

# 13 Fuzzy Systems and Computational Intelligence

*Fuzzy Systems Engineering  
Toward Human-Centric Computing*

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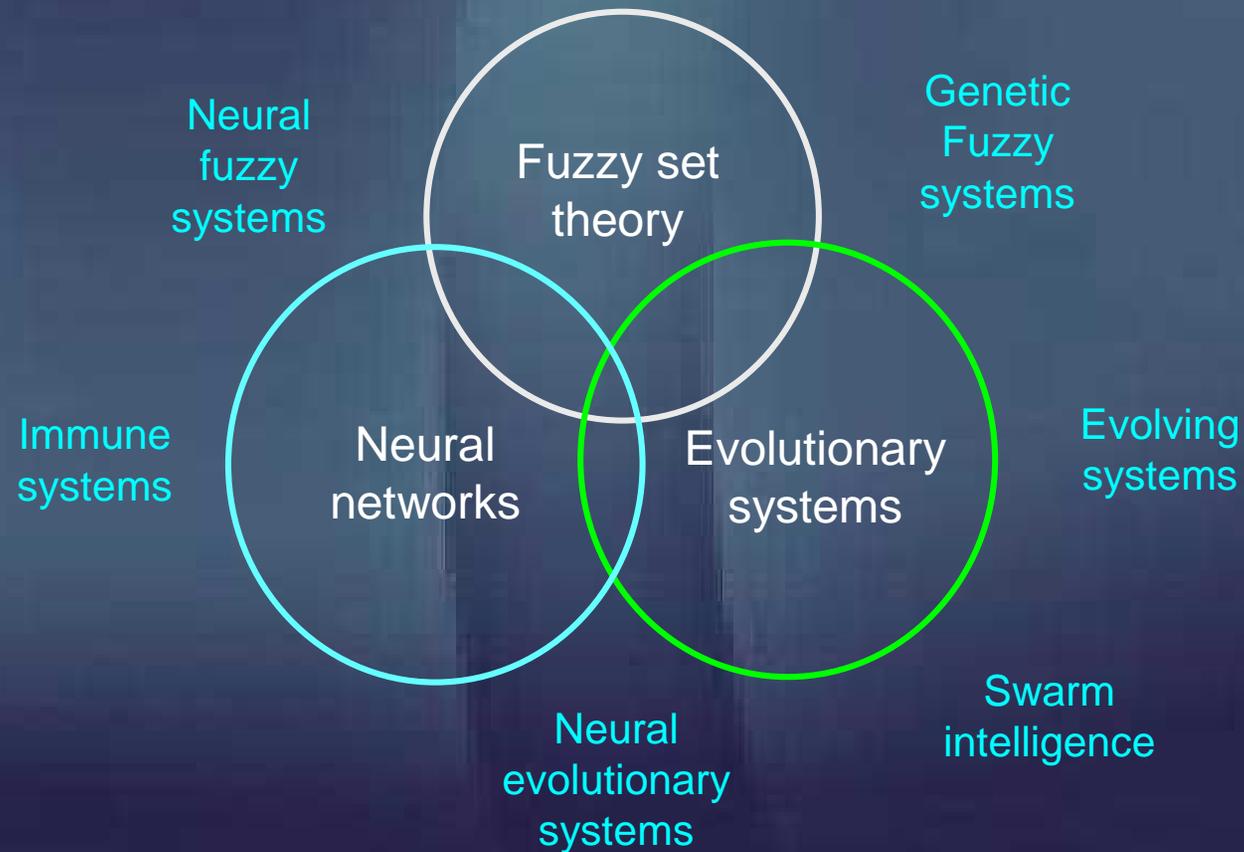
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# 13.1 Computational intelligence

# Computational Intelligence

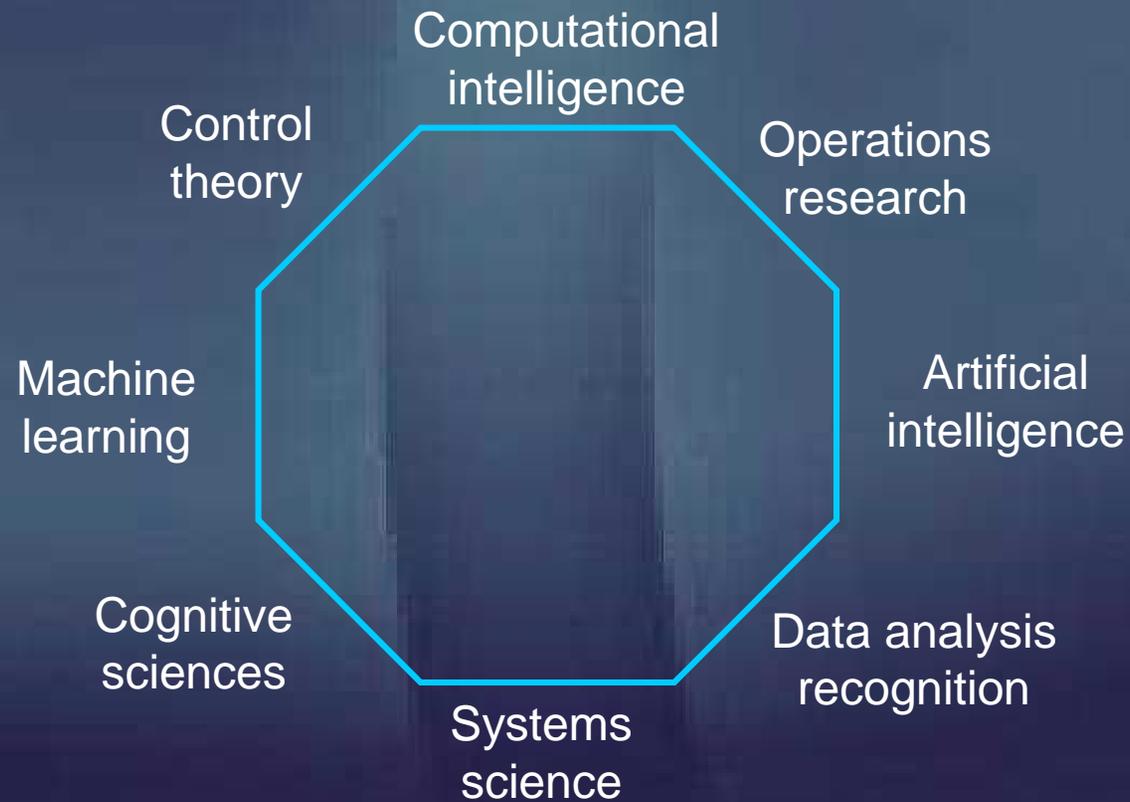


# Computational intelligence

- Data processing systems with capabilities of (Bezdek, 1992/1994)
  - pattern recognition
  - adaptive
  - fault tolerance
  - performance approximates human performance
  - no use of explicit knowledge
- Framework to design and analyze intelligent systems (Duch, 2007)
  - autonomy
  - learning
  - reasoning

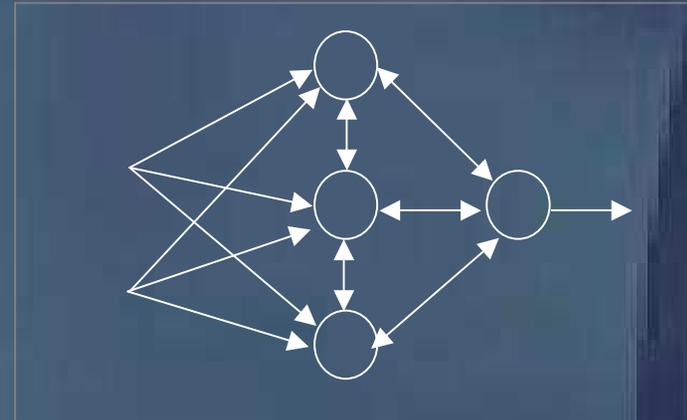
- Computing systems able to (Eberhart, 1996)
  - learn
  - deal with new situations using
    - reasoning
    - generalization
    - association
    - abstraction
    - discovering
  
- Computational intelligence
  - largely human-centered
  - forms of artificial and synthetic intelligence
  - collaboration man-machine

# Intelligent Systems

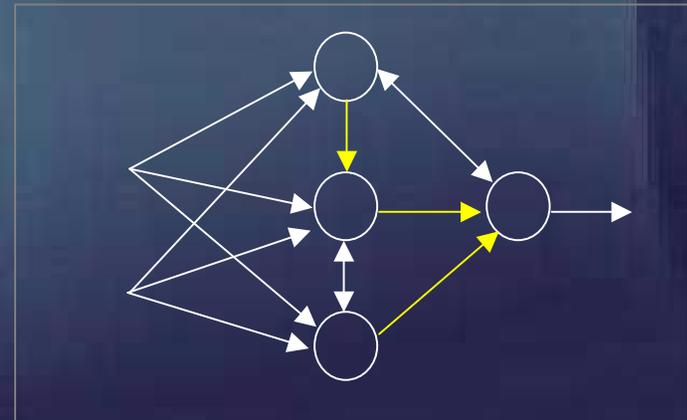


## 13.2 Recurrent neurofuzzy systems

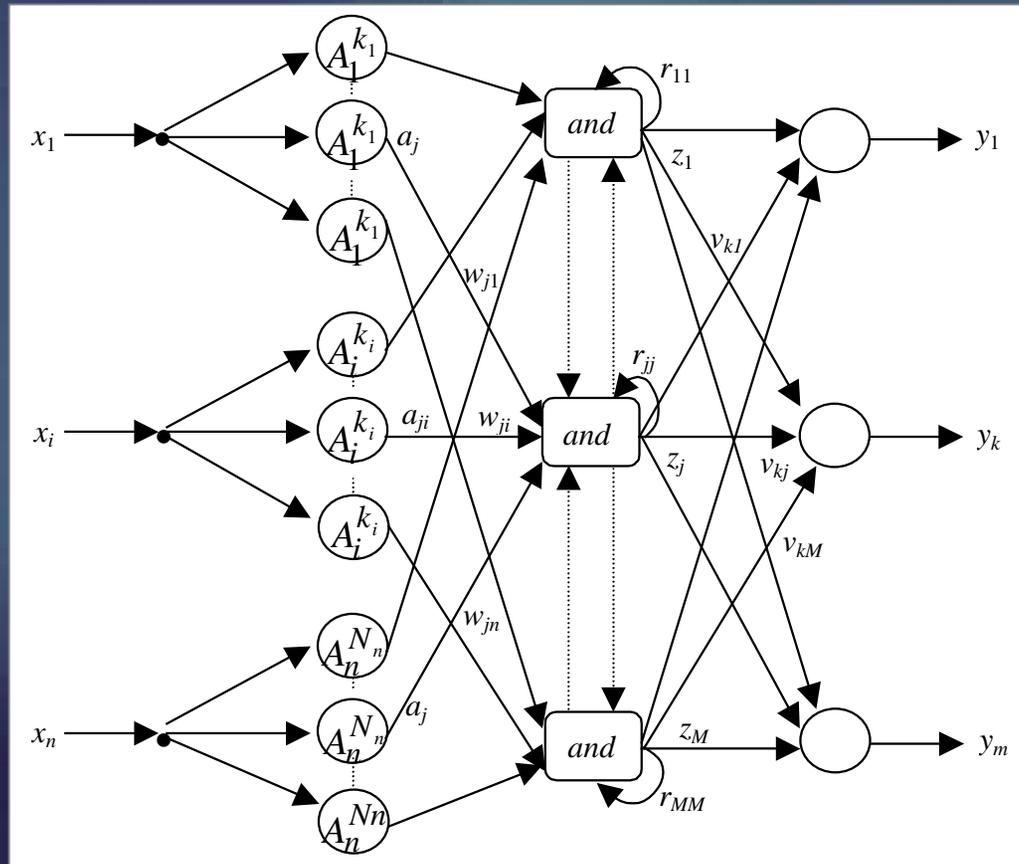
- Globally recurrent
  - full feedback connections



