

# 10 Fuzzy Modeling: Principles and Methodology

*Fuzzy Systems Engineering  
Toward Human-Centric Computing*

# Contents

**10.1 The architectural blueprint of fuzzy models**

**10.2 Key phases of the development and use of fuzzy models**

**10.3 Main categories of fuzzy models: An overview**

tabular fuzzy models

rule-based fuzzy models

fuzzy relational models and associative memories

fuzzy decision trees

fuzzy neural networks

fuzzy cognitive maps

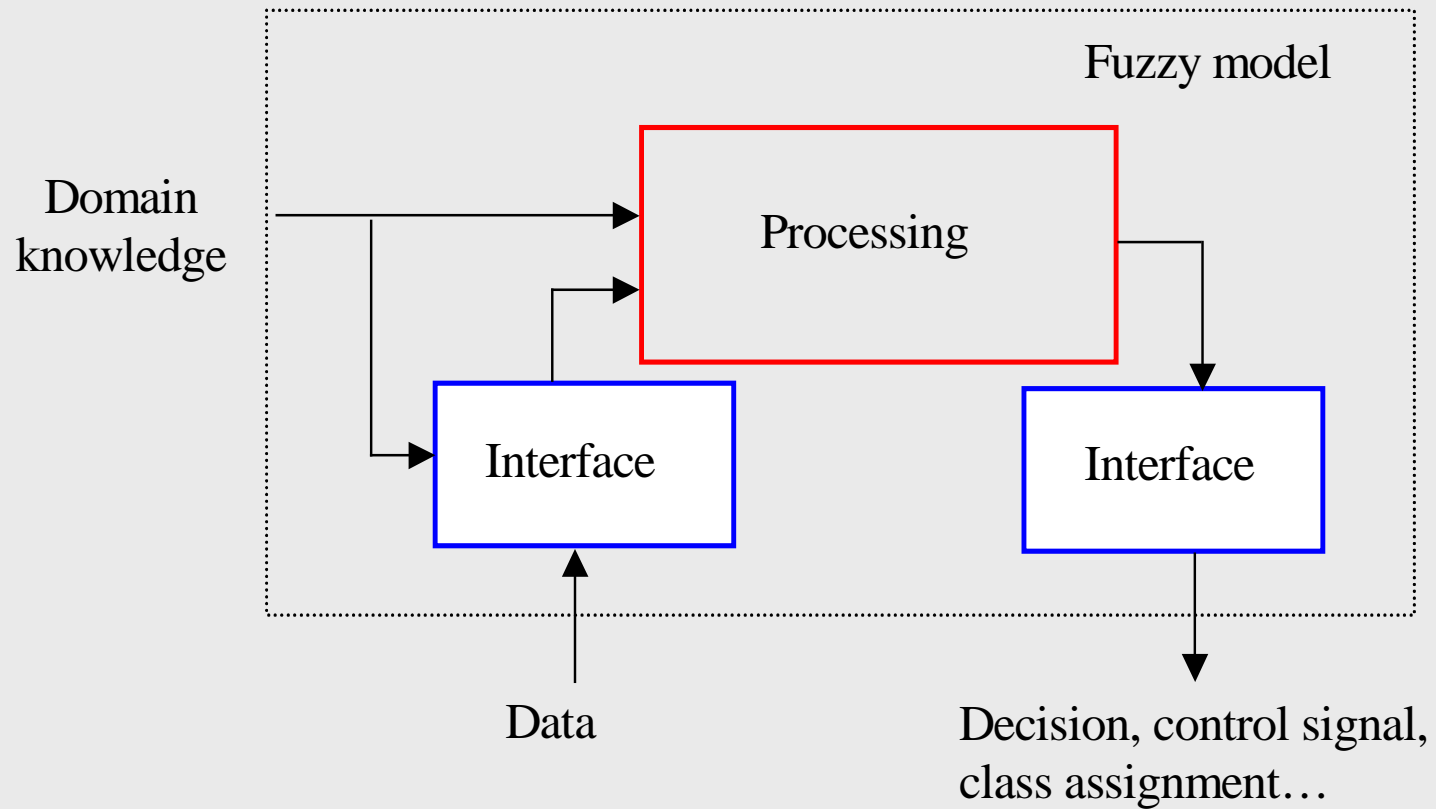
**10.4 Verification and validation of fuzzy models**

# **10.1 The architectural blueprint of fuzzy models**

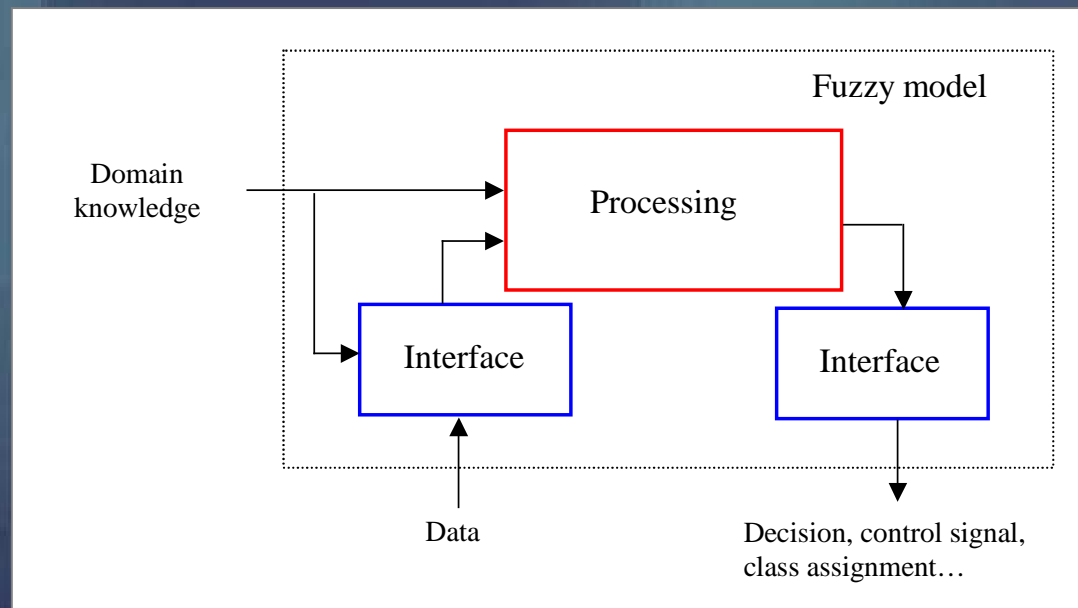
# Preamble

- Fuzzy models operate on information granules that are fuzzy sets and fuzzy relations
- Information granules are abstract realizations of concepts used in modeling
- As modeling is realized at higher, more abstract level, fuzzy models give rise to a general architecture in which we highlight three main functional modules, that is
  - input interface
  - processing module
  - output interface

# General architecture

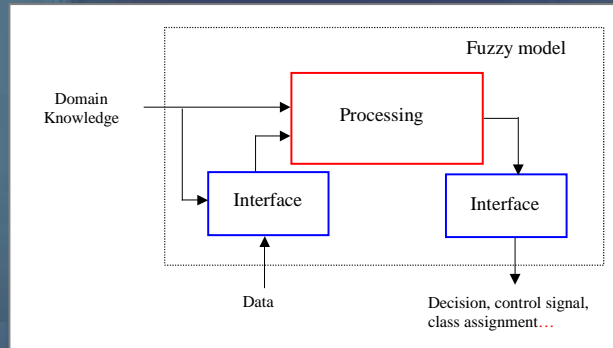


# General architecture: functional modules



- **Input interface:** accepts heterogeneous data (information granules and numeric data) and converts them to internal format where processing at the level of fuzzy sets is carried out
- **Processing module:** processing pertinent to information granules
- **Output interface:** converts results of processing information granules into the format acceptable by the modeling environment

# Functional modules of fuzzy models: rule-based systems



- Processing module: collection of rules,  $i = 1, 2, \dots, N$

**If** condition<sub>1</sub> is  $A_i$  **and** condition<sub>2</sub> is  $B_i$  **then** action (decision, conclusion) is  $D_i$

