

# 15 Emerging trends in fuzzy systems

*Fuzzy Systems Engineering  
Toward Human-Centric Computing*

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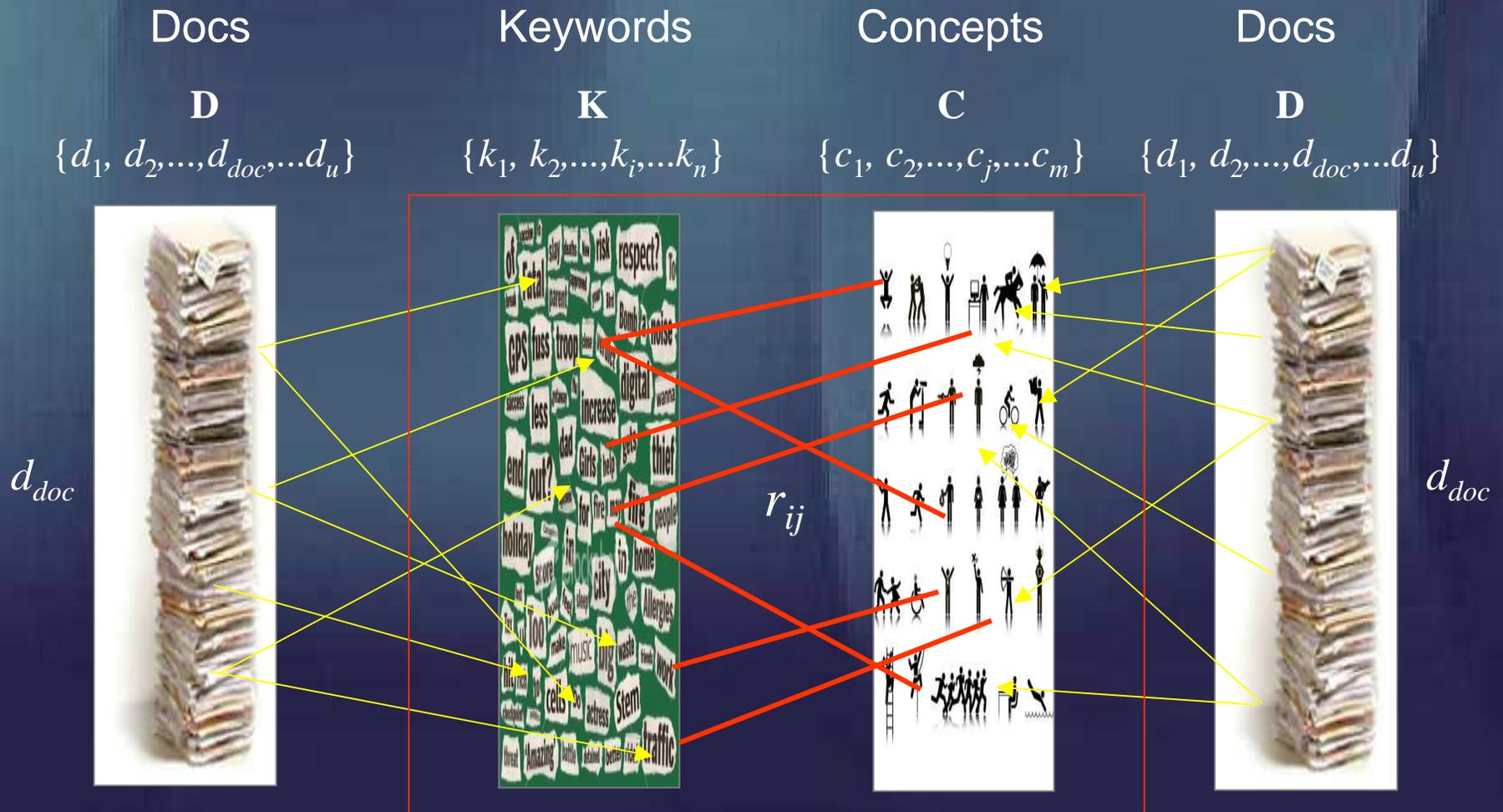
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# 15.1 Relational ontology in information retrieval

# Fuzzy relational ontological model



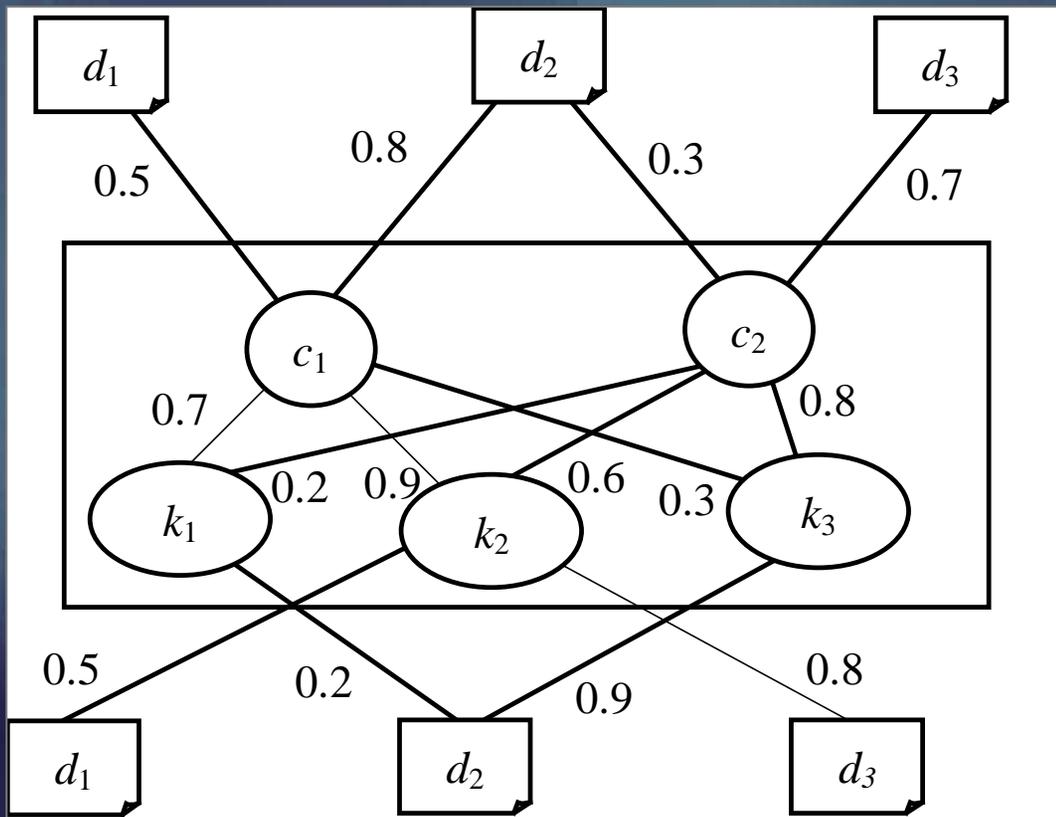
$$R = \{(k_i, c_j) \mid k_i \in \mathbf{K}, c_j \in \mathbf{C}\}$$

# Relational ontology model

- Concerns with
  - two-layer ontology
    - category names
    - keywords
  - association between category and keywords (fuzzy relation)
- Fuzzy relational ontology
  - fuzzy relation  $R = [r_{ij}]$  in  $\mathbf{K} \times \mathbf{C}$
  - $\mathbf{K} = \{k_1, k_2, \dots, k_n\}$
  - $\mathbf{C} = \{c_1, c_2, \dots, c_m\}$
  - $r_{ij}$  degree of association between  $k_i$  and  $c_j$

$$R = \begin{matrix} & c_1 & c_2 & \cdots & c_m \\ \begin{matrix} k_1 \\ k_2 \\ \vdots \\ k_n \end{matrix} & \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1m} \\ r_{21} & r_{22} & \cdots & r_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ r_{n1} & r_{n2} & \cdots & r_{nm} \end{bmatrix} \end{matrix}$$