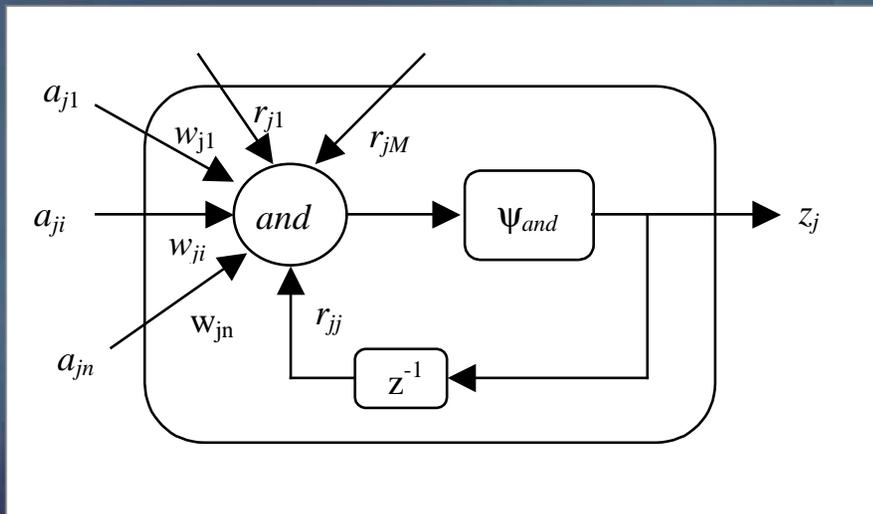
- Partially recurrent
  - partial feedback connections



# Recurrent neural fuzzy network model



## Recurrent *and* fuzzy neuron



$$z_j = \prod_{i=1}^{n+M} (w_{ji} s a_{ji})$$

$$z_j = \text{AND}(\mathbf{a}_j; \mathbf{w}_j)$$

- $N_i$  number of fuzzy sets that granulate the  $i$ th input
- $j$  indexes *and* neurons; given  $k_i$ ,  $j$  is found using

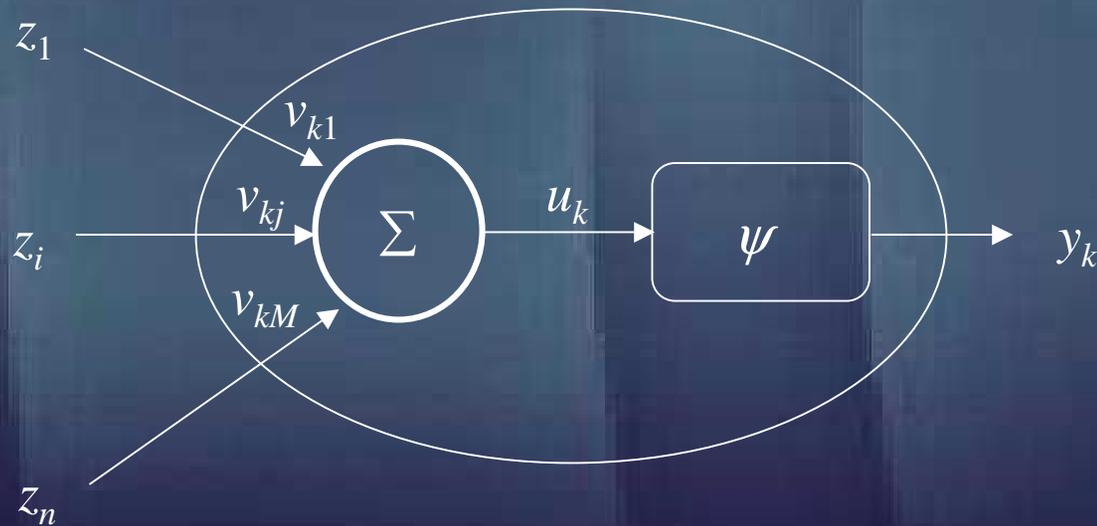
$$j = k_n + \sum_{i=2}^M (k_{(n-i+1)} - 1) \left( \prod_{r=1}^{i-1} N_{(n+1-r)} \right)$$

- $x_1, \dots, x_i, \dots, x_n$  inputs

- $a_{ji} = A_i^{k_i}(x_i)$

- $w_{ji}$  weight between  $i$ th input and  $j$ th *and* neuron
- $z_j$  output of the  $j$ th *and* neuron
- $v_{kj}$  weight  $j$ th input of the  $k$ th output neuron
- $r_{jl}$  feedback connection of the  $l$ th input of the  $j$ th *and* neuron
- $y_k = \psi(u_k)$  output  $k$ th neuron of the output layer