- Input interface:

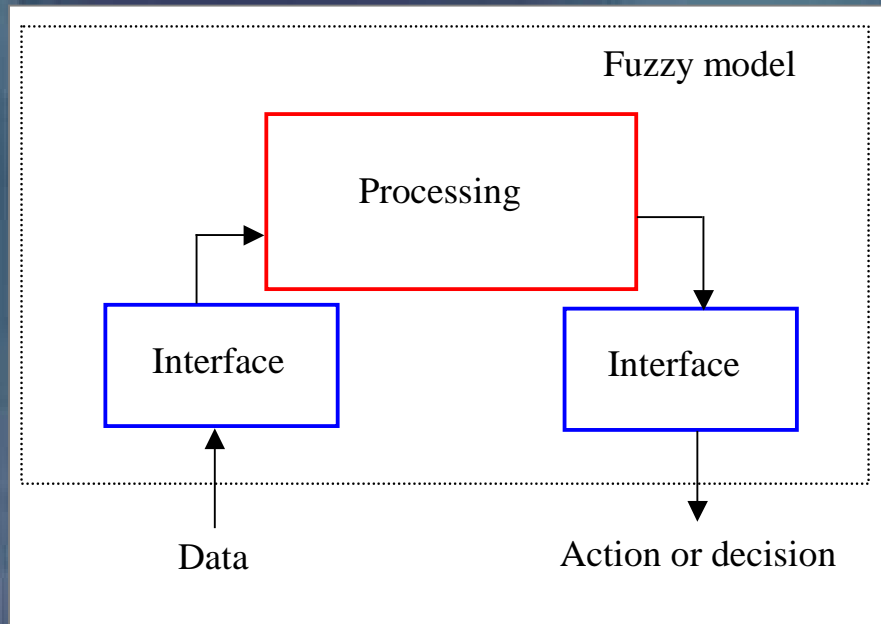
input  $X$ : express it in terms of fuzzy sets  $A_i$  present in the conditions of rules

- Output interface:

decode the result of processing, say fuzzy set  $D$ , in the format required by the modeling environment, say a single numeric entity

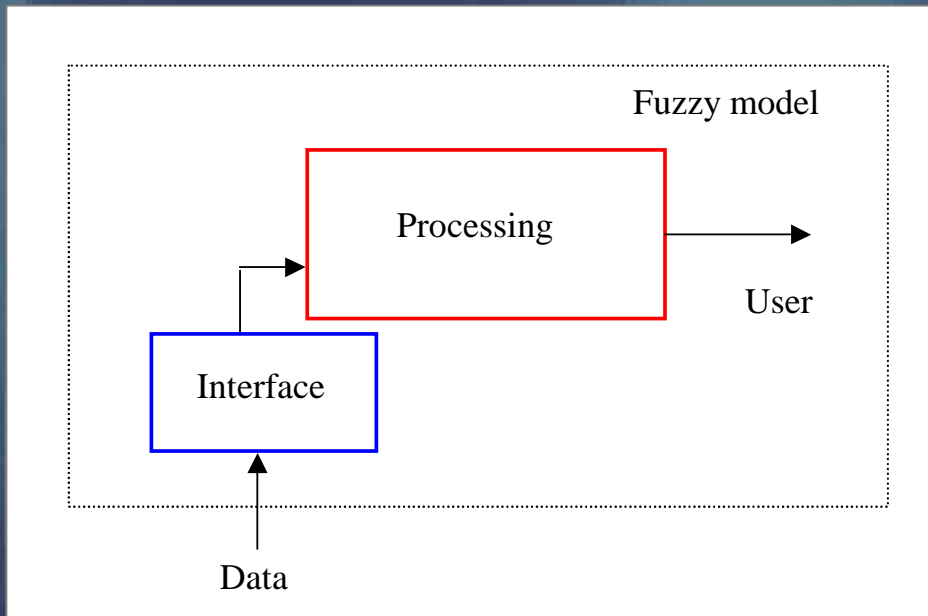
## **10.2 Key phases of the development and use of fuzzy models**

# Main modes of use of fuzzy models (a)



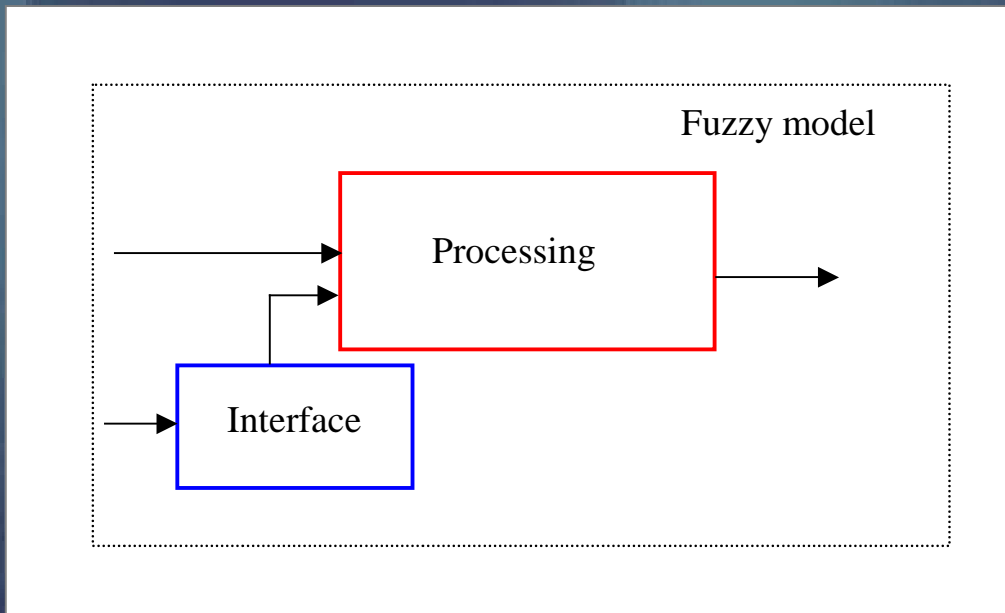
- The use of numeric data and generation of numeric results
- Module reflects a large modeling spectrum
- After development, model is used in purely numerical fashion accepts numbers and produce numbers as nonlinear I/O mappings

## Main modes of use of fuzzy models (b)



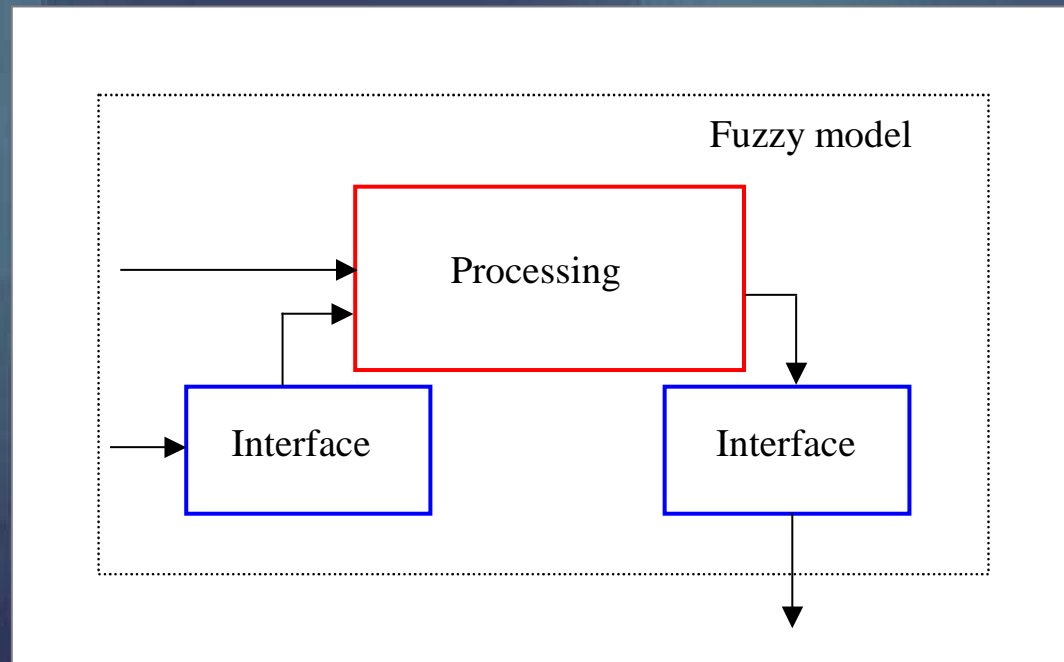
- use of numeric data and granular results (fuzzy sets)
- User centric: more informative and comprehensive than numbers
- User provided with preferences (membership degrees) associated with a collection of possible outcomes

# Main modes of use of fuzzy models (c)



- Granular input data and fuzzy sets as outputs
- Scenarios where we encounter collection of linguistic observations
- Examples: expert judgment, unreliable sensor readings, etc.