# Example



# Information retrieval model

- $IR = \langle \mathbf{D}, \mathbf{Q}, V, F(q_i, d_{doc}) \rangle$

$\mathbf{D}$  : set of document representation

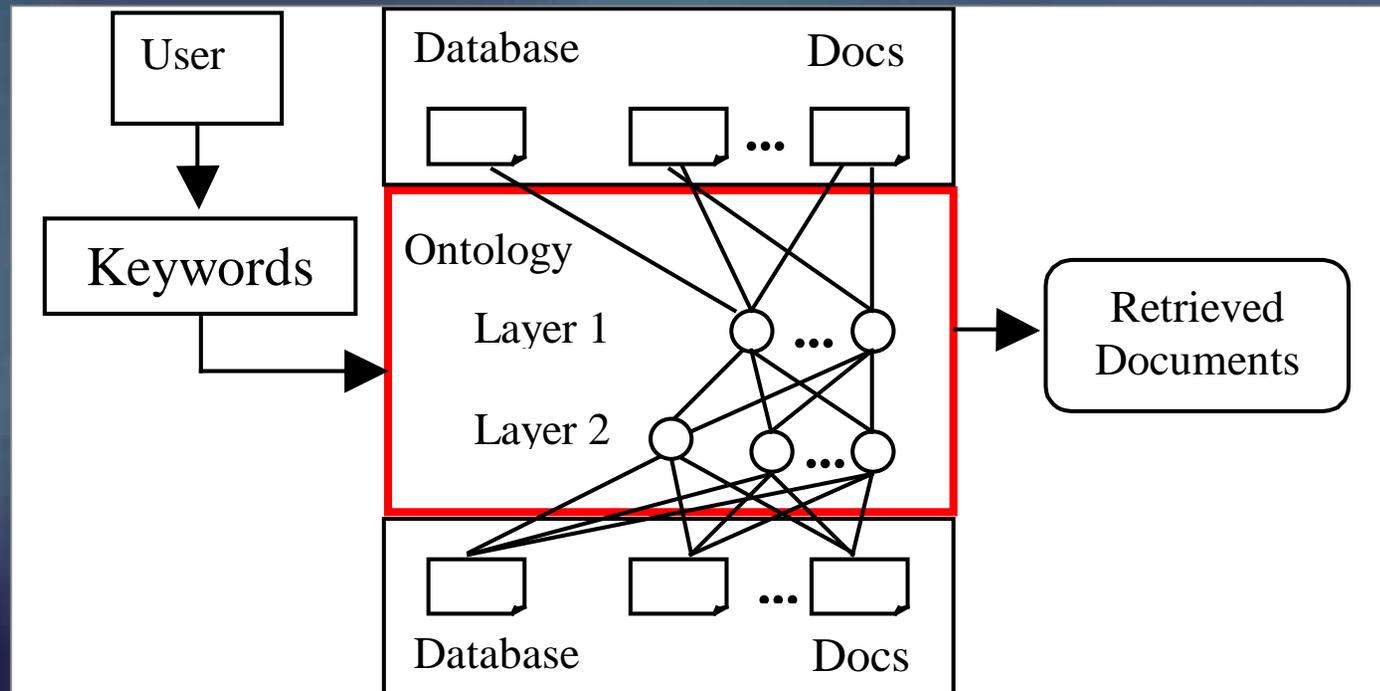
$\mathbf{Q}$  : set of query representation

$V$  : framework to represent docs, queries and their relations

$F$  : function associating a real number to  $(doc, query)$  pair

- $\mathbf{D} = \{d_1, d_2, \dots, d_{doc}, \dots, d_u\}$

# Information retrieval system structure



# Documents representation

$$T_k = \begin{matrix} & k_1 & k_2 & \cdots & k_n \\ d_1 & \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1n} \\ d_2 & \alpha_{21} & \alpha_{22} & \cdots & \alpha_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ d_u & \alpha_{u1} & \alpha_{u2} & \cdots & \alpha_{un} \end{matrix}$$

$$T_c = \begin{matrix} & c_1 & c_2 & \cdots & c_m \\ d_1 & \beta_{11} & \beta_{12} & \cdots & \beta_{1m} \\ d_2 & \beta_{21} & \beta_{22} & \cdots & \beta_{2m} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ d_u & \beta_{u1} & \beta_{u2} & \cdots & \beta_{um} \end{matrix}$$

# Query representation

- $Q = \{k_i, c_j\}$ ;  $k_i, c_j$  may be linked by *and* or *or* operators

$Q$  is represented by vectors  $\mathbf{x} \in \{0,1\}^n$  and  $\mathbf{y} \in \{0,1\}^m$

$$x_i = \begin{cases} 1 & \text{if } k_i \in Q \\ 0 & \text{otherwise} \end{cases}$$

$$y_j = \begin{cases} 1 & \text{if } c_j \in Q \\ 0 & \text{otherwise} \end{cases}$$

# Information retrieval

- $\mathbf{x}$  vector query keywords
- $\mathbf{y}$  vector query keywords
- $R$  fuzzy relational ontology

$$G_c = \mathbf{x} \circ R$$

$$G_k = R \circ \mathbf{y}$$

- $Q = (k_1 \vee k_2 \vee c_1) \wedge (k_3 \vee c_2)$
- $\mathbf{C} = \{c_1, c_2\}, \mathbf{K} = \{k_1, k_2, k_3\}$
- $F_c = [f_{c1}, \dots, f_{cj}, \dots, f_{cm}]$
- $F_k = [f_{k1}, \dots, f_{kj}, \dots, f_{kn}]$
- $z_1$  : threshold (chosen by design)

$$f_{cj} = \begin{cases} g_{cj} & \text{if } g_{cj} > z_1 \\ 0 & \text{otherwise} \end{cases}$$

$$f_{ki} = \begin{cases} g_{ki} & \text{if } g_{ki} > z_1 \\ 0 & \text{otherwise} \end{cases}$$

- Relevance degrees between docs and keywords

$$V_{DK} = T_k \circ F_k$$

- Relevance degrees between docs and concepts

$$V_{DC} = T_c \circ F_c^T$$

- Retrieval vector

$$V_D = V_{DK} \cup V_{DC}$$

- Ordering  $V_D$  produces retrieval vector  $V$

**procedure** INFORMATION-RETRIEVAL ( $Q$ ) **returns** documents

**input:** query  $Q$

**local:** thresholds:  $z_1, z_2$   
fuzzy relations:  $G_c, G_k$

set  $Q = \{k_i, c_j\}$

split  $Q$ :  $Q_1 = \{k_i\}$  and  $Q_2 = \{c_j\}$

construct queries vectors  $\mathbf{x}$  and  $\mathbf{y}$

compute  $G_c = [g_{cj}]$

compute  $G_k = [g_{ki}]$

select categories with  $g_{cj} > z_1$

select keywords with  $g_{ki} > z_1$

find database documents related with the categories  $c_j$

find database documents related with the keywords  $k_i$

find database documents related with the categories presented in  $Q_2$

find database documents related with the keywords presented in  $Q_1$

**if** sub-queries AND connected **then** select the common documents

**if** sub-queries OR connected **then** select all documents

compute  $V_{DK}$

compute  $V_{DC}$

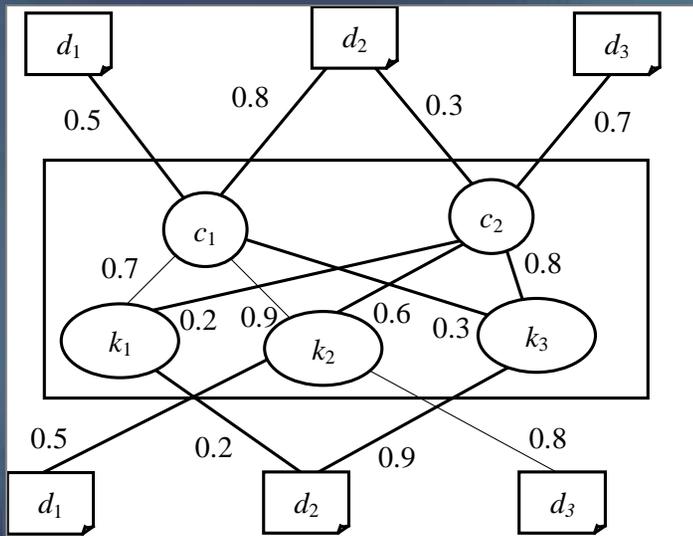
set  $V_D = V_{DK} \cup V_{DC}$

set  $V =$  rank ordered  $V_D$  according to increasing values

retrieval documents for which  $V$  component values are greater than or equal to  $z_2$

**return** documents

# Example



$$R = \begin{bmatrix} 0.7 & 0.2 \\ 0.9 & 0.6 \\ 0.3 & 0.8 \end{bmatrix}$$

$$T_k = \begin{bmatrix} 0 & 0.5 & 0 \\ 0.2 & 0 & 0.9 \\ 0 & 0.8 & 0 \end{bmatrix}$$

$$Q = \{k_2 \text{ and } c_1\}$$

$$z_1 = 0.65$$

$$z_2 = 0.4$$

$$T_c = \begin{bmatrix} 0.5 & 0 \\ 0.8 & 0.3 \\ 0 & 0.7 \end{bmatrix}$$

$$Q_1 = \{k_2\} \text{ and } Q_2 = \{c_1\}$$

$$G_c = x \circ R = [0 \ 0 \ 1] \circ \begin{bmatrix} 0.7 & 0.2 \\ 0.9 & 0.6 \\ 0.3 & 0.8 \end{bmatrix} = [0.3 \ 0.8]$$

$$G_k = R \circ y = \begin{bmatrix} 0.7 & 0.2 \\ 0.9 & 0.6 \\ 0.3 & 0.8 \end{bmatrix} \circ \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.7 \\ 0.9 \\ 0.3 \end{bmatrix}$$

$$V_{DC} = T_c \circ F'_c = \begin{bmatrix} 0.5 & 0 \\ 0.8 & 0.3 \\ 0 & 0.7 \end{bmatrix} \circ \begin{bmatrix} 0 \\ 0.8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.3 \\ 0.8 \end{bmatrix}$$

$$V_{DK} = T_K \circ F_K = \begin{bmatrix} 0 & 0.5 & 0 \\ 0.2 & 0 & 0.9 \\ 0 & 0.8 & 0 \end{bmatrix} \circ \begin{bmatrix} 0.7 \\ 0.9 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0.2 \\ 0.8 \end{bmatrix}$$

$$V_D = V_{DK} \cup V_{DC} = \begin{bmatrix} 0.5 \\ 0.3 \\ 0.8 \end{bmatrix}$$

$$V = \begin{bmatrix} 0.8 \\ 0.5 \\ 0.3 \end{bmatrix}$$

# Example

- 100 scientific papers on computational intelligence
- 61 words
- 6 concepts
- 55 keywords