# Output layer neuron



$$y = \psi(u_k) = \psi\left(\sum_{j=1}^M v_{kj} z_j\right)$$

# Learning algorithm

**procedure** NET-LEARNING ( $x, y$ ) **returns** a network

**input:** data  $x, y$

**local:** fuzzy sets

$t, s$ : triangular norms

$\varepsilon$ : threshold

GENERATE-MEMBERSHIP-FUNCTIONS

INITIALIZE-NETWORK-WEIGHTS

**until** stop criteria  $\leq \varepsilon$  **do**

choose and input-output pair  $x$  and  $y$  of the data set

ACTIVE-AND-NEURONS

ENCODING

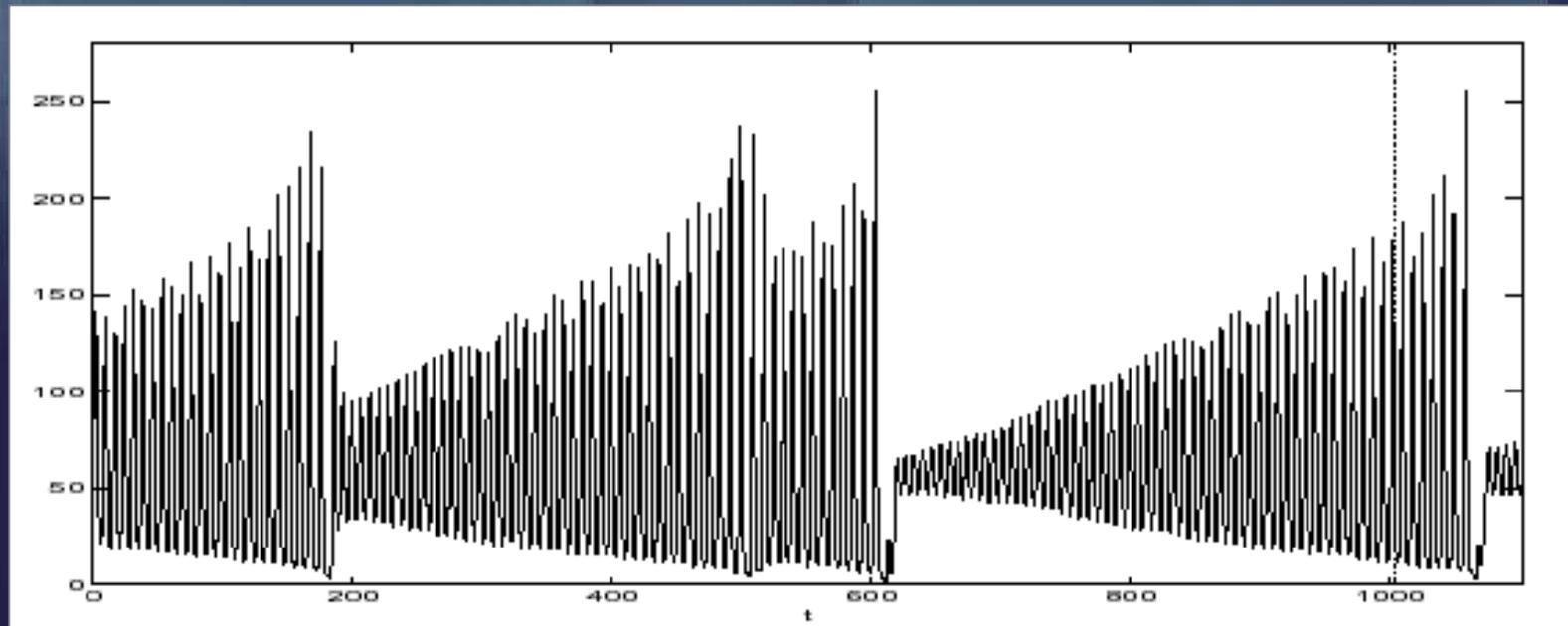
UPDATE-WEIGHTS

**return** a network

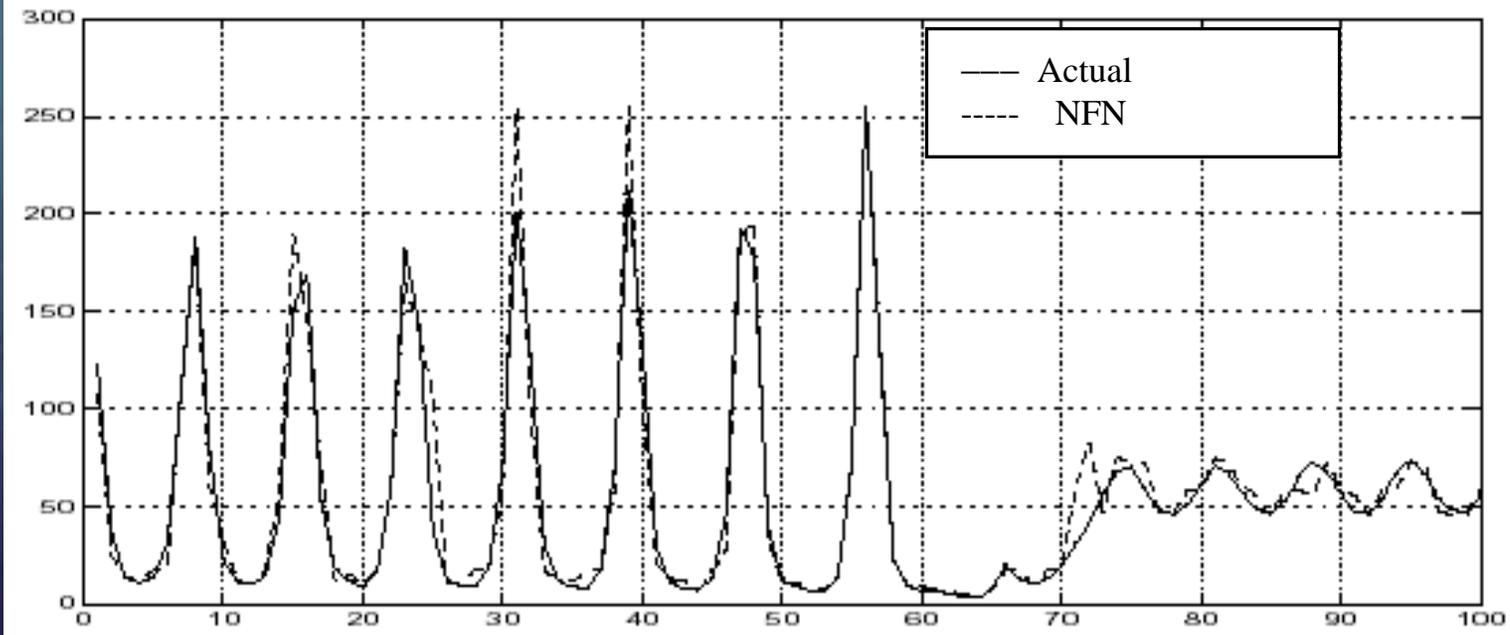
# Example

Chaotic NH<sub>3</sub> laser time series data

- first 1000 samples for learning
- predict next 100 steps



# 100 steps ahead prediction



## Normalized squared forecasting errors (NSE) NH3 laser time series

Model	1 step ahead	100 steps ahead
FIR	0.0230	0.0551
NFN	0.0139	0.0306

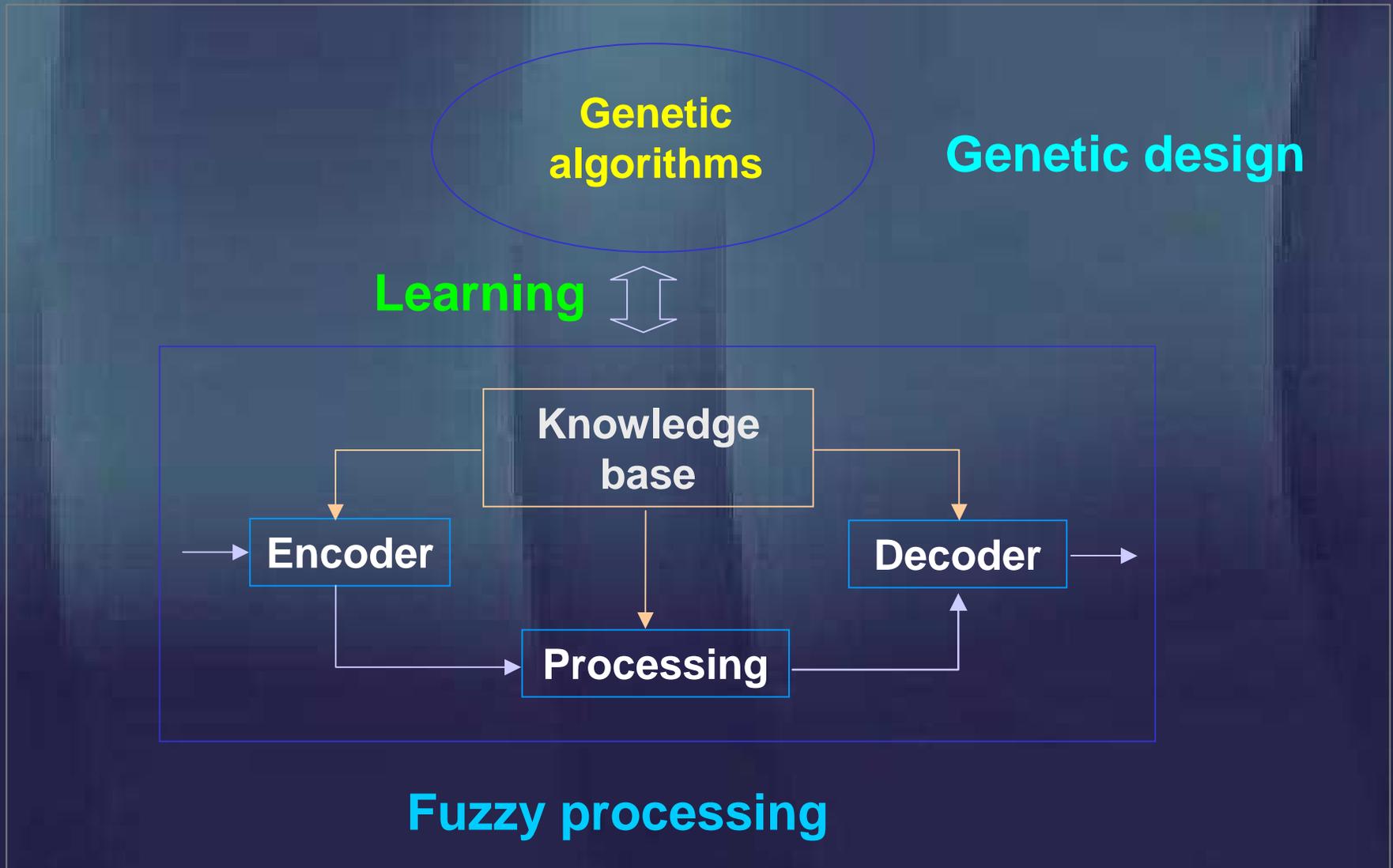
$$NSE = \frac{1}{\sigma^2 N} \sum_{i=1}^N (y_i - \hat{y}_i)^2$$

## 13.3 Genetic fuzzy systems

# Genetic fuzzy systems

- GFS is an approach to design fuzzy models and systems
- GFS = fuzzy system + learning using genetic algorithm
- Learning of models structure and parameters
  - rule base
  - fuzzy rules
  - membership functions
  - operators
  - inference procedures