# Main modes of use of fuzzy models (d)



- Use of fuzzy sets as model inputs and outputs
- Granular data forming aggregates of detailed numeric data

## **10.3 Main categories of fuzzy models: An overview**

# Main categories of models: An overview

- Diversified landscape of fuzzy models - selected categories:
  - tabular fuzzy models
  - rule-based fuzzy models
  - fuzzy relational models including associative memories
  - fuzzy decision trees
  - fuzzy neural networks
  - fuzzy cognitive maps
  - ....

# Main categories of models: Some design considerations

- Expressive power
- Processing capabilities
- Design schemes and ensuing optimization
- Interpretability
- Ability to deal with heterogeneous data
- ....

# Tabular fuzzy models

- Table of **relationships** between the variables of the system granulated by some fuzzy sets.
- Easy to build and interpret
- Limited processing capabilities (not included as a part of the model)

	$B_1$	$B_2$	$B_3$	$B_4$	$B_5$
$A_1$					
$A_2$			$C_3$		
$A_3$					$C_1$

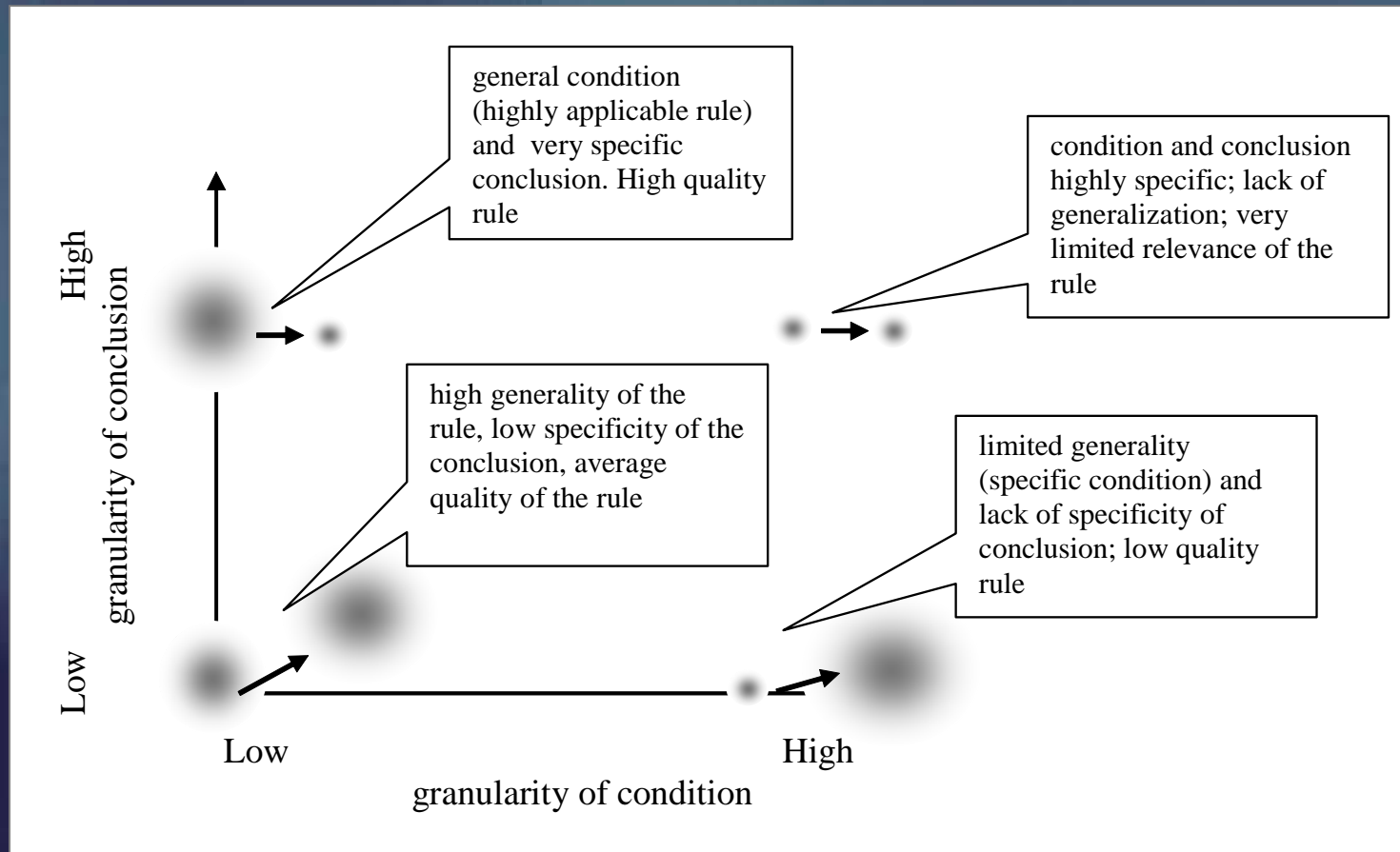
# Rule-based fuzzy models

- Highly modular and easily expandable fuzzy models
- Composed of a family of conditional (**If – then**) statements (rules)
- Fuzzy sets occur in their conditions and conclusions
- Standard format

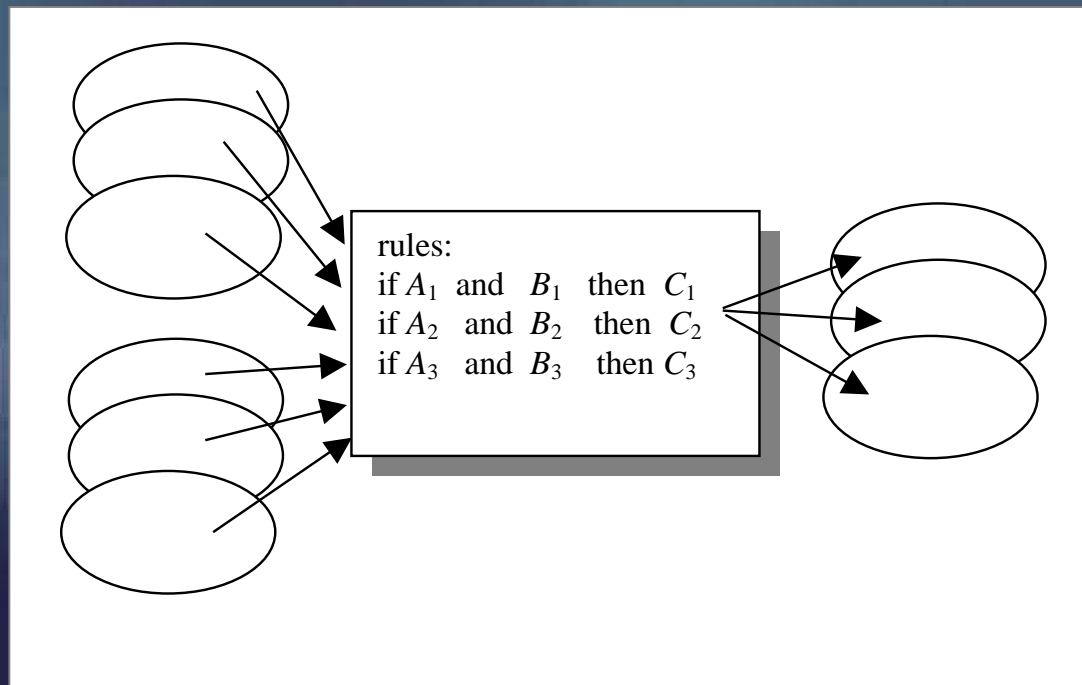
**If** condition<sub>1</sub> is *A* *and* condition<sub>2</sub> is *B* *and* ... *and* condition<sub>*n*</sub> is *W*  
**then** conclusion is *Z*