# Performance measures

$$\text{Recall} = \frac{\text{Number of relevant documents retrieved}}{\text{Total number of relevant documents in database}}$$

$$\text{Precision} = \frac{\text{Number of relevant documents retrieved}}{\text{Number of documents retrieved}}$$

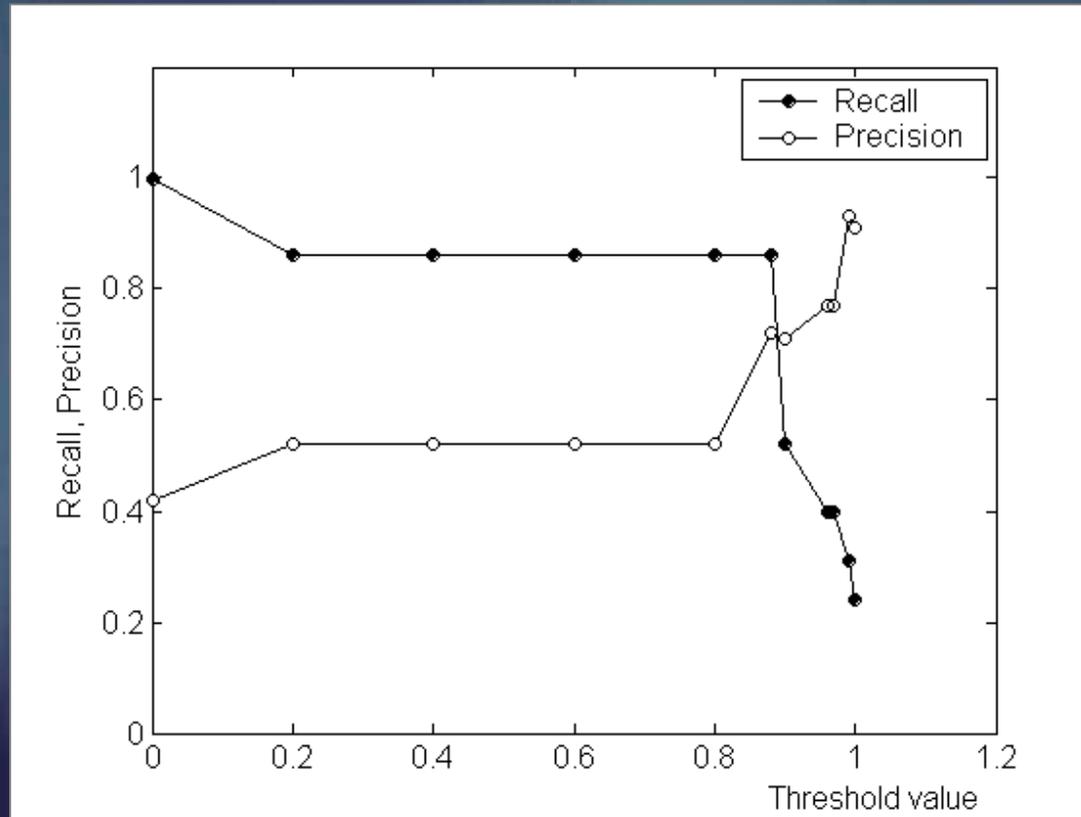
$$\overline{P}(r) = \sum_{i=1}^{N_q} \frac{P_i(r)}{N_q}$$

$\overline{P}(r)$ : average precision at recall level  $r$

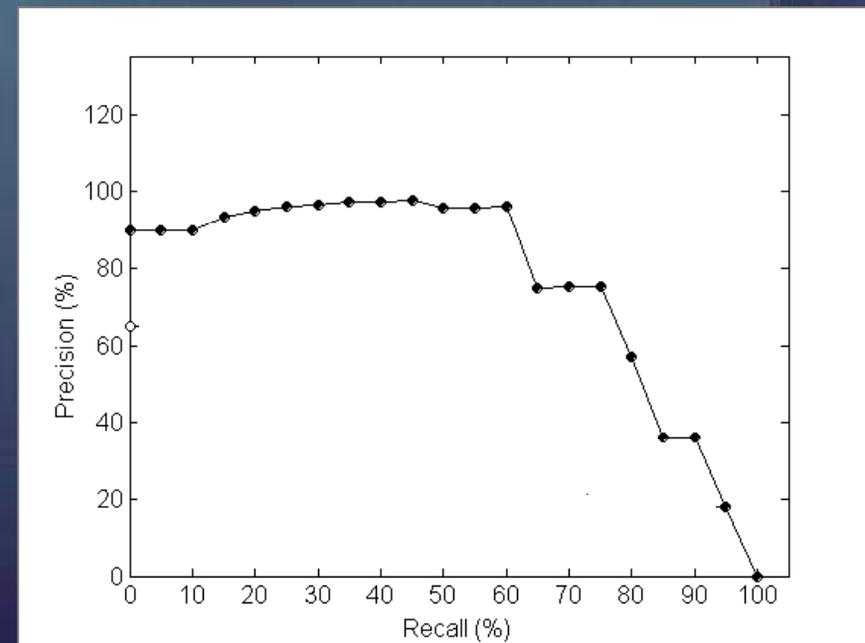
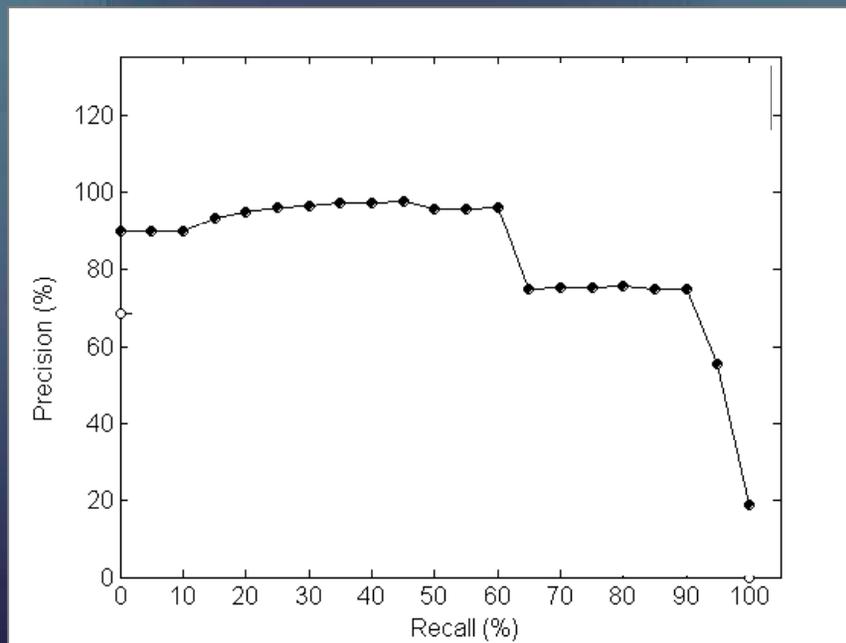
$N_q$ : number of queries

$P_i(r)$ : precision at recall level  $r$  for the  $i$ -th query

# Threshold $\times$ recall/precision



# Precision $\times$ recall for composite queries



## 15.2 Multiagent fuzzy systems

# Agents and multiagents

- Agent: anything that can be viewed as perceiving its environment through sensors and acting upon that environment through actuators
- Rational agent: one that does the right thing
- Rationality depends on:
  - performance measure to define success
  - agent's prior knowledge of the environment
  - actions that the agent can perform
  - agent percept history (sequence) to date

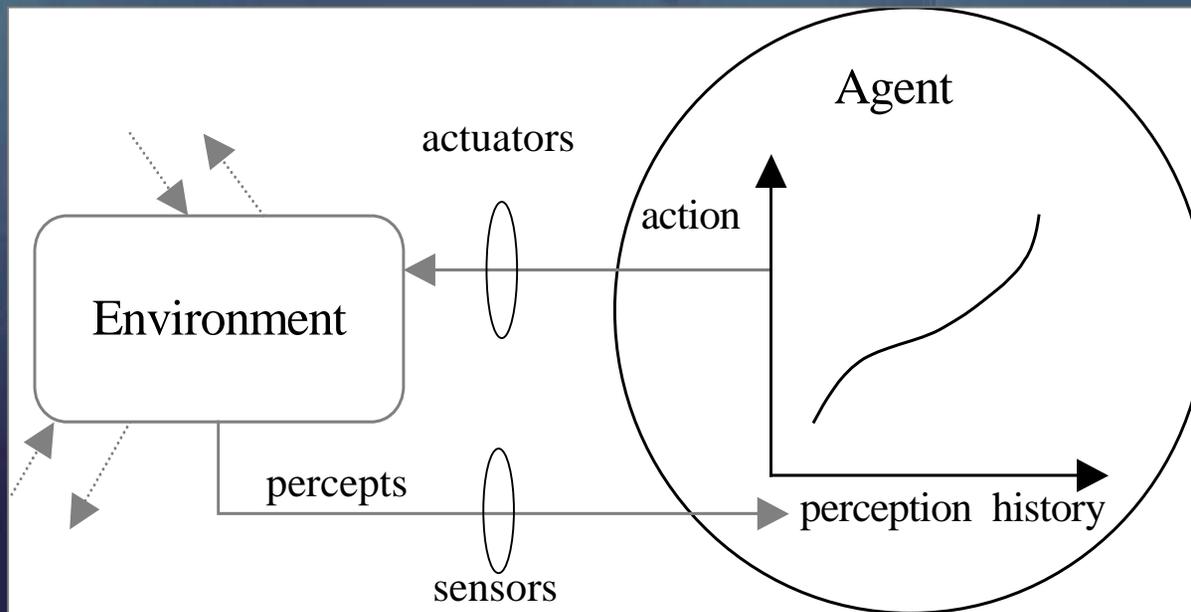
(Russel & Norvig, 2003)