# Genetic Fuzzy Systems

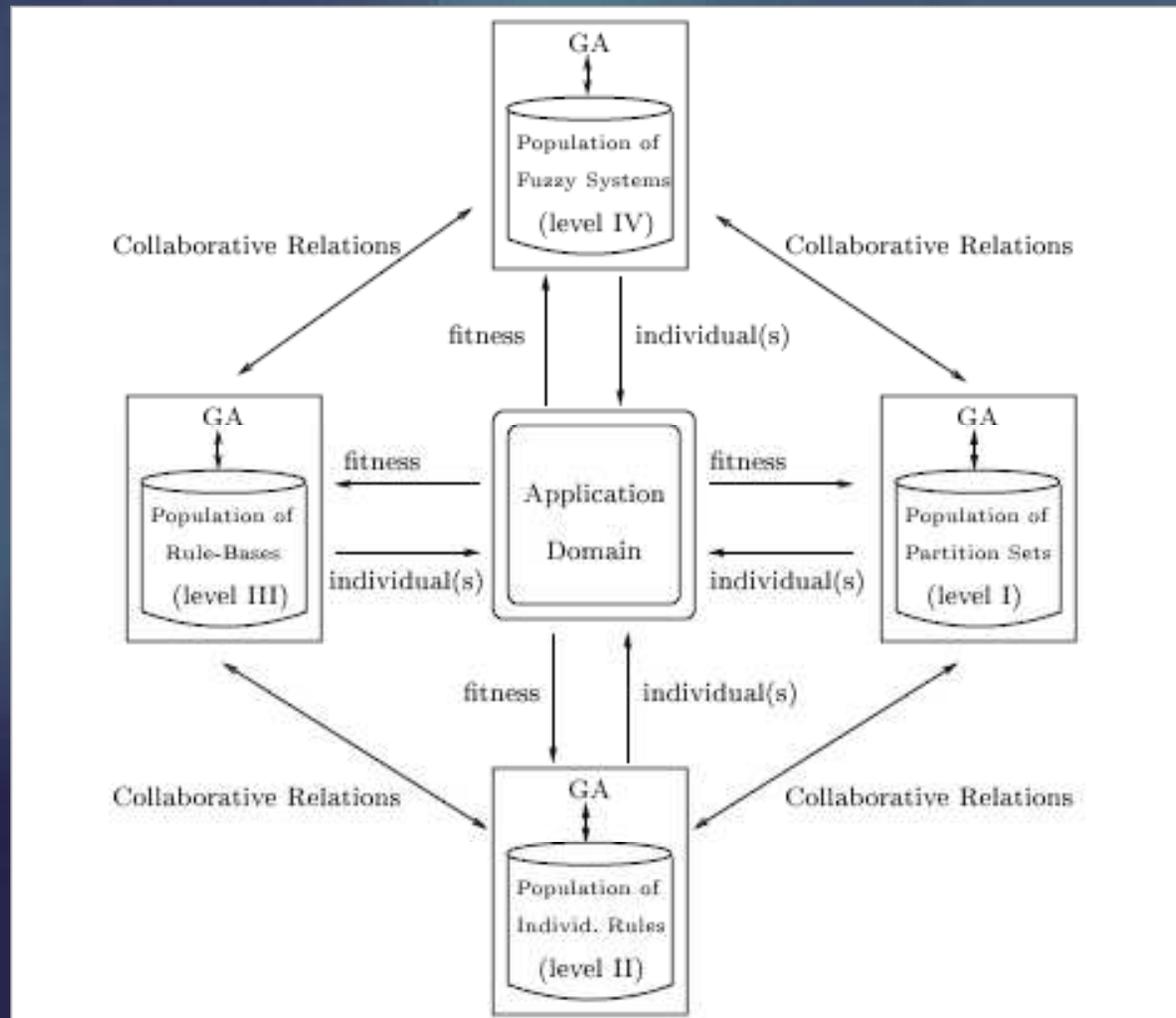


# 13.4 Coevolutionary hierarchical genetic fuzzy system

# Coevolution

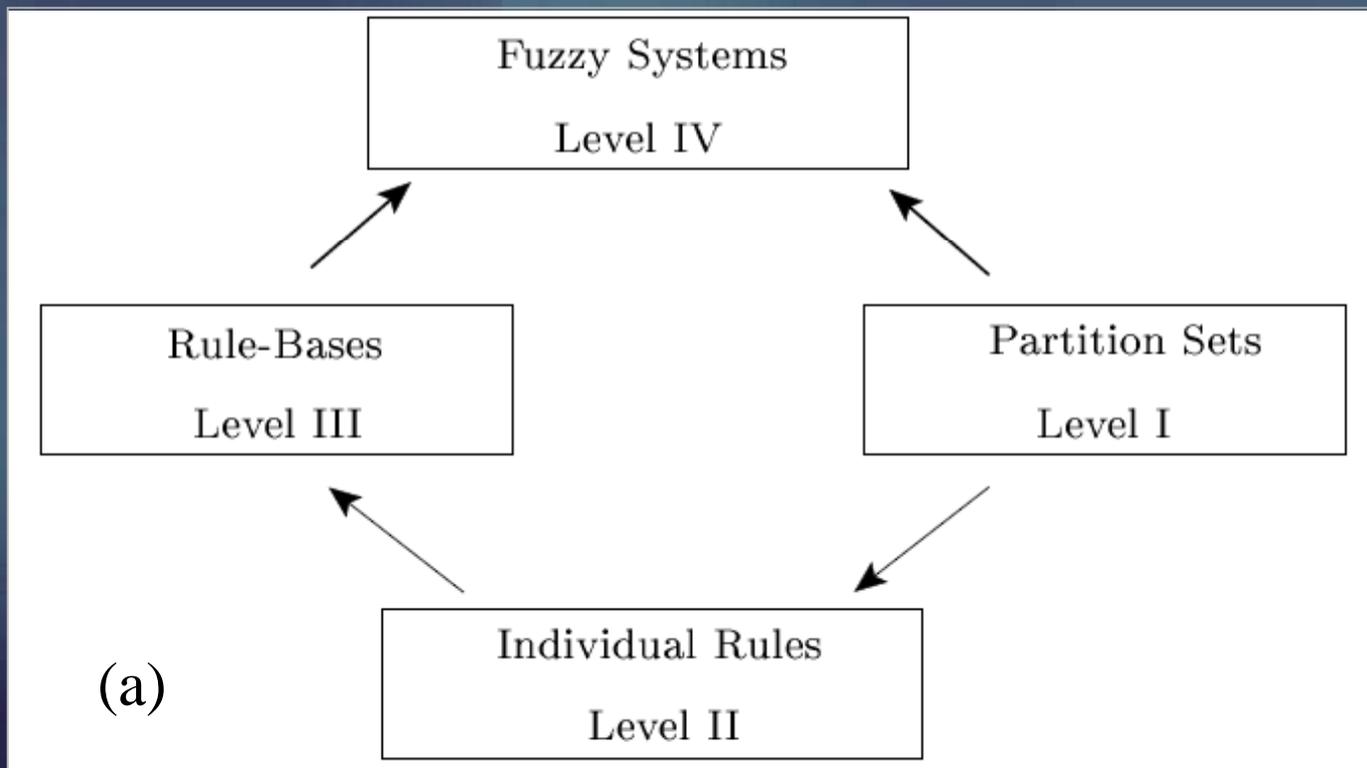
- Considers interactions between population members
- Populations hierarchically structured
- Hierarchy levels associated with partial solutions of the problem
  - individuals of different populations keep collaborative relations
  - collaboration depends on the fitness of the individuals
  - hierarchical levels:
    - I : membership functions
    - II : fuzzy rules
    - III: rule bases
    - IV: fuzzy systems (models)

# Coevolutionary GFS approach

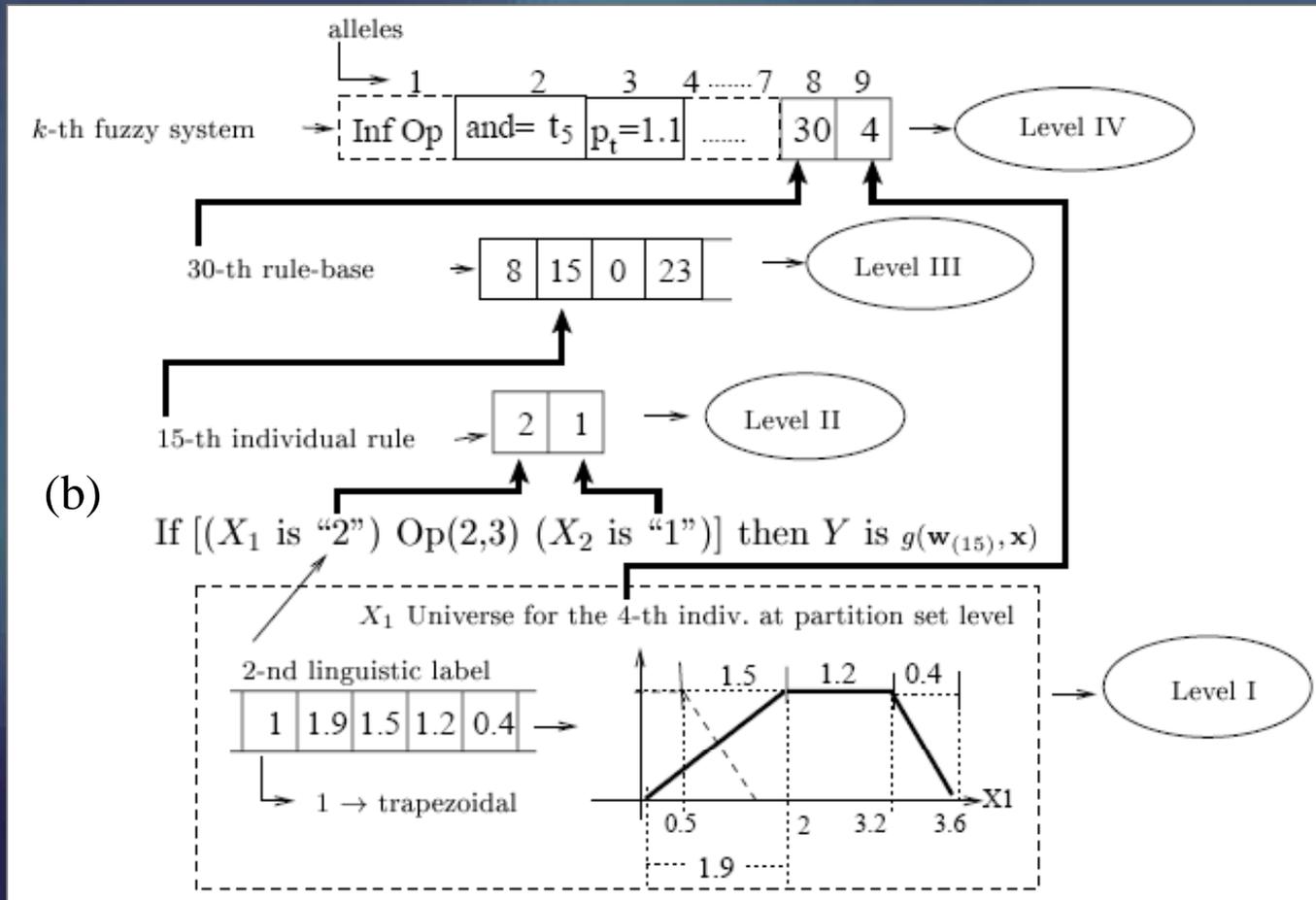


# 13.5 Hierarchical collaborative relations

# Collaboration between species

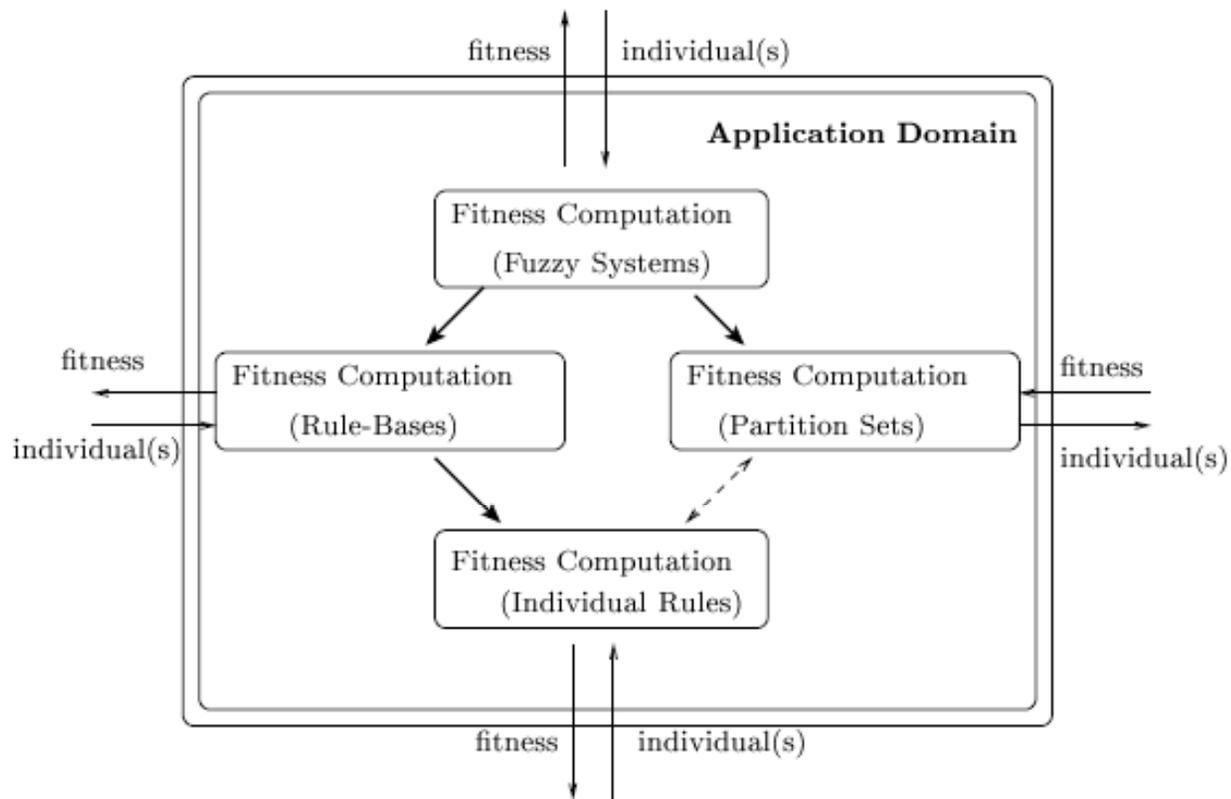


# Collaboration between individuals



$$R_j: \text{If } x_1 \text{ is } A_1^j \text{ and ...and } x_n \text{ is } A_n^j \text{ then } y = g(w_j, \mathbf{x})$$