- Conditions  $\equiv$  rule antecedent
- Conclusions  $\equiv$  rule consequent

# Rule-based fuzzy models: Granularity and quality of rules

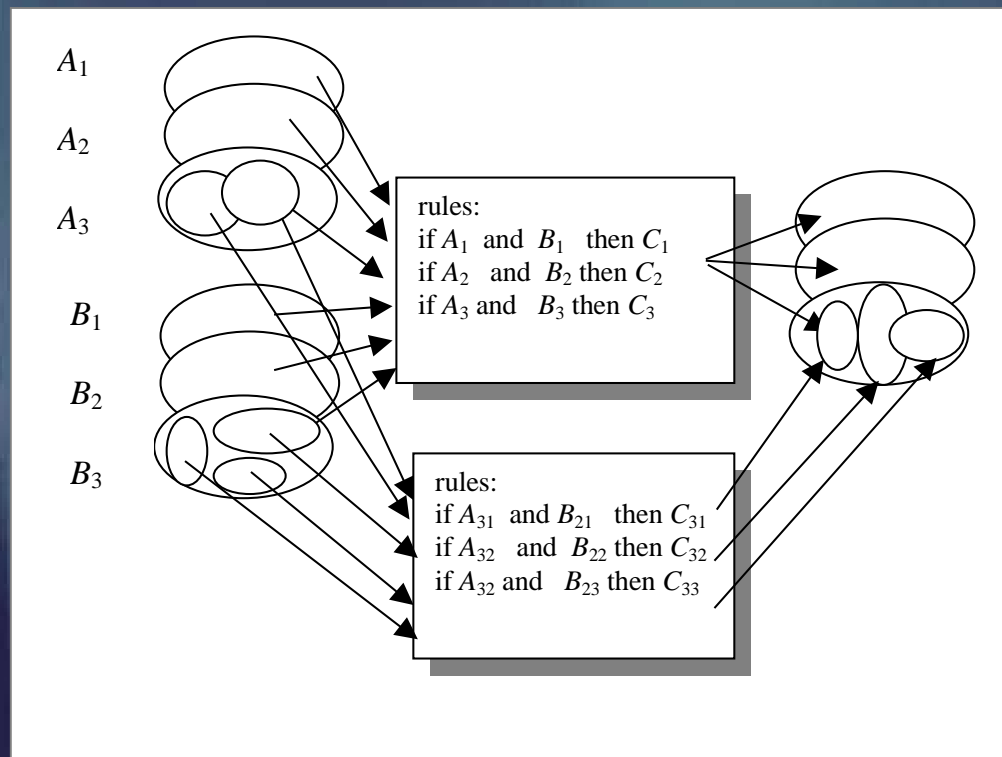


# Granularity of information in rule-based systems



Same level of  
granularity

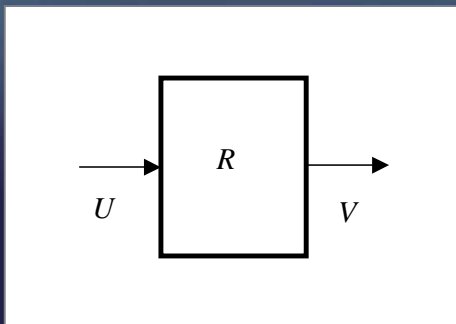
# Granularity of information in rule-based systems



Different levels  
of granularity

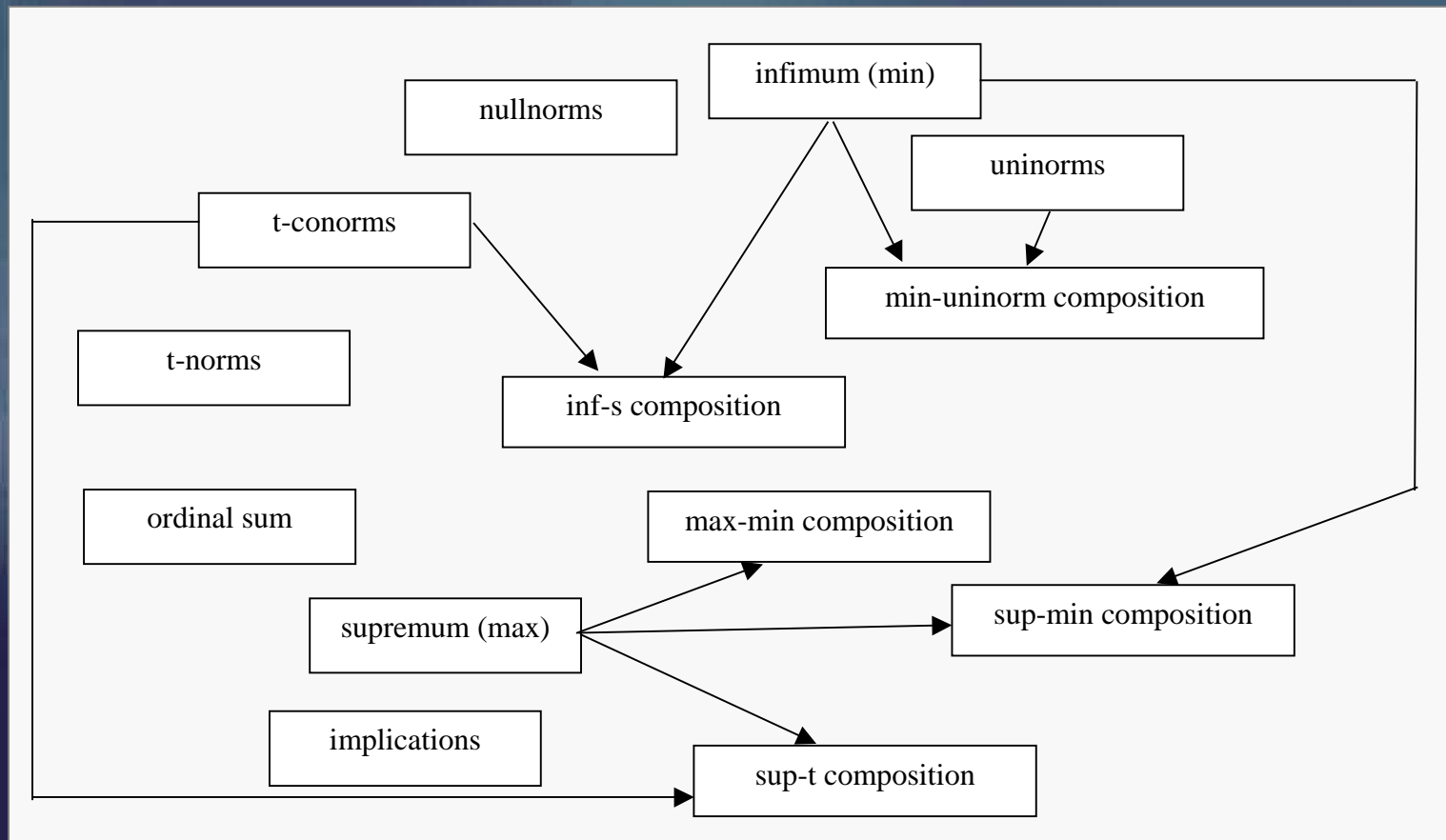
# Fuzzy relational models and associative memories

- Relational transformation of fuzzy sets
- Two main modes
  - construction of fuzzy relations-storing
  - inference-recall