# Agents

- Computer systems that is situated in some environment, and that is capable if autonomous action in this environment to meet its design objectives
- Agents: autonomy is central
- Intelligent agents
  - reactive
  - proactive
  - social ability
  - learning

(Wooldridge, 2002)

- Agent function: abstract mathematical (computable) description
- Agent function of an artificial agent is implemented by an agent program



Agent function  
(strategy)

# Structure of agents

- Agent = architecture + programs
- Agent programs
  - table driven
  - reflex
  - model-based
  - goal-based
  - utility-based
  - learning agents

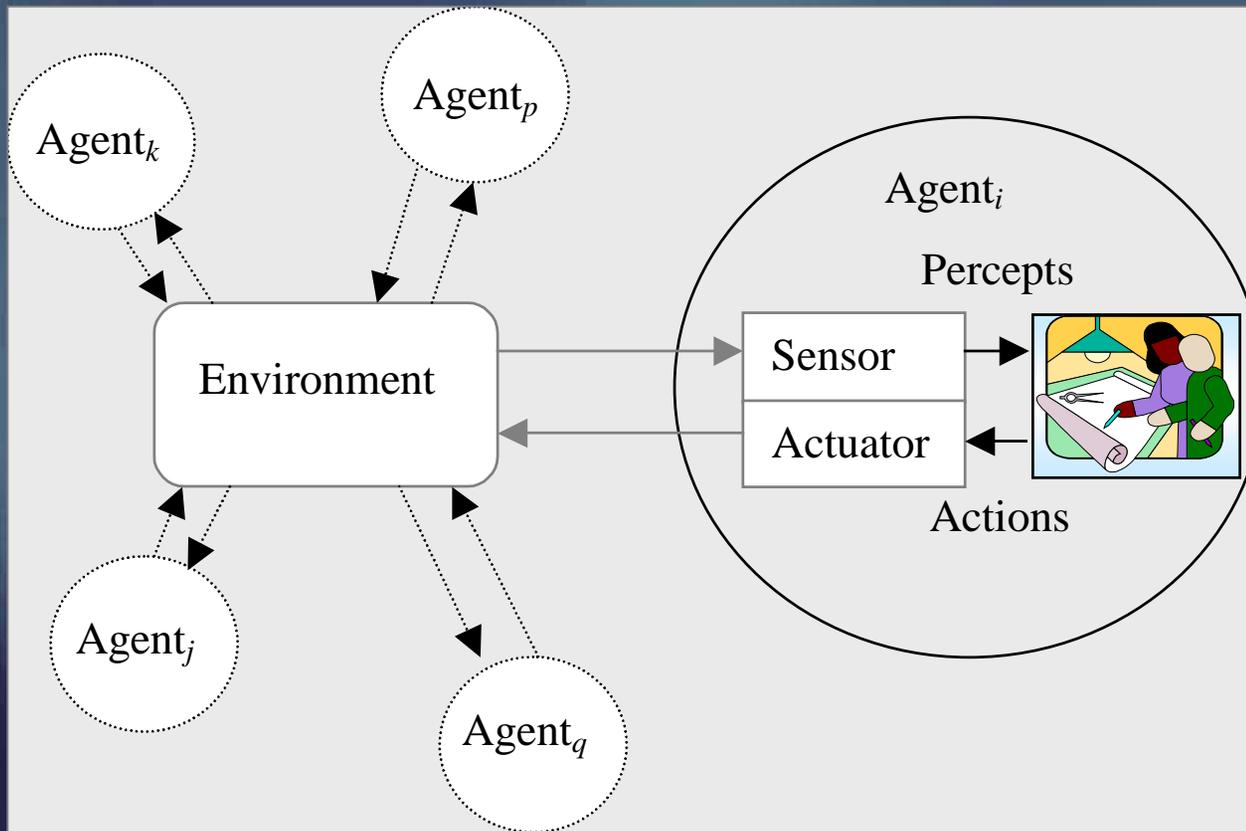
(Russel & Norvig, 2003)

# Multiagents

- Systems composed of multiple interacting agents
- Multiagent systems concerns
  - individual gents
  - collection of these agents
  - interaction between agents
- Interactions involve
  - cooperation
  - negotiation
  - coordination

(Wooldridge, 2002)

# Architecture of multiagent systems

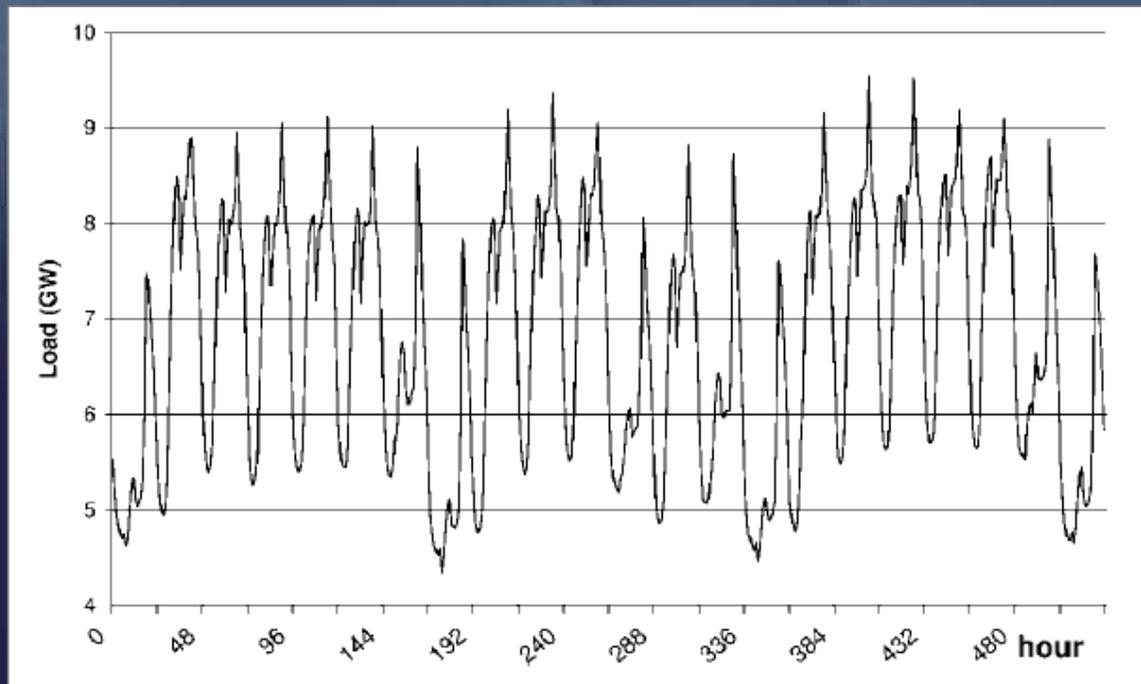


# Electricity markets

- Power industry worldwide
  - auctions
    - resource allocation
    - systems coordination
- Power supplier in competing markets must
  - decide bidding strategy
  - maximize expected profit
  - market share

# Demand

- Publicly known
- Auction is an ex-ante mechanism to allocate power hourly



Hourly load  
profile

# Running cost function

- Pool of thermal plants
- Cost of coal, gas, oil power plants
- Cost of nuclear plants: linear cost function
- Supplier cost function:

$$F(g_{jh}) = a + bg_{jh} + cg_{jh}^2$$

$$C(g_{jh}) = \alpha + \beta g_{jh} + \gamma g_{jh}^2$$

$g_{jh}$  : active power supplied by plant  $j$  at hour  $h$

# Uniform price auction

1. Open the auction
2. Publish day ahead hourly load forecast
3. Accept bids from suppliers
4. Stop receiving bids
5. Apply a pricing procedure (e.g. merit order)
6. Publish the hourly price  $\pi_h, h = 1, \dots, 24$
7. Inform each supplier the power to be produced for 24 h
8. Close the auction

- Power demand inelastic with price
- Auctioneer must assure  $D_h$  is met at all  $h$
- Conservative agent bid pairs (MW, \$):  $(q_{jh}, p_{jh})$
- Intelligent agent: free to choose price and amount

$$D_h = \sum_{j=1}^{T_h} g_{hj}$$

$$P_{jh} = \pi_h g_{hj} - C_j(g_{jh})$$

$$MC_j = \left. \frac{\partial C_j(g_{jh})}{\partial g_{jh}} \right|_{g_{jh} = G_j}$$

Supply = demand (MW)