# Fitness evaluation in hierarchical collaborative evolution



# Example: function approximation

$R_j$ : If  $x_1$  is  $A_1^j$  *and* ...*and*  $x_n$  is  $A_n^j$  then  $y = g(w_j, \mathbf{x})$

*and* = t-norm

$$a \text{ t } b = \frac{ab}{p_t + (1 - p_t)(a + b - ab)}$$

$p_t$  : obtained by coevolution

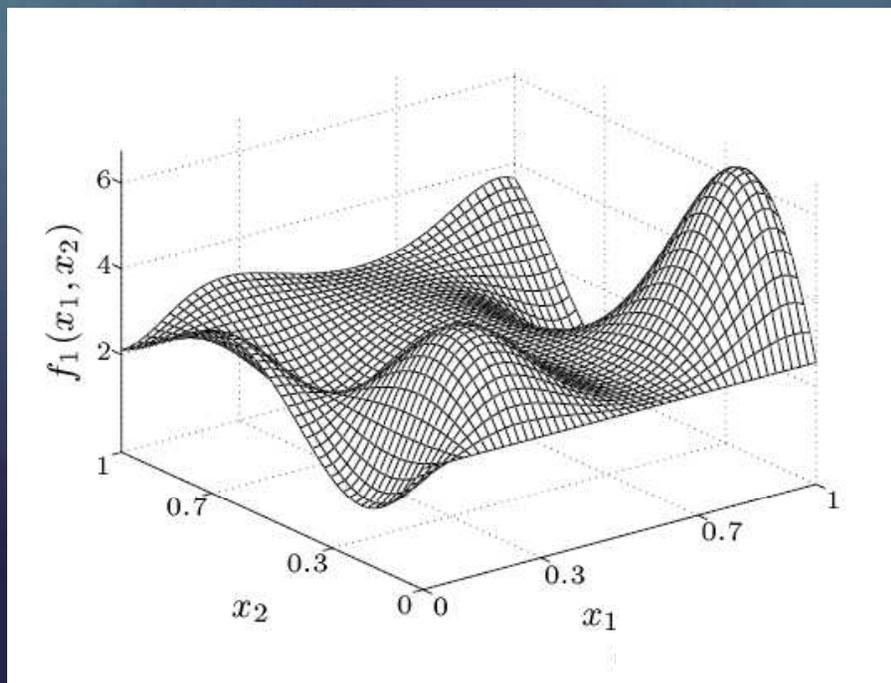
$g(w_j, \mathbf{x})$  : least squares + pruning

$$F_1 : \Omega \rightarrow R$$

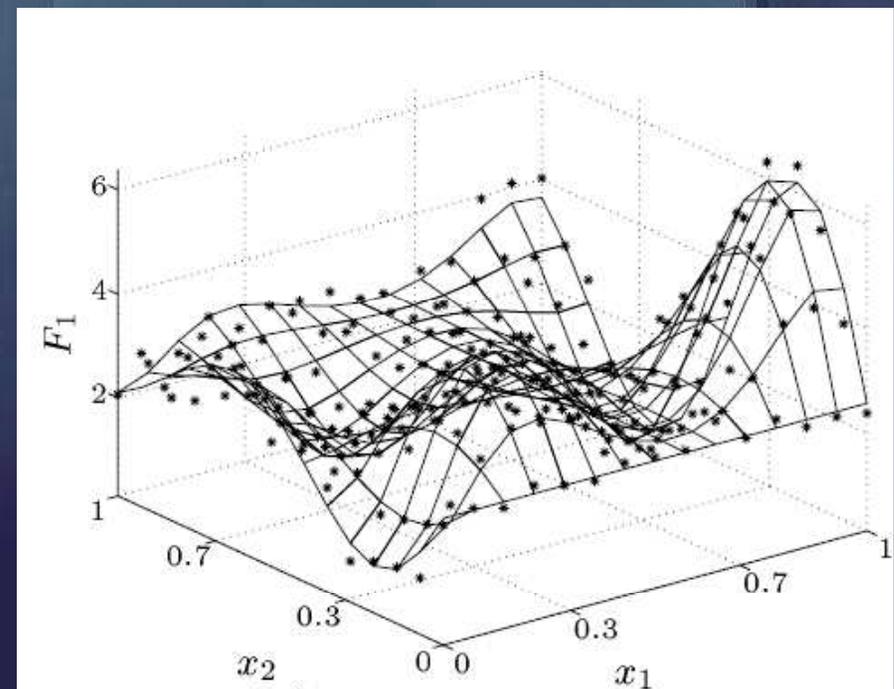
$$F_1(x_1, x_2) = f_1(x_1, x_2) + N(m, \sigma)$$

$$f_1(x_1, x_2) = 1.9(1.35 + \exp(x_1) \sin[13(x_1 - 0.6)^2 \exp(-x_2) \sin(7x_2)])$$

$$\Omega = [0,1], m = 0, \sigma = 0.3$$

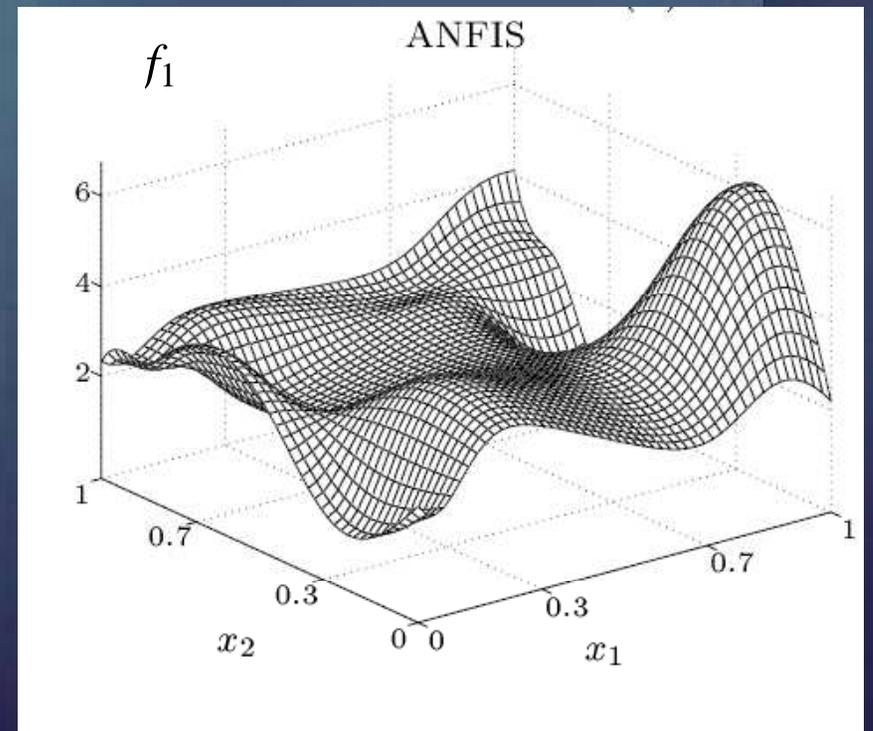
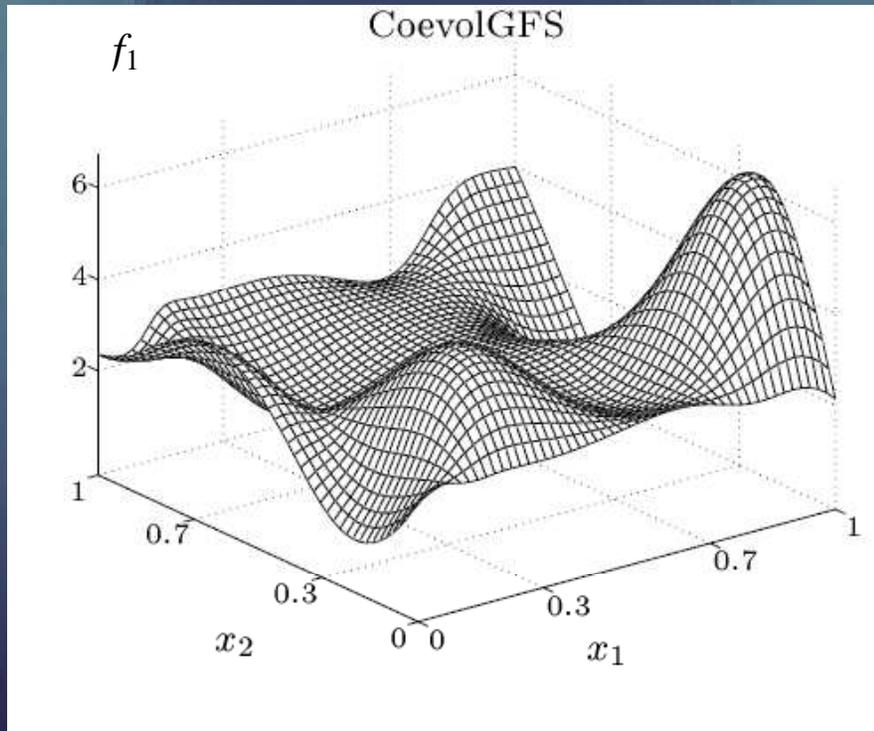