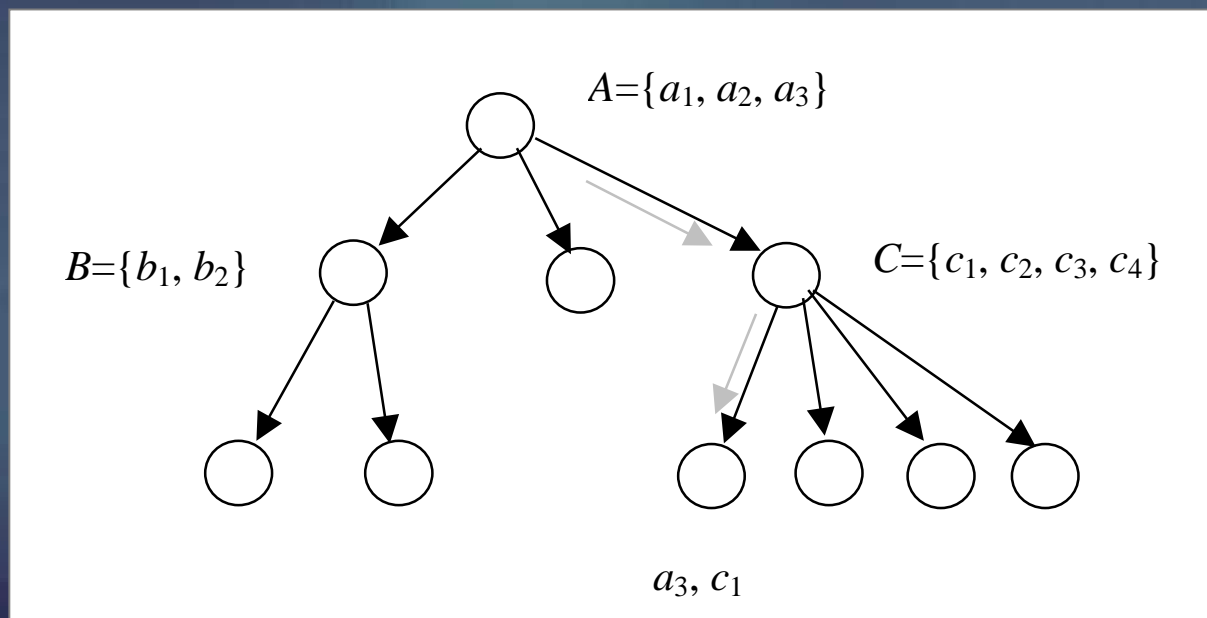
$$R = \bigcup_{k=1}^N (A_k \times B_k)$$
$$V = U \circ R$$

# Fuzzy relational structures: A general taxonomy



# Fuzzy decision trees

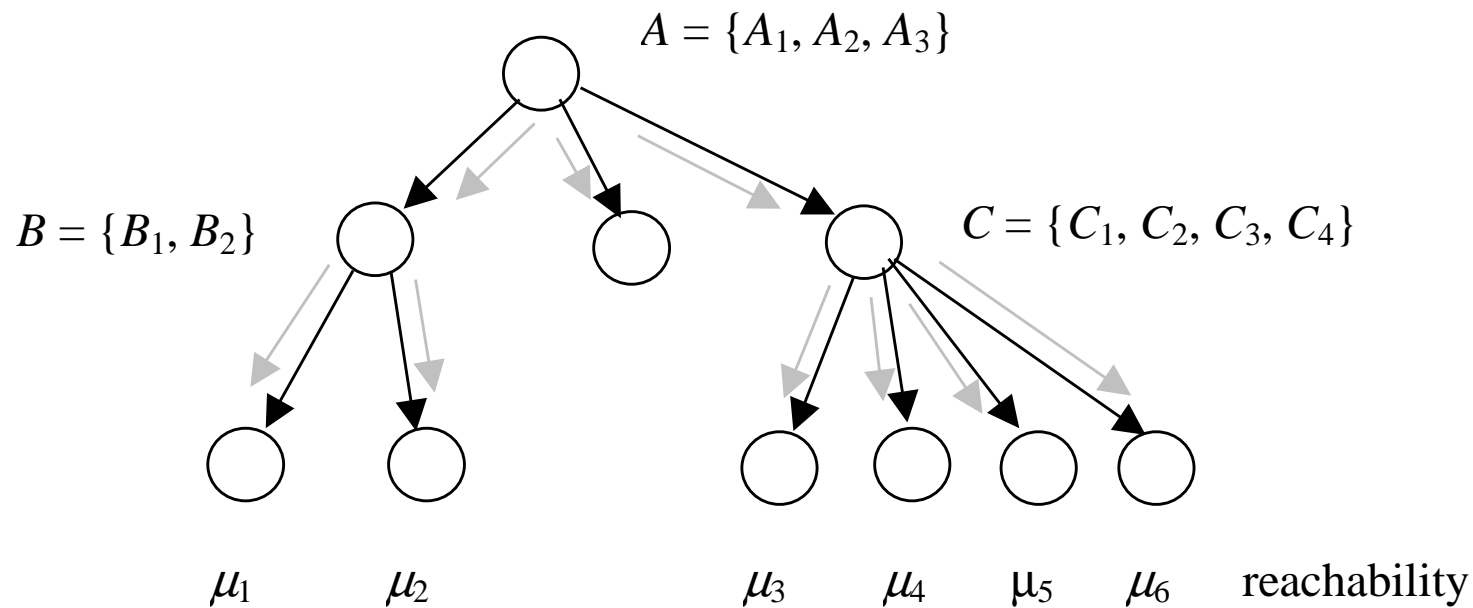
- Generalization of decision trees



- Traversal of tree depending on the values of the attributes: only a single path traversed and a single terminal node reached

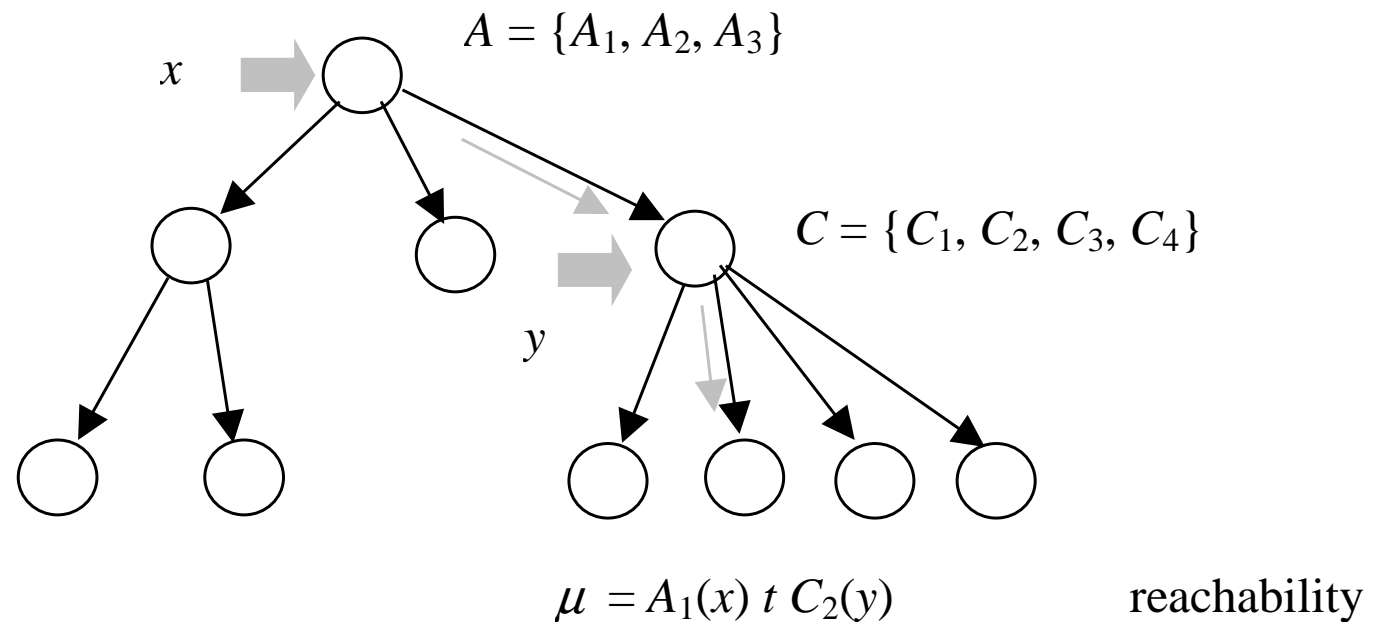
# Fuzzy decision trees

- Traversal of a number of paths leading to a number of terminal nodes (reachability levels)