Supplier profit (\$)

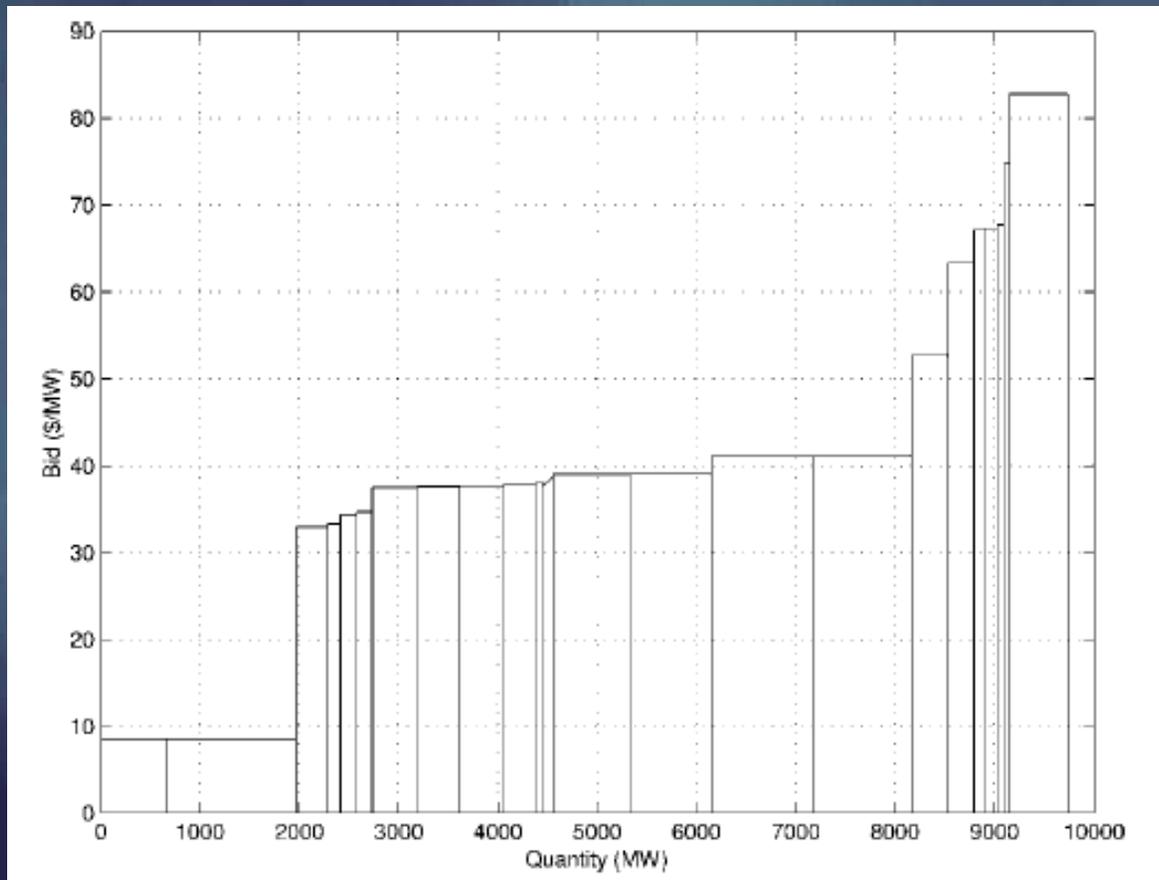
Marginal cost (\$/MW)

# Market players

**Table 1** Thermal plants characteristics

Plant	Type	$G_j$	$MC_j(G_j)$	$C_j(.)$
Angra 1	Nuclear	657	8.5	8.5g
Angra 2	Nuclear	1309	8.5	8.5g
P.Medici 3-4	Coal	320	32.95	$865.3 + 28.914g + 0.0063g^2$
P.Medici 1-2	Coal	126	33.33	$343.34 + 28.53g + 0.01905g^2$
TermoBahia	Gas	171	34.38	$580.54 + 30.985g + 0.00992g^2$
TermoCeara	Gas	153	34.72	$505.29 + 30.558g + 0.0136g^2$
Canoas	Gas	450	37.54	$1575.22 + 33.869g + 0.00408g^2$
N.Fluminense	Gas	426.6	37.63	$1484.92 + 33.759g + 0.00454g^2$
Araucaria	Gas	441.6	37.70	$1505.65 + 33.782g + 0.00443g^2$
Tres Lagoas	Gas	324	37.76	$1115.29 + 33.595g + 0.00643g^2$
Corumba	Gas	79.2	38.03	$278.97 + 33.256g + 0.03016g^2$
Juiz de Fora	Gas	103	38.73	$323.68 + 33.088g + 0.0274g^2$
Ibirite	Gas	766.5	39.07	$3632.08 + 31.966g + 0.00463g^2$
TermoRio	Gas	824.7	39.11	$3904.05 + 31.912g + 0.00436g^2$
Argentina I	Gas	1018	41.04	$4459.61 + 32.775g + 0.00406g^2$
Argentina II	Gas	1000	41.05	$4379.82 + 32.774g + 0.00414g^2$
J.Lacerda C	Coal	363	52.64	$1547.15 + 45.962g + 0.00919g^2$
J.Lacerda B	Coal	262	63.30	$1407.65 + 56.198g + 0.01356g^2$
J.Lacerda A1-2	Coal	100	67.10	$549.89 + 57.895g + 0.04605g^2$
J.Lacerda A3-4	Coal	132	67.35	$728.6 + 57.65g + 0.03674g^2$
Charqueadas	Coal	69.1	67.72	$414.59 + 60.037g + 0.05559g^2$
FAFEN	Gas	57.6	74.78	$417.18 + 66.857g + 0.06879g^2$
Uruguaiana	Gas	582	82.77	$4306.82 + 76.729g + 0.00519g^2$

# Market supplier function



agents bid marginal prices at full capacity)

# Intelligent agent $\equiv$ genetic fuzzy system

**procedure** GFRBS-ALGORITHM ( $X, Y, f$ ) **returns** a rule base

**input** : universes  $X, Y$

fitness function:  $f$

**local**: population: set of individuals

crossover rate, mutation rate

max: maximum number of generations

INITIALIZE(population, number individuals)

**repeat**

evaluate each individual using  $f$

select parents in population using relative fitness

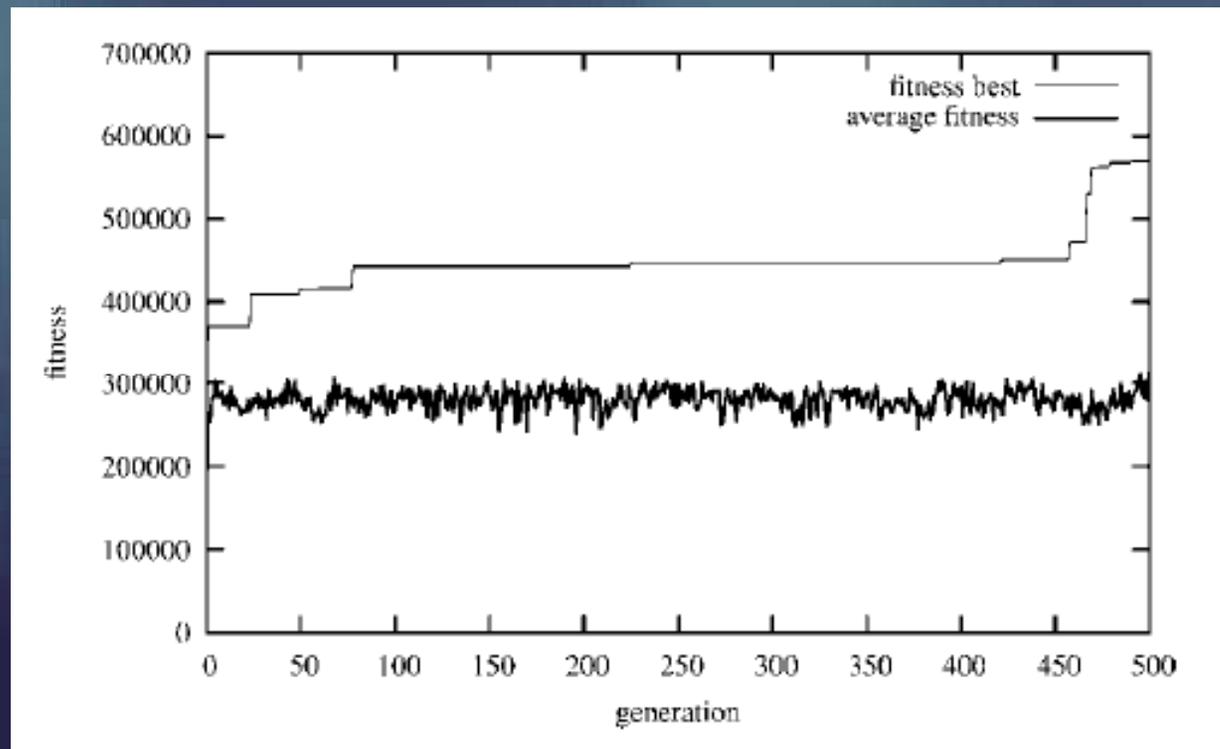
apply crossover and mutation on parents

create new population

**until** number generations  $\geq$  max

**return** rule base

# Example



## Rule base

10010  $\Rightarrow$  01010101 111011

11111  $\Rightarrow$  01100000 110011

11011  $\Rightarrow$  11100010 010110

00010  $\Rightarrow$  01010101 011011

00011  $\Rightarrow$  01011010 010111

01010  $\Rightarrow$  11110110 011111

If  $X_1$  is ( $A_{11}$  or  $A_{14}$ )  
then  $Y_1$  is ( $C_{12}$  or  $C_{14}$  or  $C_{16}$  or  $C_{18}$ )  
and  $Y_2$  is ( $C_{21}$  or  $C_{22}$  or  $C_{23}$  or  $C_{25}$  or  $C_{26}$ )