Original function



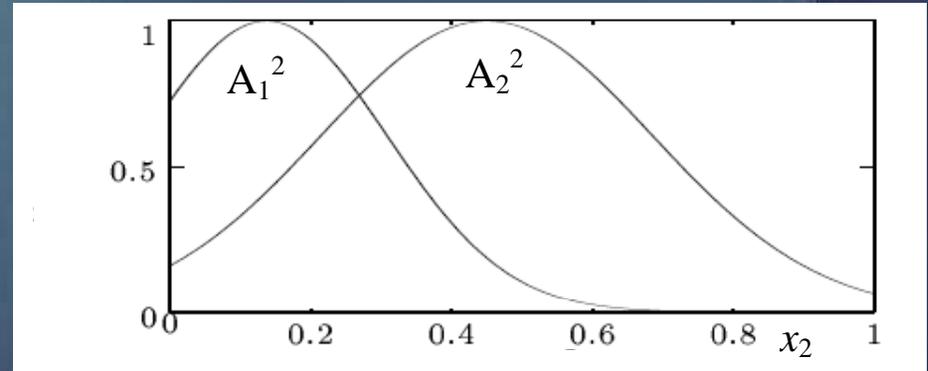
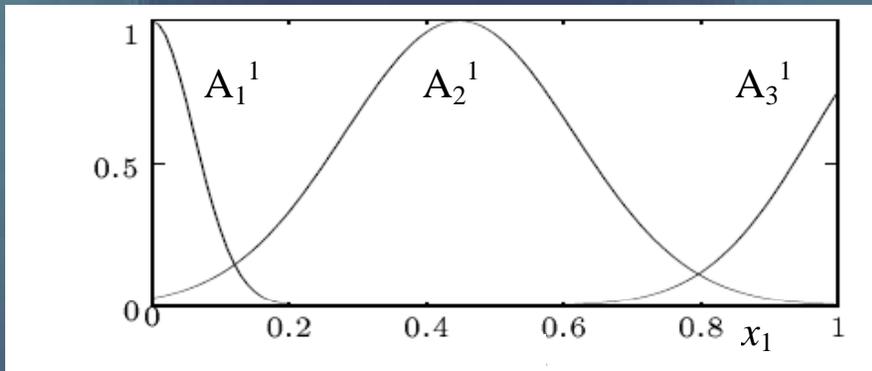
Training data

# Result

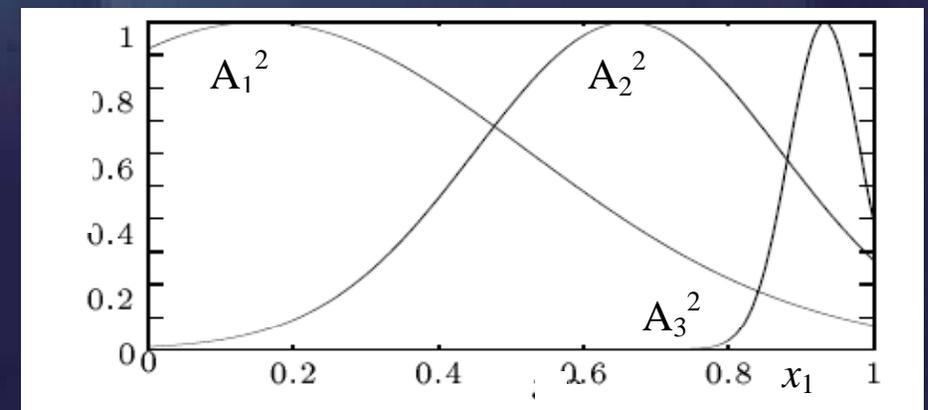
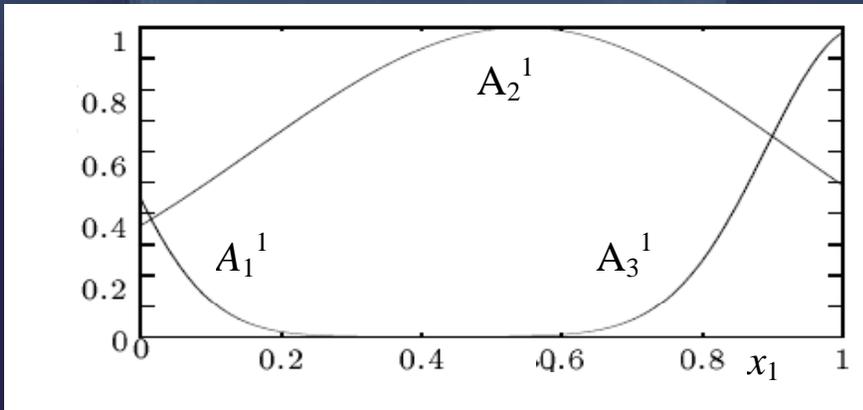


# Partitions

## CoevoIGFS



## ANFIS

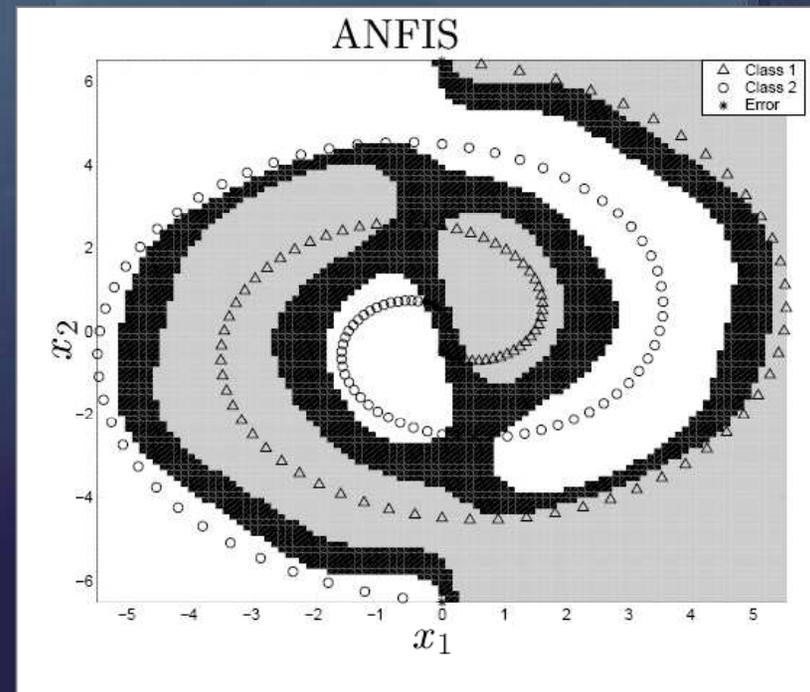
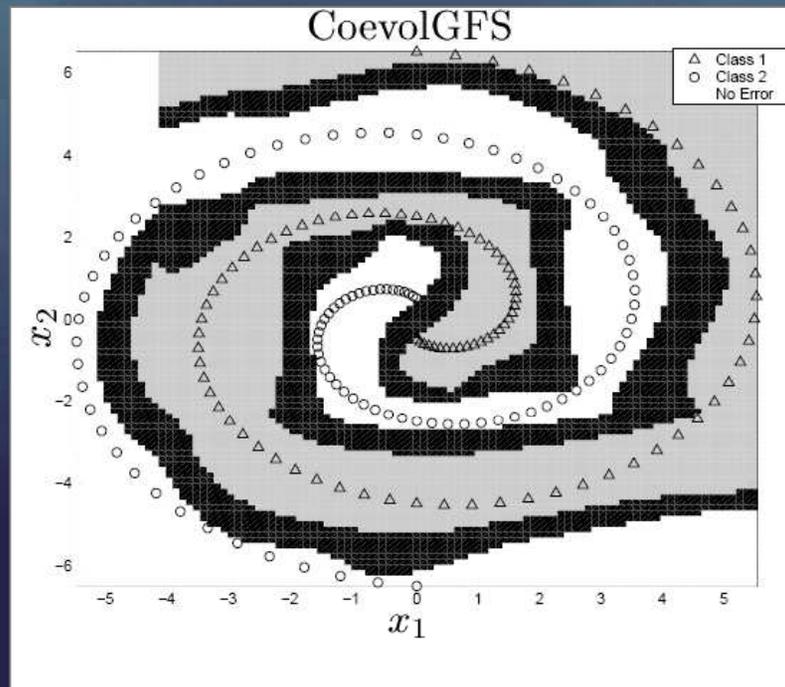


## RME for function approximation example

Approach	Training RME	Test RME	Number of Rules
CoevoIGFS	0.25	0.13	8
ANFIS	0.32	0.21	9

# Example: classification

- Intertwined spirals



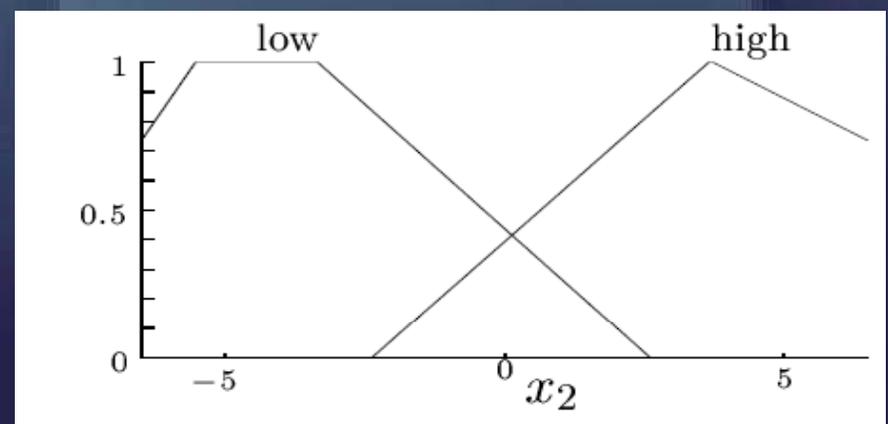
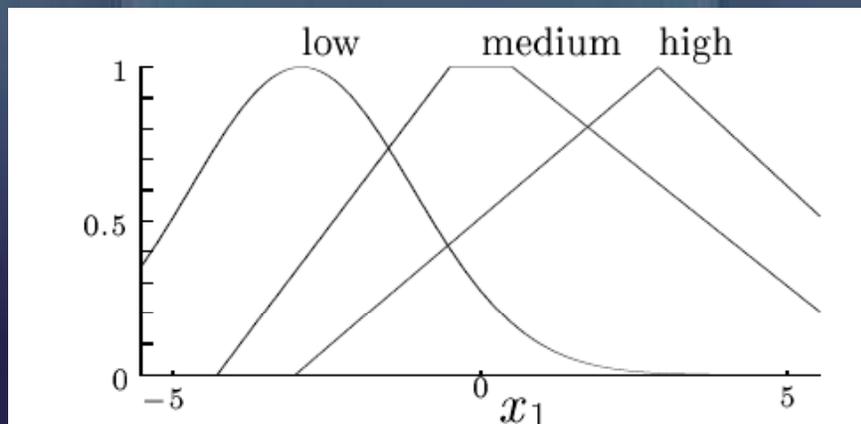
# Classification rules

$R_1$  :If  $x_1$  is low *and*  $x_2$  is low then  $y = -0.31 + 1.6x_1 - 0.26x_2 + 0.34x_1^2 + 0.17x_2^2 - 0.1x_1x_2$

$R_2$  :If  $x_1$  is medium *and*  $x_2$  is low then  $y = 15.3 - 1.3x_1 + 7.7x_2 - 0.05x_1^2 + 0.84x_2^2 - 0.46x_1x_2$