# Fuzzy decision trees

- Traversal of a number of paths leading to a number of terminal nodes (reachability levels)

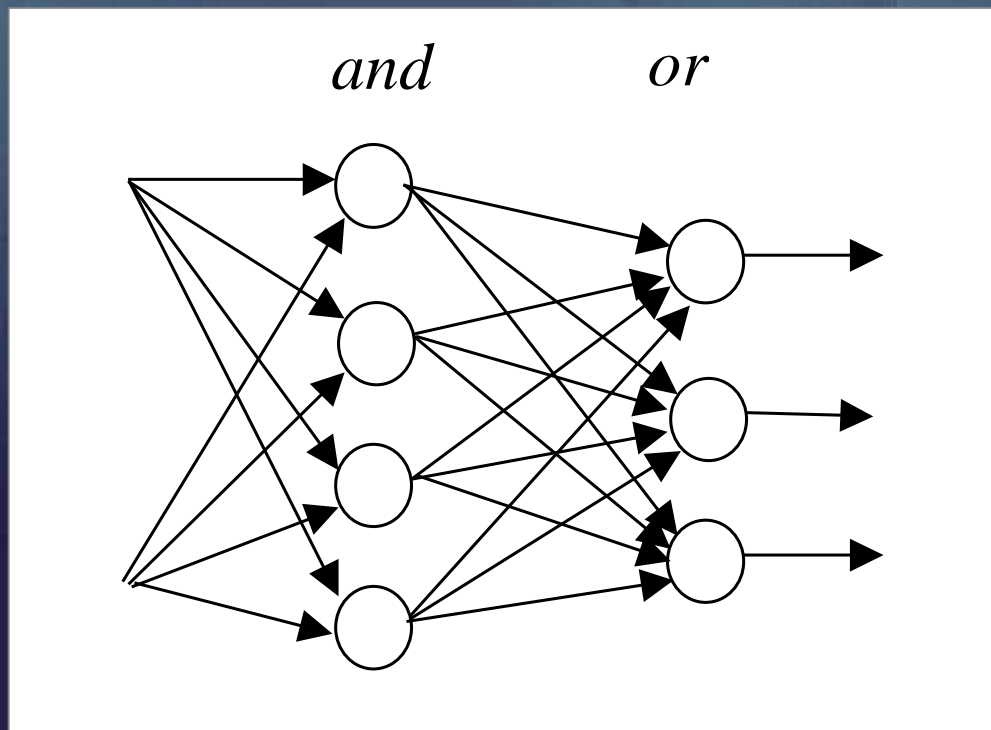


# Fuzzy neural networks

- Architectures in which we combine adaptive properties of neural networks with interpretability (transparency) of fuzzy sets
- A suite of fuzzy logic neurons:
  - aggregative neurons (*and*, *or* neurons)
  - referential neurons (dominance, equality, inclusion...)
- Learning mechanisms could be applied to adjustment of connections of neurons
- Each neuron comes with a well-defined semantics; the network could be easily interpreted once the training has been completed

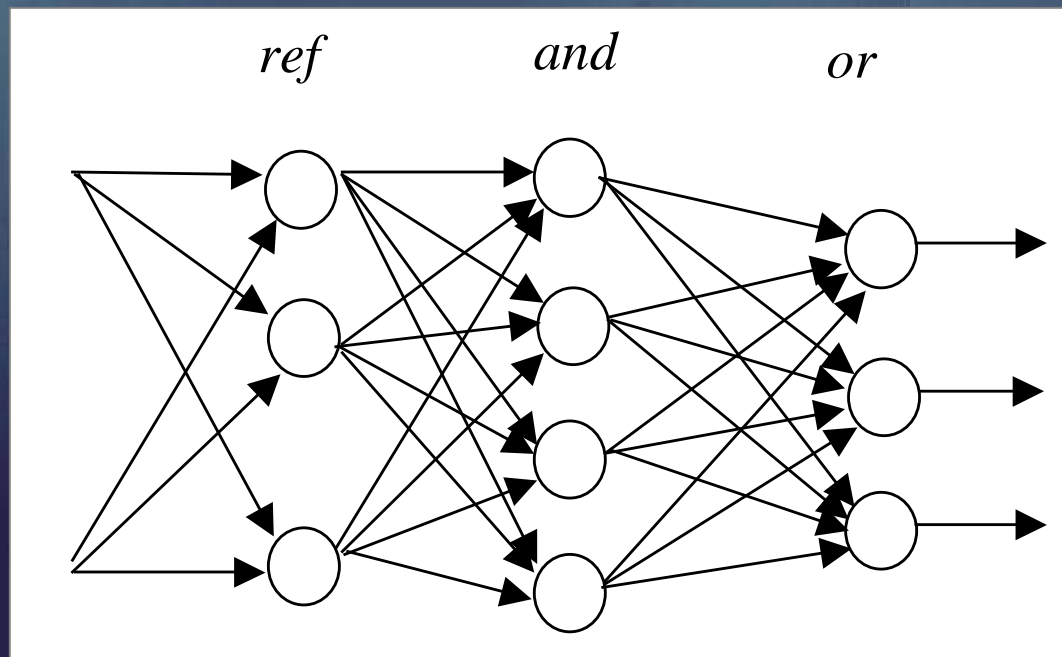
# Fuzzy neural networks: Examples of architectures

- Use of *and* and *or* neurons (logic processor)



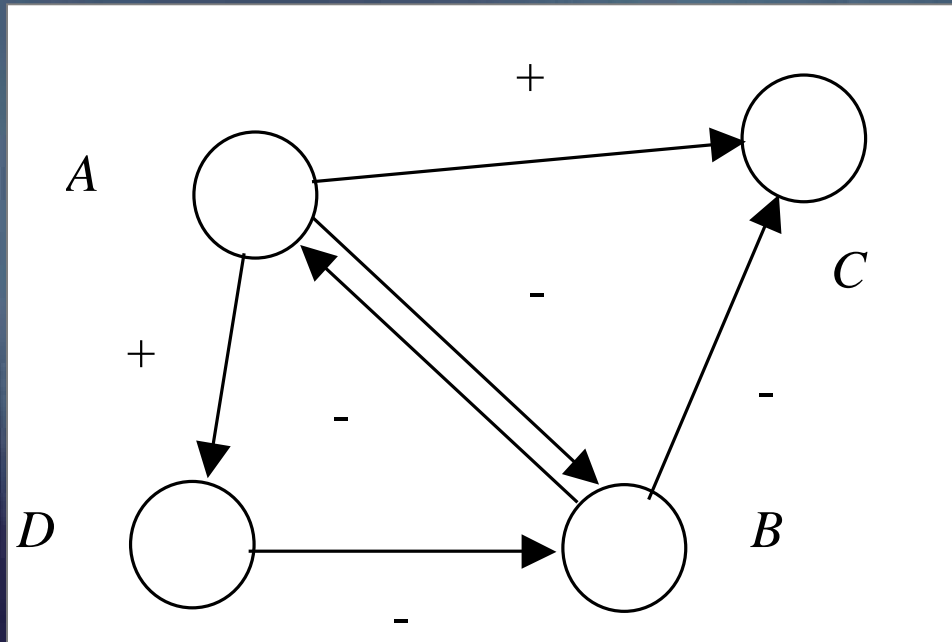
# Fuzzy neural networks: Examples of architectures