First rule

- Best bid strategy: two weeks test period
  - profit 36.7% higher
  - 91.3% more energy produced than conservative strategy
  
- Rule-base semantics
  - when demand is low and price below its marginal cost at full capacity, the agent bids lower price and a quantity that minimizes loss
  
  - increases price when he has opportunity to be the marginal supplier

# 15.3 Distributed fuzzy control

# Resource allocation

$$\max J(\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_n)$$

$$s.t. \sum_{i=1}^n \mathbf{u}_i \leq r$$

$$\mathbf{u}_i \geq 0, i = 1, \dots, n$$

# Economic system

- Agents
  - producers
  - consumers
- Price system
- commodities

# Artificial market

- Agents
  - producers: maximize profit
  - consumers: maximize utility
- resource vector  $u \in \mathbb{R}^\ell$
- consumer agents:  $n, f_i(u_i^c), u_i^c \in \mathbb{R}^\ell$  ( $f_i$  = utility)
- producer agents:  $m, f_j(u_j^p), u_j^p \in \mathbb{R}^\ell$  ( $f_j$  = profit)

# Optimal market allocation

$$\max_{\mathbf{u}_i^c} f_i(\mathbf{u}_i^c)$$

$$\max_{\mathbf{u}_j^p} f_j(\mathbf{u}_j^p)$$

$$s.t. \sum_{i=1}^n \mathbf{u}_i^c = \sum_{j=1}^m \mathbf{u}_j^p$$

$$\mathbf{u}_i^c, \mathbf{u}_j^p \geq 0, i = 1, \dots, n; j = 1, \dots, m$$

# Control systems and economy

$$\mathbf{x}_i(t+1) = f_i(\mathbf{x}(t), \mathbf{u}_i(t), t)$$

$$\mathbf{y}_i(t) = g_i(\mathbf{x}(t), t), \quad \mathbf{x}_i(0) = \mathbf{x}_{i0}$$

$$\mathbf{x} \in R^q, \quad \mathbf{u}_i \in R^\ell, \quad \mathbf{y}_i \in R^s$$

Coupled dynamic  
systems



$$\min_u F_i(|u_1(t)|, \dots, |u_n(t)|, x_0)$$

$$s.t. \quad \mathbf{x}_i(t+1) = f_i(\mathbf{x}(t), \mathbf{u}_i(t), t)$$

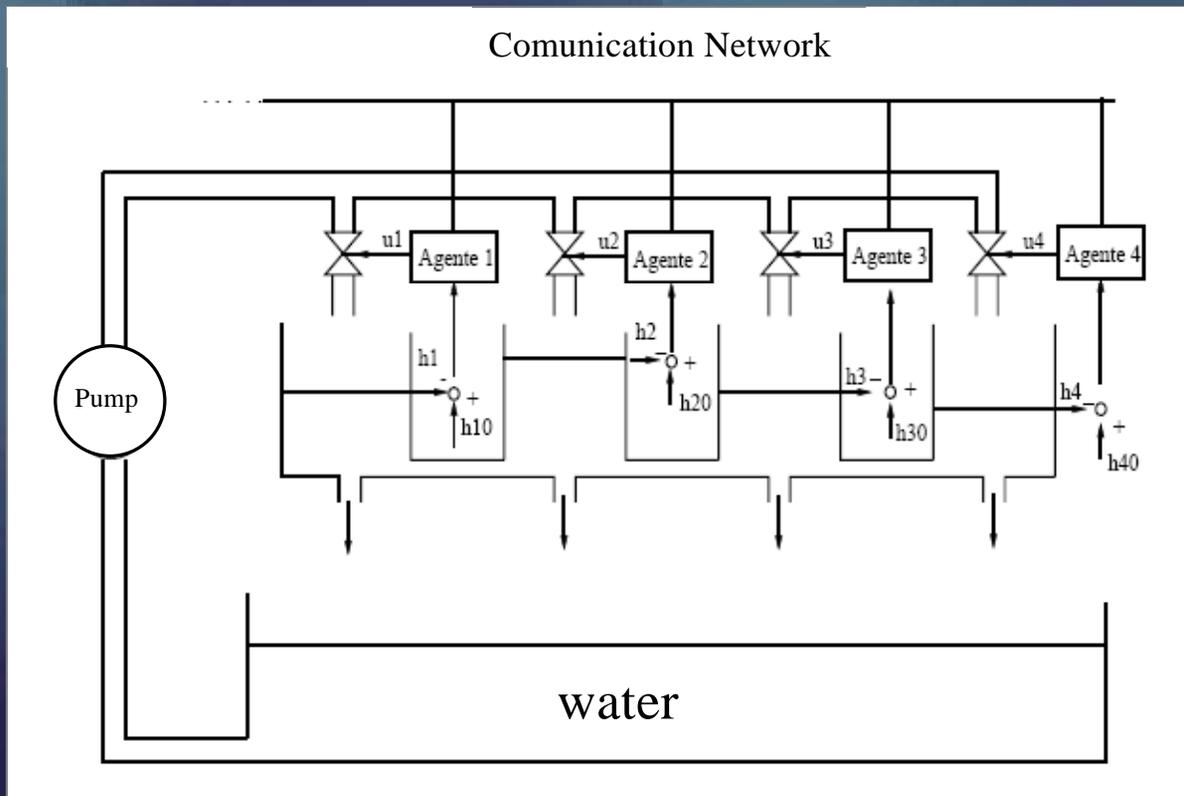
$$\mathbf{y}_i(t) = g_i(\mathbf{x}(t), t)$$

$$|\mathbf{u}_i(t)| \geq 0$$

$$\mathbf{x}_i(0) = \mathbf{x}_{i0}$$

Distributed resource  
allocation problem

# Fuzzy market-based control



$\mathbf{h}_s \equiv$  set-points

$\mathbf{u}_o \equiv$  operation point

$\Delta h_i(t)$  input variable for  $i^{th}$  agent

$u_i(t) = u_{io} + \Delta u_i(t)$  decision of  $i^{th}$  agent

# Market-based control algorithm

**procedure** DISTRIBUTED-FUZZY-AUCTIONEER ( $d,s$ ) **returns** price

**input** : demand:  $d$

supply:  $s$

**for** each agent **do**

**if** demand agent **then** get demand

**if** consumer agent **then** get supply

    compute equilibrium price

    run auctions for the training period

    store individual fitness  $\phi$

    remove individual from the market

**return** price

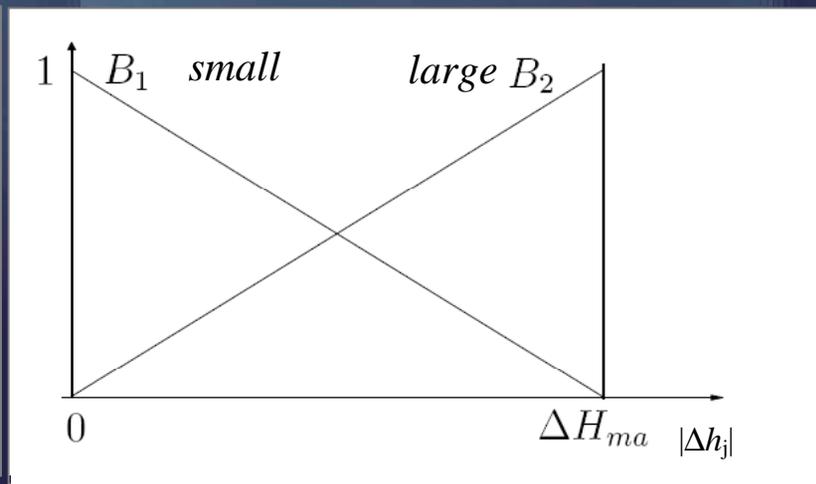
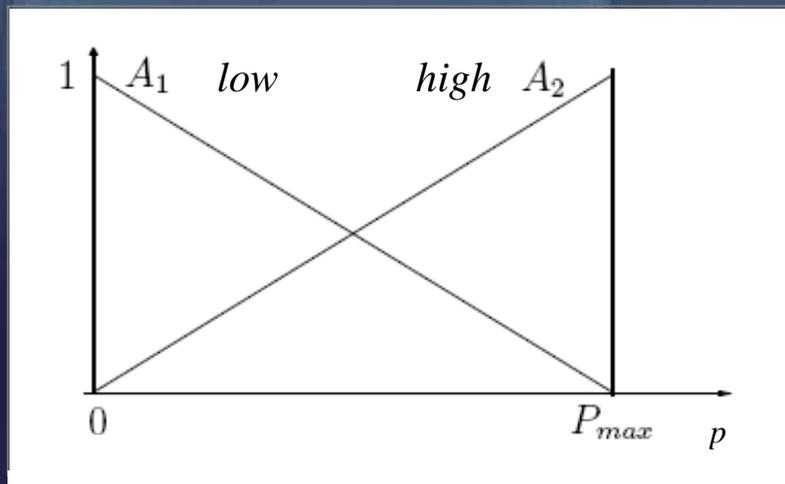
$$r_{pp}(t) + \sum_{j=1}^m r_j^p(t) = r_{cp}(t) + \sum_{i=1}^q r_i^c(t)$$