$R_3$  :If  $x_1$  is medium *and*  $x_2$  is high then  $y = -17.2 - 2.2x_1 + 7.6x_2 - 0.08x_1^2 - 0.78x_2^2 + 0.45x_1x_2$

$R_4$  :If  $x_1$  is high *and*  $x_2$  is high then  $y = 1.14 + 2.0x_1 + 1.24x_2 - 0.25x_1^2 - 0.28x_2^2 - 0.34x_1x_2$



# Classification performance: Intertwined spirals

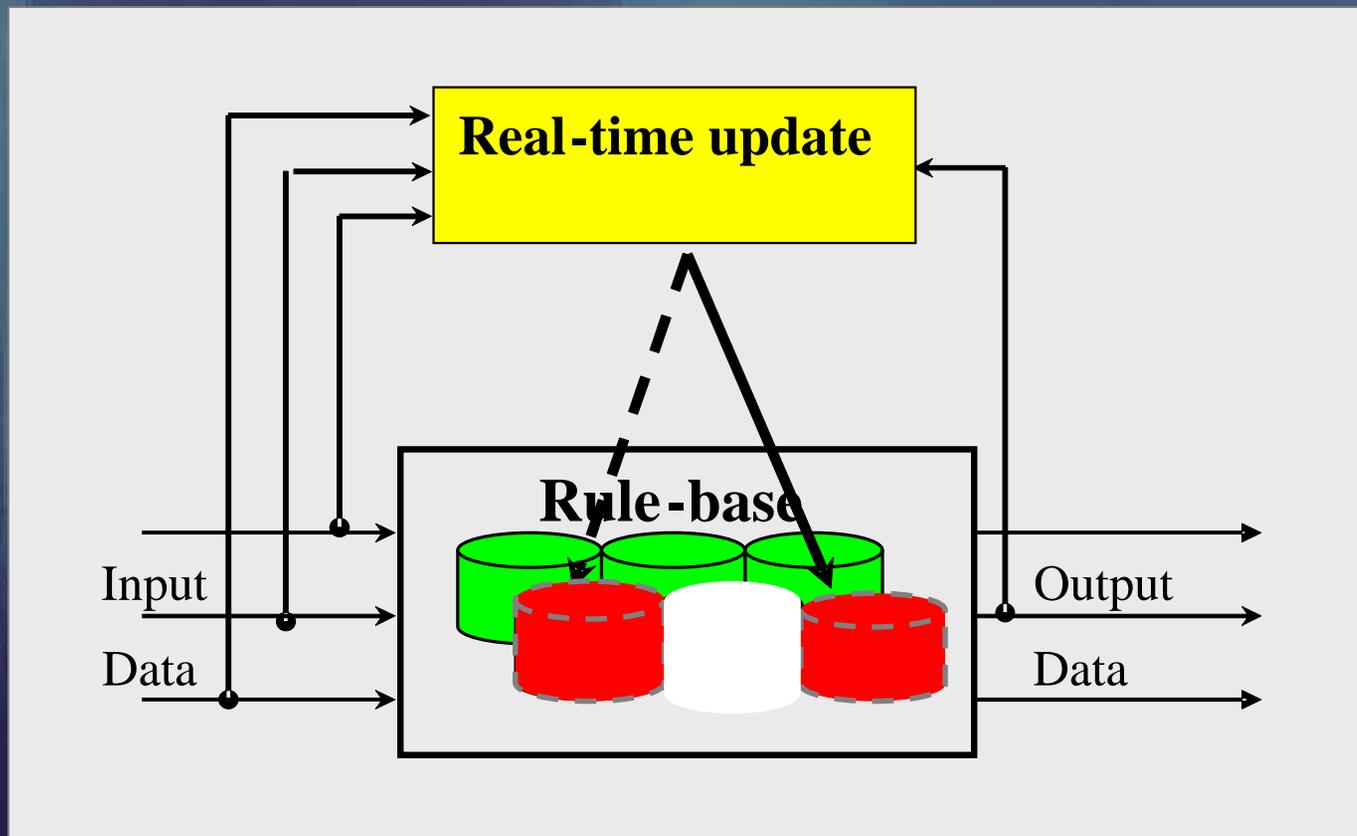
Approach	Cycles	Misclassification	Number of Rules
CoevoIGFS	529	18	9
ANFIS	1000	0.21	9

# 13.6 Evolving fuzzy systems

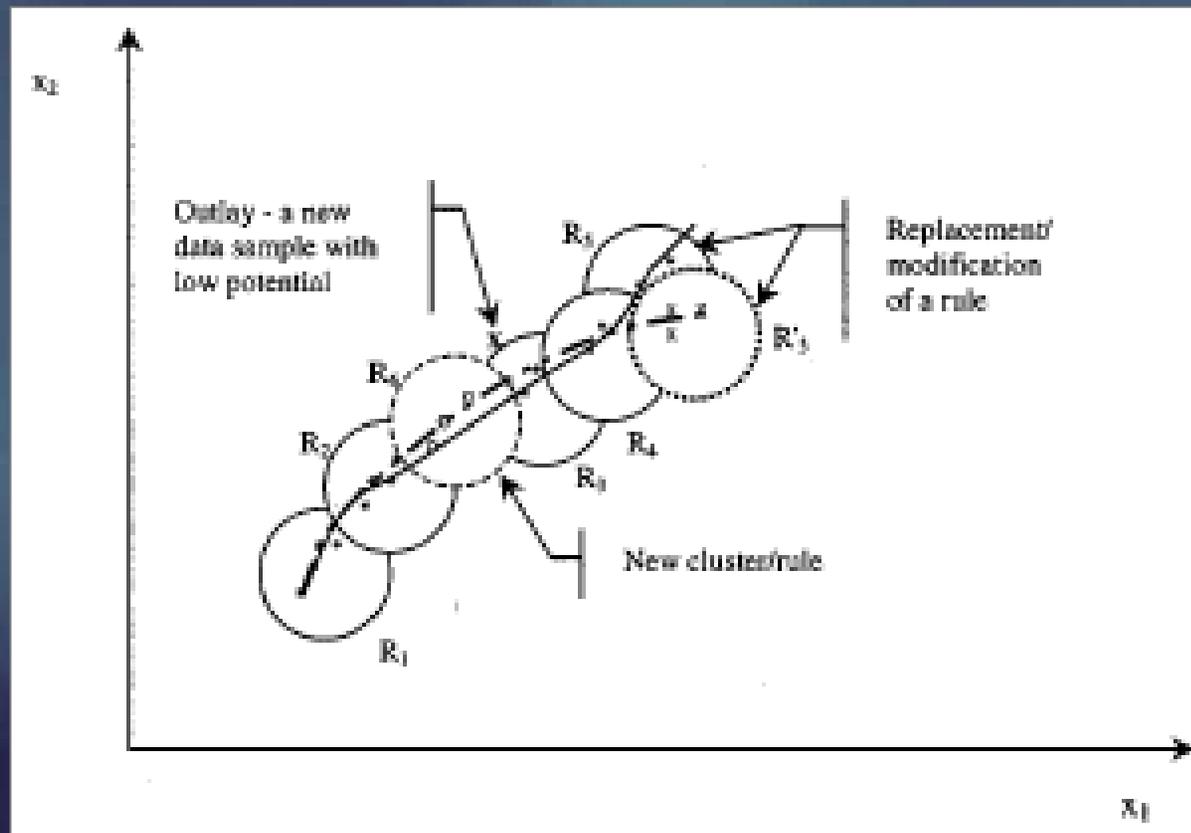
# Evolving fuzzy systems

- Evolving systems: an approach to develop adaptive fuzzy models
- Evolving modeling targets nonstationary process and systems
- Main properties
  - inherit new knowledge
  - gradual changes
  - life-long learning
  - self organization of the system structure
  - complements GFS approach
  - may act online

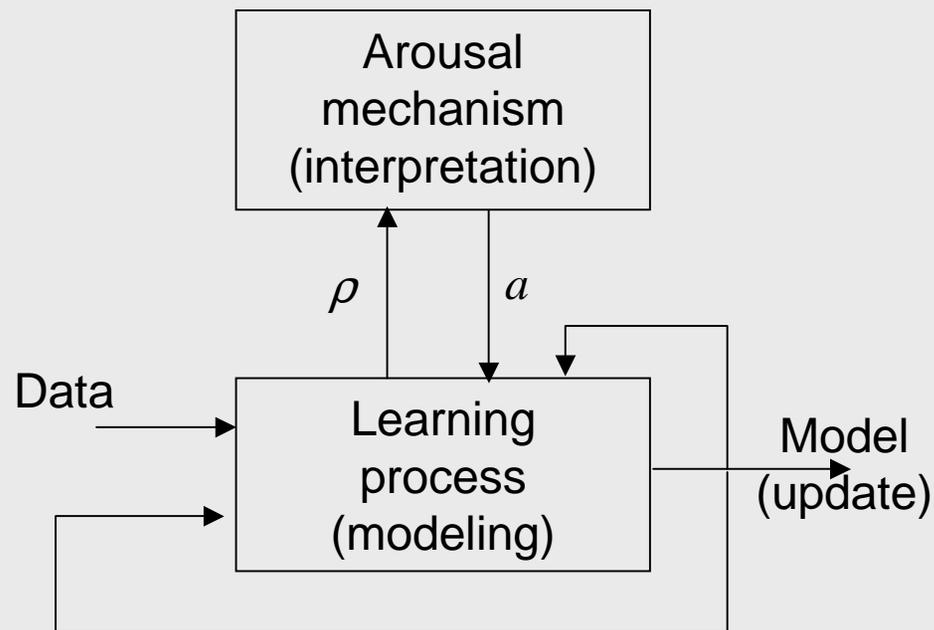
# Rule base evolution



# Recursive clustering



# Participatory learning



(Details in Chapter 14)

# Functional fuzzy models

$$R_i : \text{if } \mathbf{x} \text{ is } \mathbf{A}_i \text{ then } y_i = a_{i0} + \sum_{j=1}^n a_{ij}x_j$$

$$A_j^i(x_j) = \exp[-k_{ij}(x_j - v_{ij})^2]$$

$$y = \sum_{i=1}^c w_i y_i$$

$$w_i(x) = \frac{\lambda_i(x)}{\sum_{i=1}^c \lambda_i(x)}$$

$$\lambda_i = A_1^i(x_1) \cdot A_2^i(x_2) \cdot \dots \cdot A_n^i(x_n)$$

# Evolving participatory learning algorithm

**procedure** EVOLVE-PARTICIPATORY- LEARNING (x,y) **returns** an output

**input** : data x,y

**local:** antecedent parameters  
consequent parameters

INITIALIZE-RULES-PARAMETERS

**do forever**

read x

PL-CLUSTERING

UPDATE-RULE-BASE

RUN-LEAST-SQUARES(x,y)