- Use of *and*, *or* and referential (*ref*) neurons



# Network of fuzzy processing units

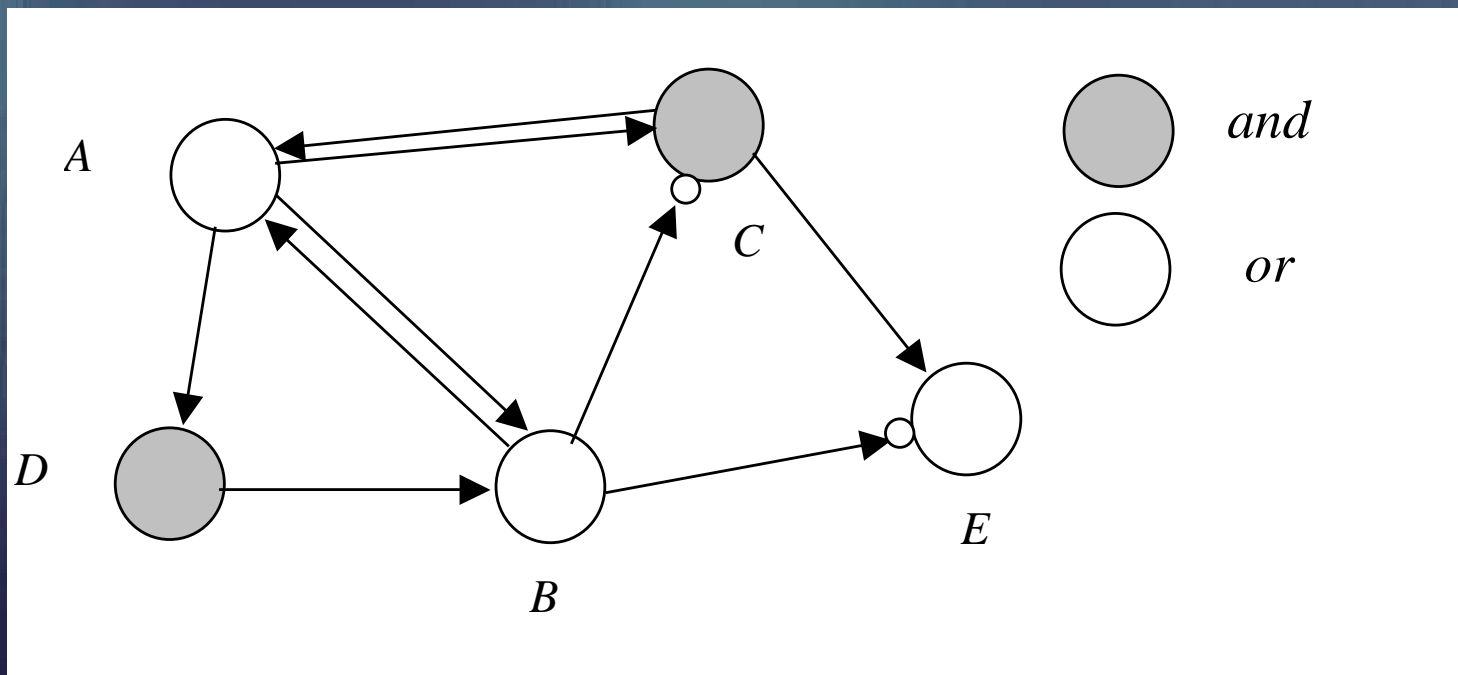
- Representation of concepts and linkages between concepts
- Directed graph: concepts are nodes; linkages are edges



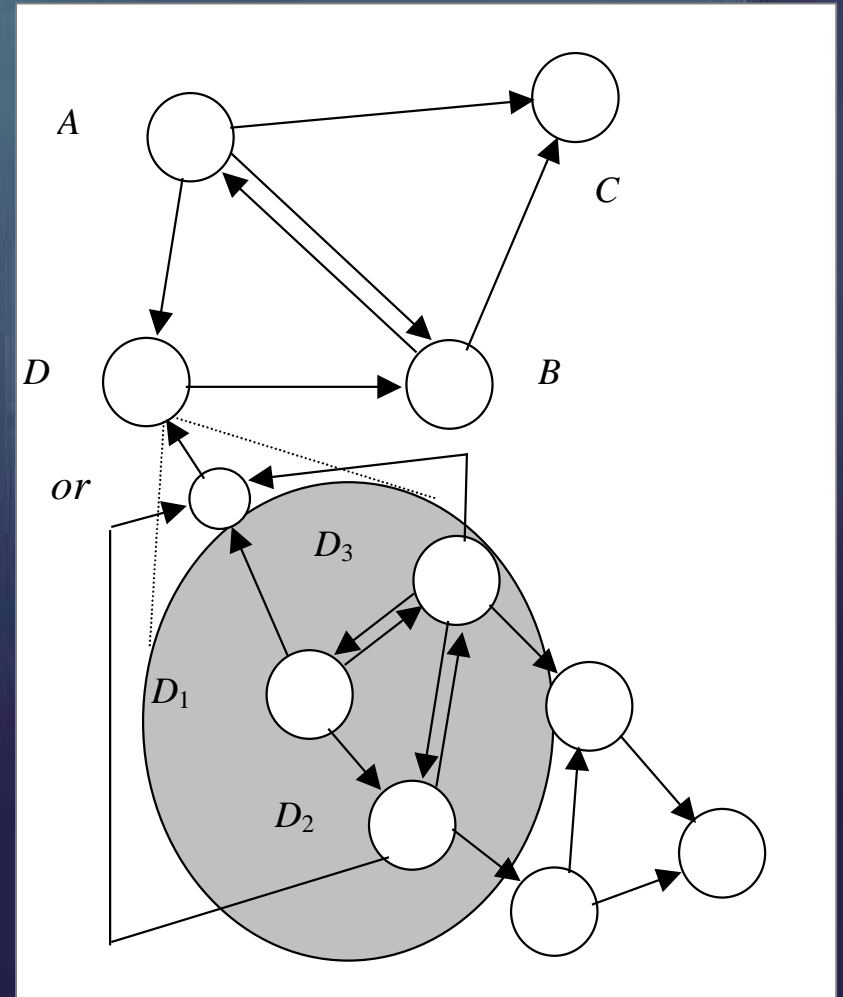
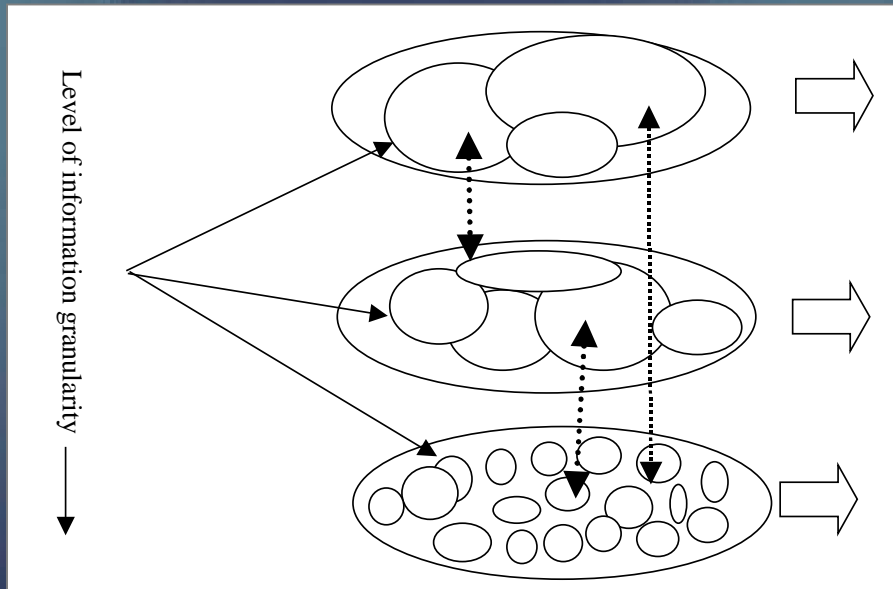
Fuzzy cognitive maps

- $A$ ,  $B$ ,  $C$ , and  $D$  = concepts.
- Inhibition (-) or excitation (+) between the concepts (nodes)

# Fuzzy cognitive maps: extensions



# Fuzzy cognitive maps: hierarchy



## **10.4 Verification and validation Of fuzzy models**

# Verification and validation of fuzzy models

- **Verification and Validation** (V&V) are concerned with the development of the model and assessment of its usefulness
- **Verification** is concerned with the analysis of the underlying processes of constructing the fuzzy model do we follow sound design principles ?

