, t)$ is the contribution of Agent A_i to the state on place p in time t. This component is equivalent to the one in equation (6). Component R_1 is a component of firstness. Component R_2 is a component of secondness and component R_3 is a component of thirdness. With this, our mathematical reality fully instantiates the Peircean notion of reality. The defined mathematical reality is illustrated in figure 3.

The other component of our mathematical universe U is a set of Agents A. Each agent A_i in A is able to sense and actuate on mathematical reality R (see figure 3). This means that it will be able to contribute to the new values of states within reality R, according to equation (10). This contribution may be a function of some states in reality R. But each agent is limited to just a limited subset of places in P, for sensing, and another subset of P for actuation. These subsets may change in time. Let us define then the function Π :

$$\Pi : A \times T \to 2^P \tag{11}$$

which we call the Perceptive Scope function, which defines for each A_i and time t a set of places which can be measured by the agent. As a complement, let us define the function Γ :

$$\Gamma : A \times T \to 2^P \tag{12}$$

which we call the Actuative Scope function, which defines for each A_i and time t a set of places which can be actuated over by the agent.

Let us now make some simplifications in order to give some ground for the next development. Let us imagine that P is given by:

$$P = \overset{n}{\times} \mathbb{Z} \tag{13}$$

where $\mathbb{Z}$ is the set of integer numbers and P is the cartesian product of it n times. In this case, each $p = (p_1, \dots, p_n)$ will be a place in an n-dimensional grid.

In this case, we will define a region G to be any subset of P:

$$G \subset P \tag{14}$$

We will then define an *Attention Window* G^S, to be a region generated by a given place and a set of rules for including other places into the region:

$$G^S = S(p) \tag{15}$$

where $S: P \to 2^P$ and S is a function which for a given place

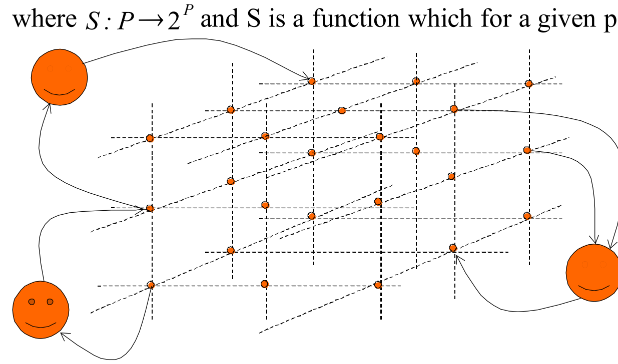


Figure 3 – Mathematical Reality Actuated by Agents

p, determines a set of other places relative to p, which may be a part of the region which is said to be located in place p. Usually S can be given by a script which given p, generates the other places member of the region G^S.

We may now restrict our framework, such that every Perceptive Scope and Actuative Scope of every agent in our mathematical universe are Attention Windows. This means that the sources and sinks of information processed by agents will always have the same structure all around the mathematical reality.

Now, let us make some generalizations. Even though R is a function of single places p in time t, we will use the same notation to denote a function of a region G in time t. So, let us understand $R(G,t)$ as the set $\{R(p,t)\}, \forall p \in G$.

We will then call the tuple

$$\sigma(p,t) = \left(G^S(p), R(G^S(p),t) \right) \quad (16)$$

as being a *Signal* located in place p.

Given these premises, we may now proceed to define a *Mathematical Semiosis*. Mathematical Semiosis is a process where a signal $\sigma_s(p_s, t_s)$ is said to represent another signal $\sigma_o(p_o, t_o)$. The process consolidates when $\sigma_s(p_s, t_s)$is used to generate another signal, $\sigma_i(p_i, t_i)$, for $t_i > t_s \geq t_o$. In this case, $\sigma_s(p_s, t_s)$is said to be a *Sign* of $\sigma_o(p_o, t_o)$, which is called to be its *Object*, and $\sigma_i(p_i, t_i)$ is said to be the *Interpretant* of the sign. But this is not enough that $\sigma_i(p_i, t_i)$ to be generated by $\sigma_s(p_s, t_s)$. In order for this to be a semiosis, there is a further condition. And this condition is that $\sigma_i(p_i, t_i)$should maintain the relation that $\sigma_s(p_s, t_s)$had to $\sigma_o(p_o, t_o)$. In other words, it should be able to generate a further interpretant, that should also maintain this relation to the object. This is the way we guarantee that $\sigma_s(p_s, t_s)$ really "represents" $\sigma_o(p_o, t_o)$.

This process is basically performed by the agents in the mathematical universe, and do have its realization within its mathematical reality, in possible different places and times.