# Consumer agent

If price is *low* and deviation is *small* then demand is  $g_{do}$

If price is *low* and deviation is *large* then demand is  $g_{dmax}$

If price is *high* then demand is  $g_{do}$



$$g_{do}(p(t), \Delta h_i(t)) = 0, \forall p(t), \Delta h_i(t)$$

$$g_{d \max}(p(t), \Delta h_i(t)) = r_{d \max}, \forall p(t), \Delta h_i(t)$$

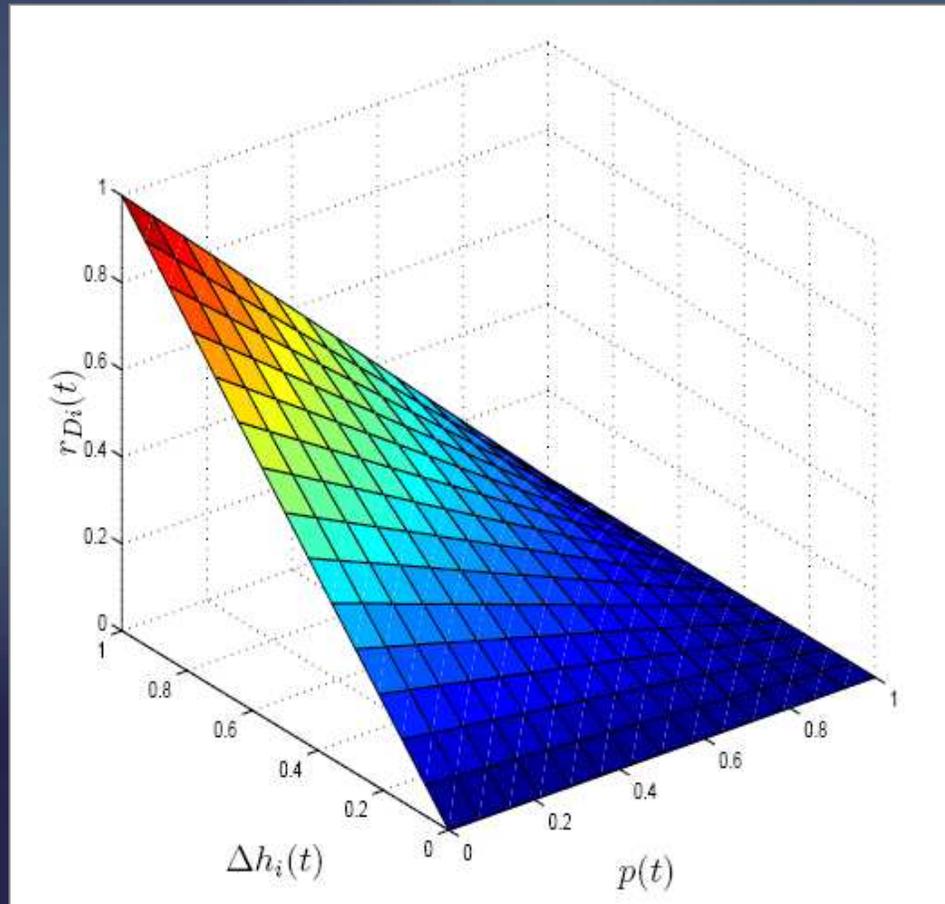


$$r_i^c(t) = (k_{d1} - k_{d2}p(t))\Delta h_i(t)$$

$$k_{d1} = (r_{d \max} / \Delta H_{\max})$$

$$k_{d2} = (k_{d1} / P_{\max})$$

# Demand function of a consumer agent

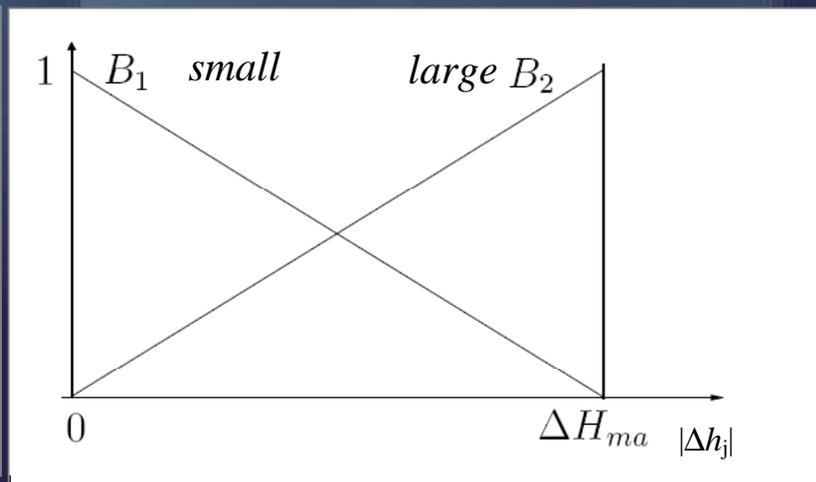
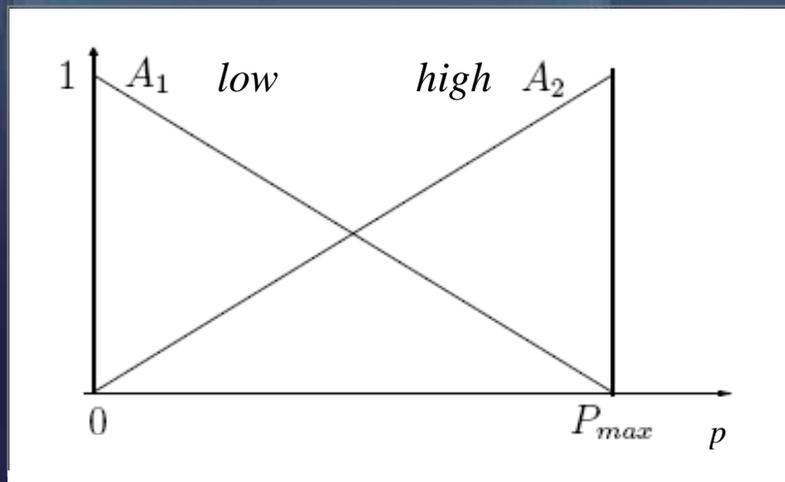


# Producer agent

If price is *low* then supply is  $g_{so}$

If price is *high* and deviation is *small* then supply is  $g_{so}$

If price is *high* and deviation is *large* then supply is  $g_{smax}$



$$g_{s0}(p(t), \Delta h_i(t)) = 0, \forall p(t), \Delta h_i(t)$$

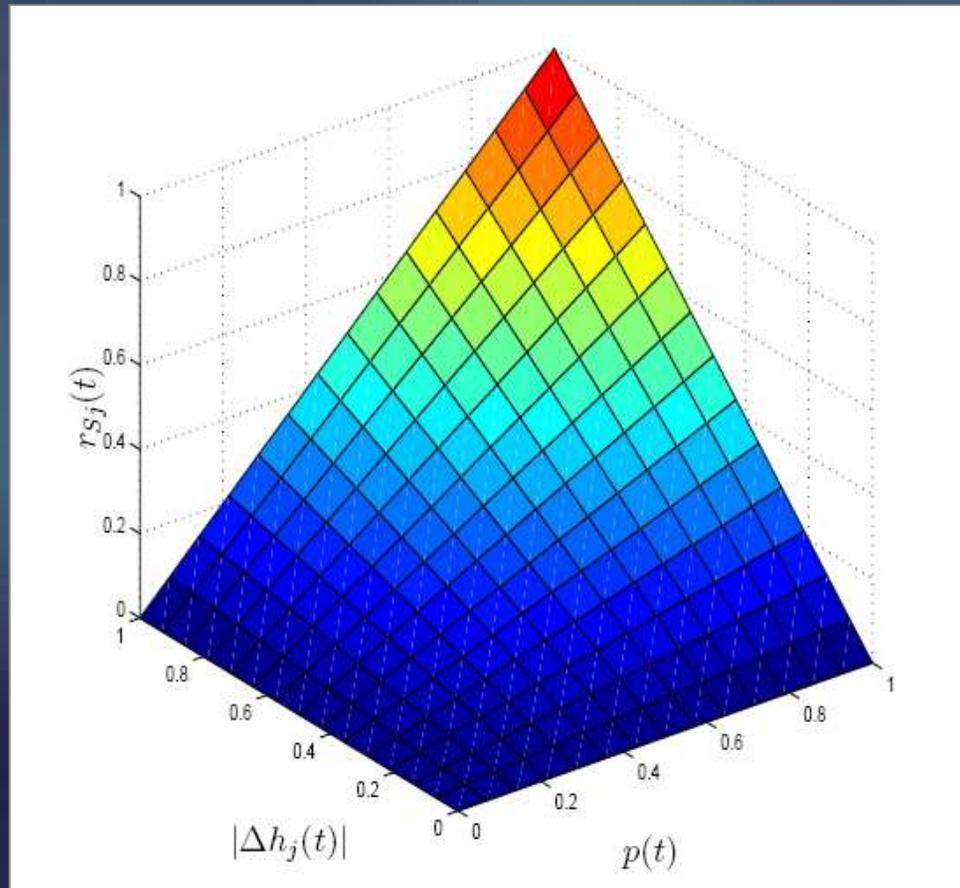
$$g_{s\max}(p(t), \Delta h_i(t)) = r_{s\max}, \forall p(t), \Delta h_i(t)$$