“Are we building the product *right*?”

- **Validation** is concerned with ensuring that the model (product) meets the requirements of the customer

“Are we building the *right* product?”

# Verification of fuzzy models

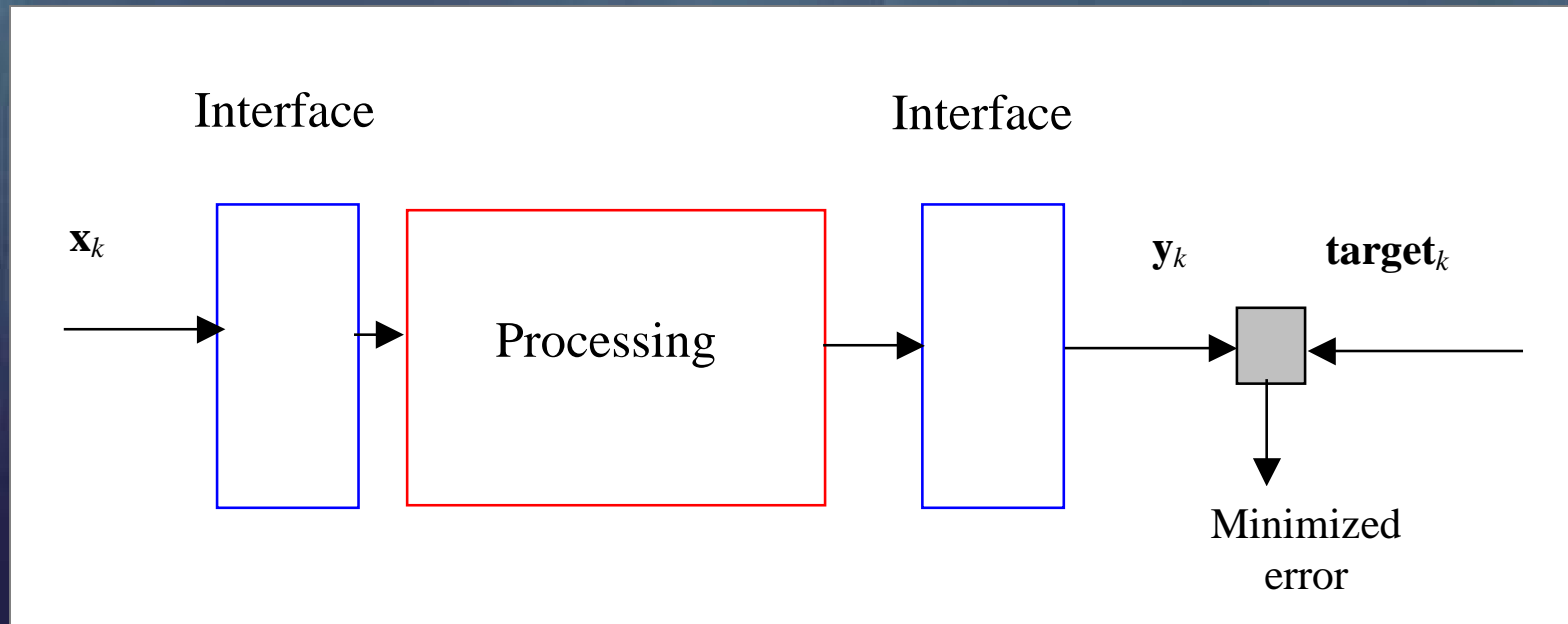
- Sound design principles
  - iterative development process
  - assessment of accuracy
  - generalization capabilities
  - complexity of the model (Occam's principle)
  - high level of autonomy of the model

# Fuzzy models: accuracy

- Two ways of expressing accuracy
  - numeric level
  - internal level

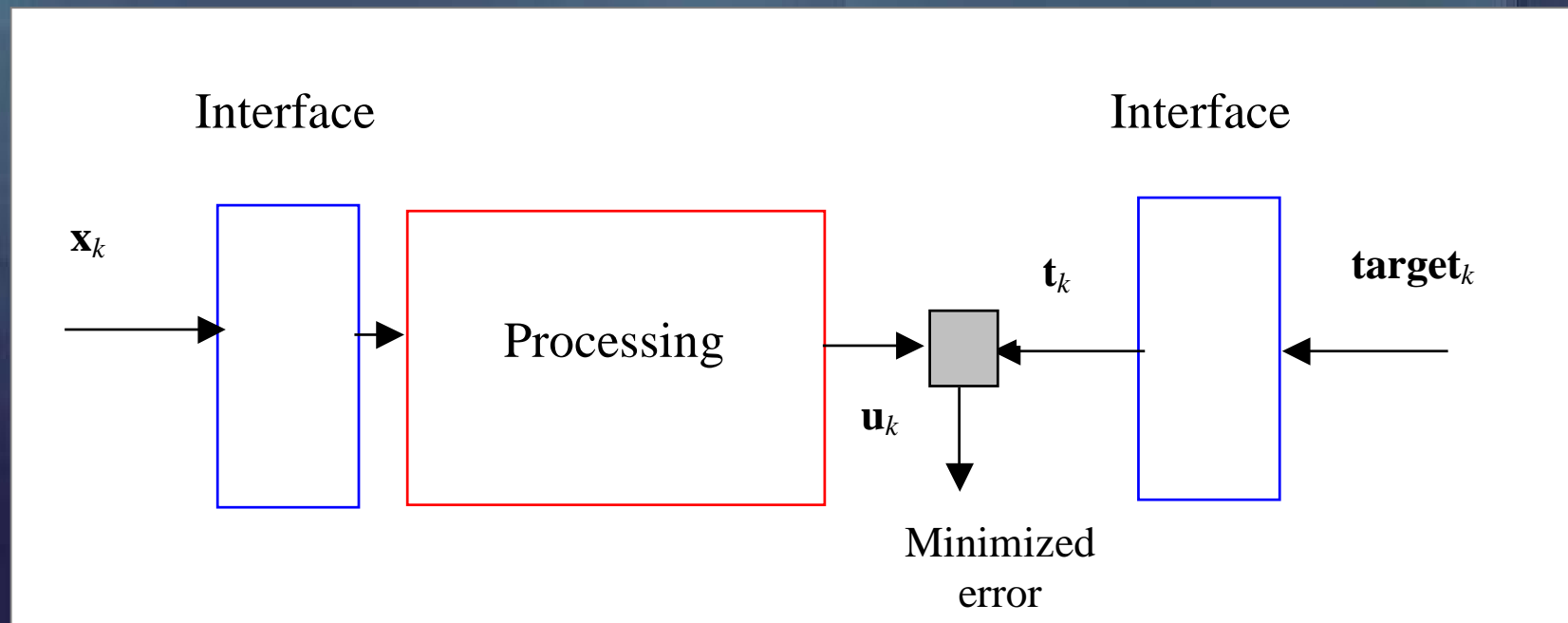
# Fuzzy models: accuracy

- Numeric level of expressing accuracy



# Fuzzy models: accuracy

- Accuracy expressed at the level of fuzzy sets



# Training, validation, and testing data

- To avoid potential bias in assessment of accuracy, data are split into
  - training
  - validation
  - testing subsets
- Training - testing
  - typically 60-40% split
  - 10 fold cross-validation (90-10% split)
  - leave one out strategy

# Validation of fuzzy models

- Are we building the *right* model?
- More difficult to quantify:
  - transparency of fuzzy models
  - stability of the fuzzy model
- ....
- Very often validation criteria are in conflict