So, Mathematical Semiosis is a process by which an agent reads a signal from a Mathematical Reality, and generates output to this same Mathematical Reality in a future time. The effect produced by the Agent is the Interpretant of the sign.

The most simple kind of semiosis is the copy. In this case, the signal $\sigma_s(p_s, t_s)$ is an exact copy of $\sigma_o(p_o, t_o)$, for a possible different place and time. In being a copy, we may assure that it is always possible to generate another copy $\sigma_i(p_i, t_i)$in a different place and time in the future. So, an exact copy is the most simple representation of something. But there may be more sophisticated kinds of representation (or semiosis). This copy should not be an exact copy, but just share a partial set of attributes. Both the copy and a partial copy will be called "*icons*", according to Peirce. But there may be the case that $\sigma_s(p_s, t_s)$ and $\sigma_o(p_o, t_o)$do not share any kind of attribute in common. But even in this case, they may give rise to a process of semiosis. How ? By using the mathematical reality in which $\sigma_s(p_s, t_s)$ is realized in order to obtain either $\sigma_o(p_o, t_o)$ or a copy of it. In this case, proximity in space (place) and time could be used to create the interpretant. This kind of signs are called *indexes*, according to Peirce. But there may be a more radical kind of sign, where there may be no sharing of attributes nor a space-time connection between sign and object. This kind of sign is called a *symbol* by Peirce. To understand how symbols are possible is a very challenging exercise in the interpretation of Peirce's work. A symbol, according to Peirce, is a totally arbitrary connection between a sign and an object. But, if it is totally arbitrary, how can an agent generate an interpretant, that is still related to this same object ? We don't have a final answer to this challenge, in a strict Peircean view. But we have some hypothesis on how to solve this puzzle, that we will show in the sequence. A possible way of solving this, is to allow the agents in the mathematical universe to have *inner mathematical realities*. An inner mathematical reality is just like a mathematical reality in the mathematical universe, but instead of being a shared space and time, (i.e. Places where all the agents are able to perceive and actuate) are internal, private instances where only the own agents are able to perceive and act. So, if we consider this inner mathematical reality, in addition to the standard mathematical reality, we may have a clue on how symbols are possible. When Peirce says that a symbol is totally arbitrary, i.e., that there may be no sharing of attributes and no physical connection (in space and time) between the sign and the object, he may be considering this statement related only to the standard reality, the one which in our case is shared by all the agents – the mathematical reality of our mathematical universe. So, considering just this mathematical reality, there is no sharing of attributes neither a physical connection in terms of space and time. But, there may be either a sharing of attributes or a physical connection in terms of space and time, considering this inner mathematical reality that each agent should possess. This may solve the puzzle and may allow for the construction of symbol-processing capabilities by our agents. This is not a final word on this, but it is a first attempt of dealing with this problem.

5. Conclusion

This paper was written with the aim of putting forward a first sketch of a general mathematical framework which should allow us to discuss seriously the possibility of artificial agents to be fully capable of understanding their surrounding world, acquiring a comprehension of it. In order to obtain this, we preconize the use of Peircean semiotics as the main theory to ground the concepts of representation and understanding.

The reader should consider this work as a first attempt in such a goal, which should be object of future enhancements and/or modifications. The reason for publishing it here is to collect feedback from the scientific community involved with this quest, and to set up a common framework where this issue of understanding and comprehension by artificial devices should be discussed with a more common ground. It is usual to find different authors dealing with the notions of "meaning", "representation" and "understanding" without a more proper technical use of these terms, using them on their own personal perspective of what they account for. The community around Artificial Intelligence, since its "foundation" in 1964 implicitly is using a somewhat obsolete model for what is meaning and representation, sometimes without taking notice of it. We feel this is time to introduce a more powerful account of this issue of meaning and representation, and it is clear that the current practices are not enough. Peircean semiotics is our proposal for a new perspective on how to develop agents which may be fully capable of understanding their surrounding world, and being capable of comprehending it. As Peircean semiotics "in the wild" can be a very abstract theory, more akin to philosophers than to engineers, this work is a first step in the creation of an interpretation of Peirce's theory more suitable for being adopted by engineers and computer scientists. The reader should have noticed that this common framework is very open, suitable to be addressed by different proposals on how to implement agents internal algorithms. This was done on purpose. Our aim is on attracting attention to this issue, and establishing a community with a common interest and goals. More than a final word on the theme, this is an invitation for a community reflection on the theme, a dialog with an organized common structure to build on. This should be a community work, and only as a community we will succeed in this quest. The first steps are here.

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