$$r_j^p(t) = k_s p(t) |\Delta h_i(t)|$$

$$k_s = (r_{s\max} / (P_{\max} \Delta H_{\max}))$$

# Supply function of a producer agent



# Market equilibrium

Pump as a permanent producer :

$$r_{pp}(t) = \delta p(t)$$

Pump as a permanent consumer:

$$r_{cp}(t) = \alpha_1 \left( 1 - \frac{p(t)}{\alpha_2} \right)$$

Equilibrium prices

$$p(t) = \frac{\alpha_1 + k_{d1} \sum_{i=1}^q \Delta h_i(t)}{\delta + (\alpha_1 / \alpha_2) + k_s \sum_{j=1}^m |\Delta h_j(t)| + k_{d2} \sum_{i=1}^q \Delta h_i(t)}$$

Controls

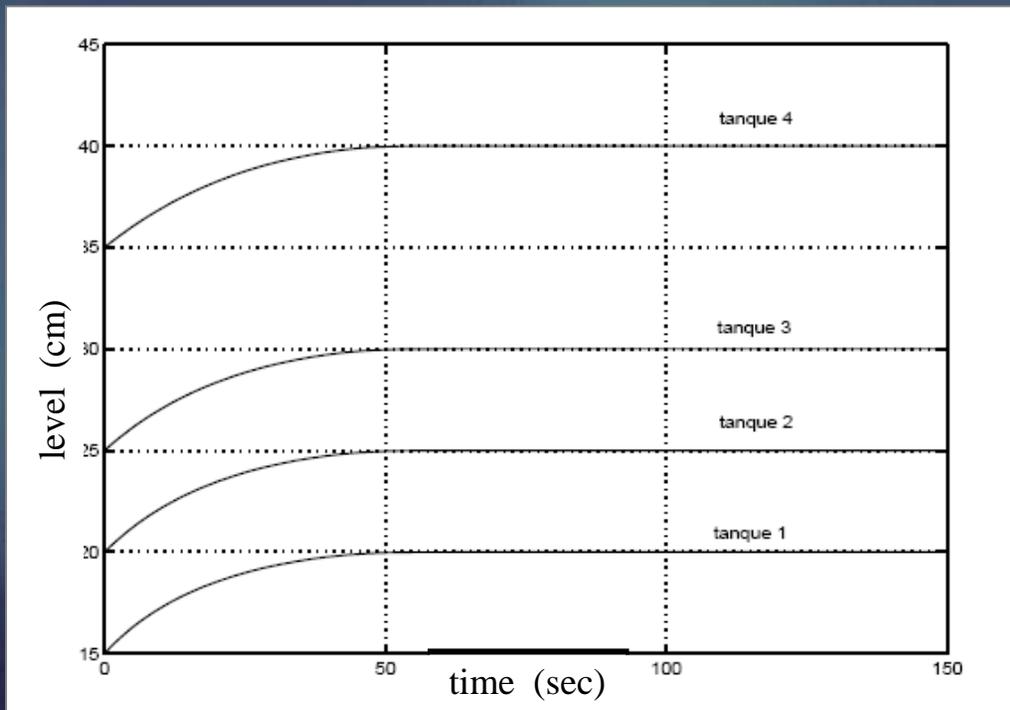
$$\Delta u_i(t) = (k_{d1} - k_{d2} p(t)) \Delta h_i(t)$$

consumer

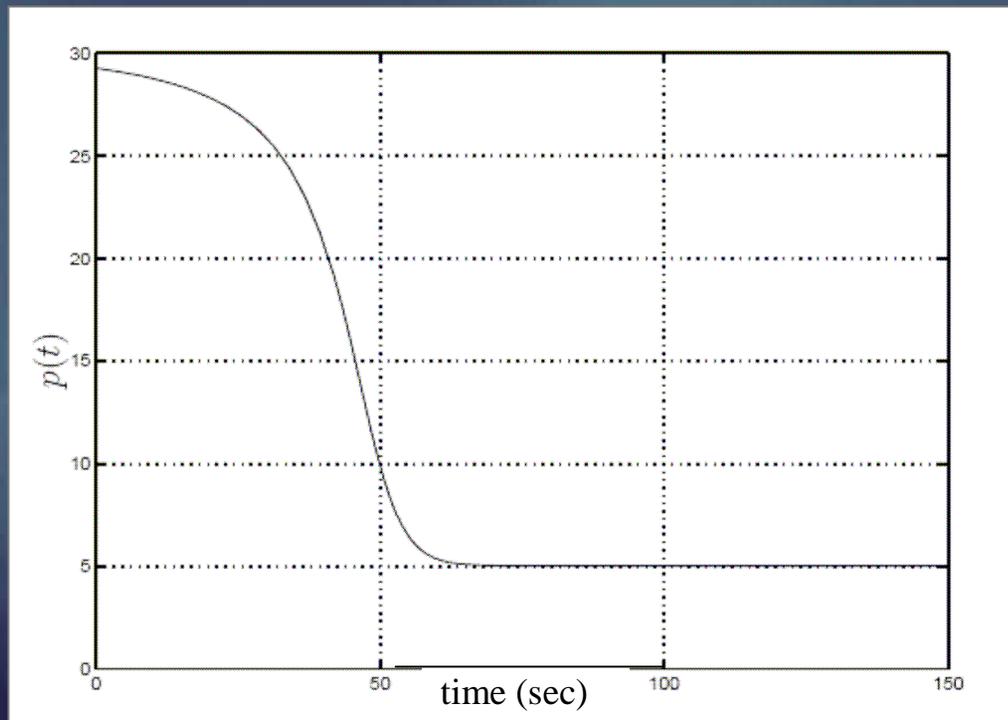
$$\Delta u_s(t) = k_p p(t) \Delta h_j(t)$$

producer

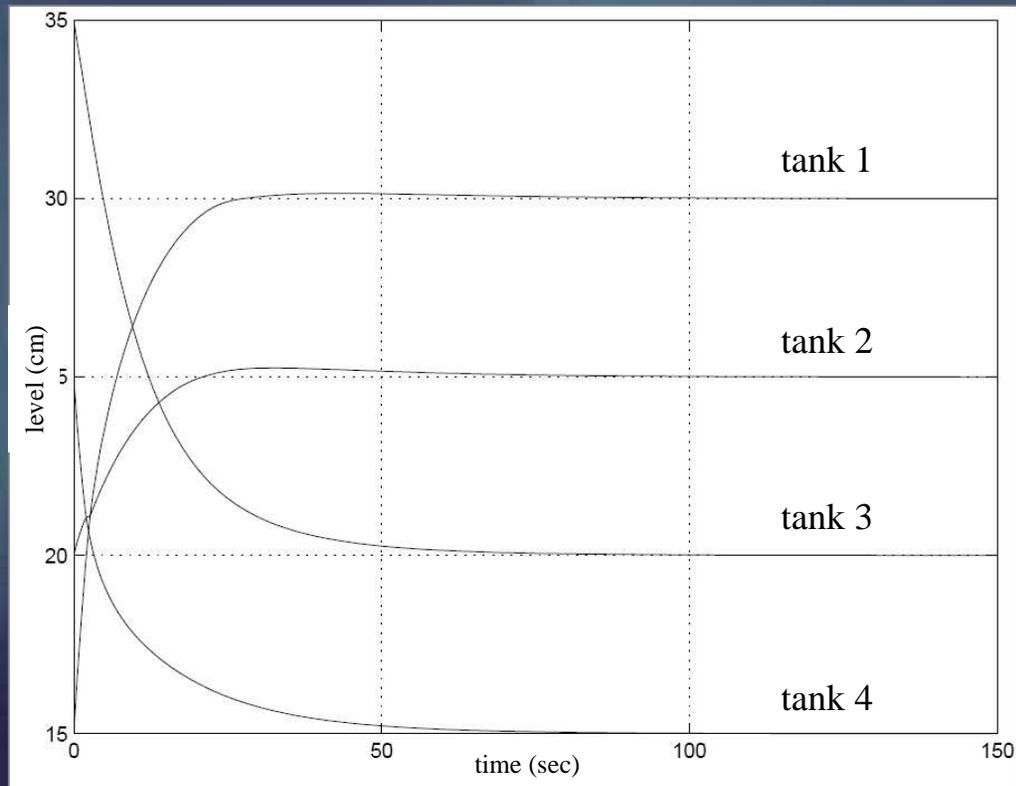
# Example 1: Tank level



# Equilibrium prices



## Example 2: Tank levels



# Equilibrium prices