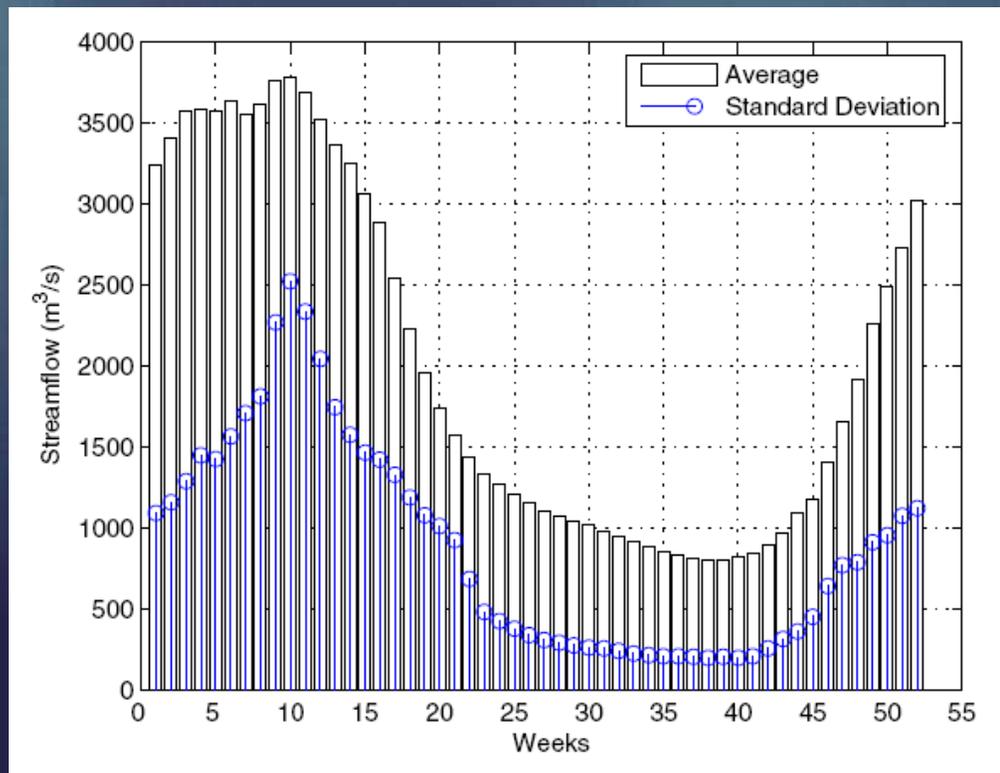
COMUTE-RULE-ACTIVATION

COMPUTE-OUTPUT

**return** y

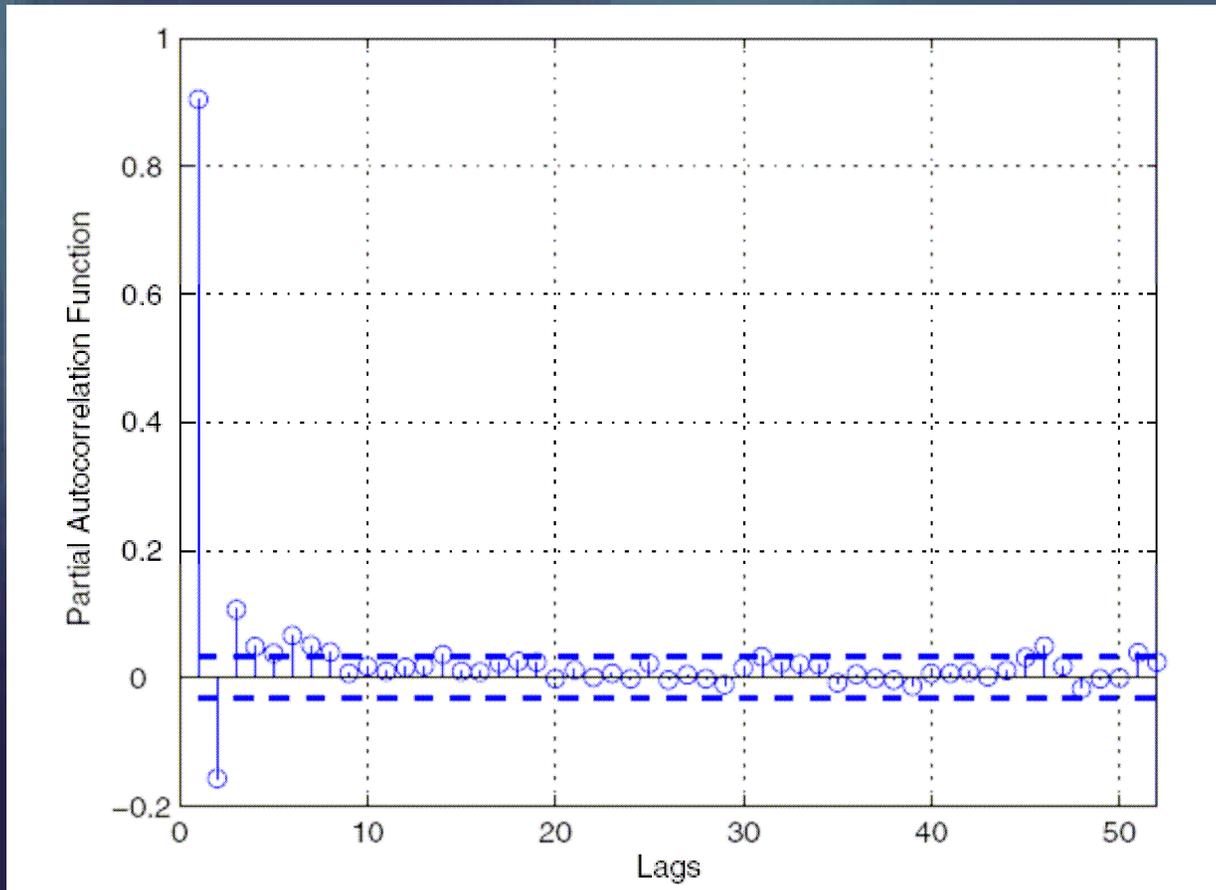
# Example

## Time series forecasting



Average weekly inflows of a power plant

# Estimated partial correlation



# Performance measures

$$RMSE = \sqrt{\frac{1}{P} \sum_{k=1}^P (x^k - x_d^k)^2}$$

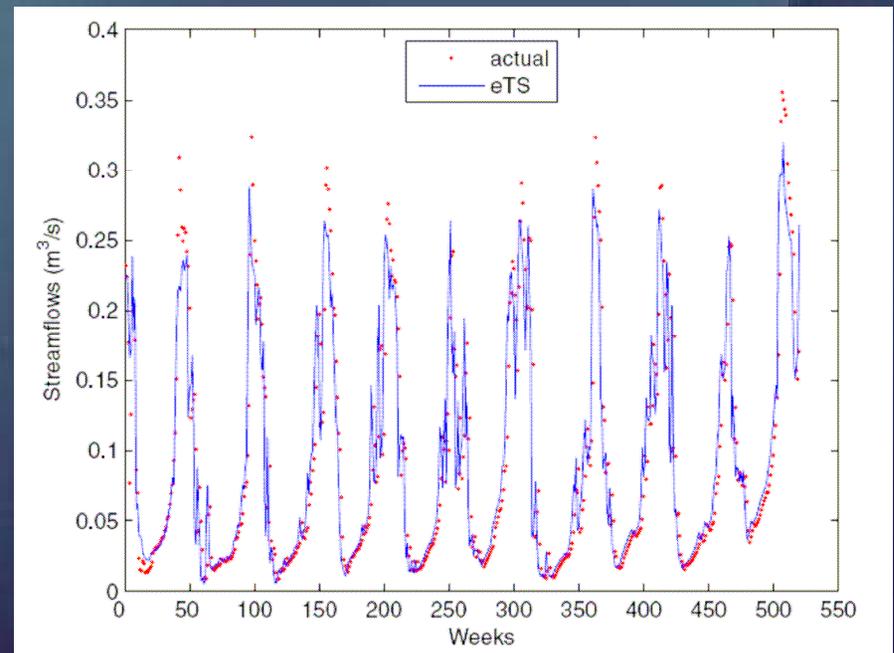
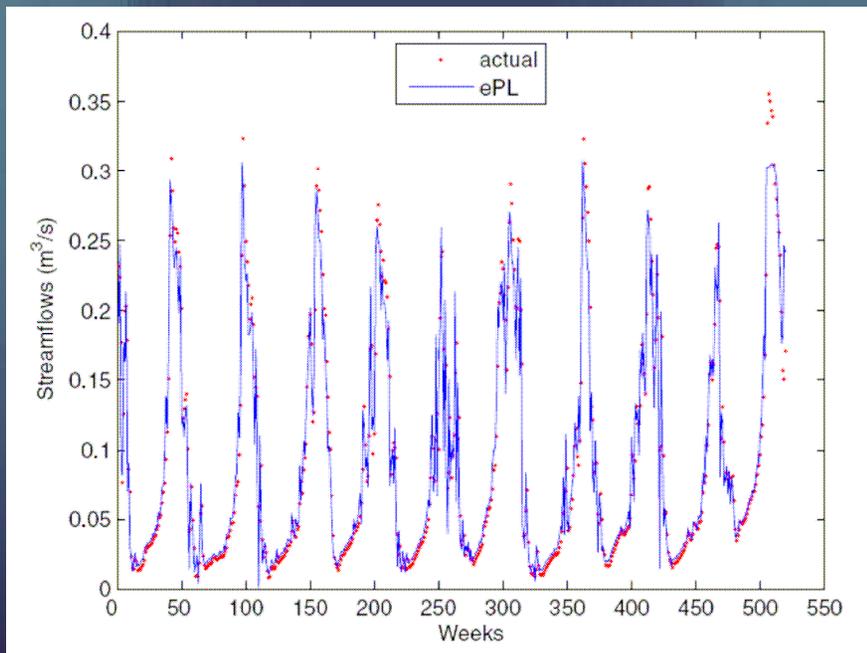
$$MRE = \frac{100}{P} \sum_{k=1}^P \frac{|x^k - x_d^k|}{x_d^k}$$

$$MAD = \frac{1}{P} \sum_{k=1}^P |x^k - x_d^k|$$

$$RE_{\max} = 100 \max \left( \frac{|x^k - x_d^k|}{x_d^k} \right)$$

$$\rho = \frac{\sum_{k=1}^P (x_d^k - \bar{x}_d)(x^k - \bar{x})}{\sqrt{\sum_{k=1}^P (x_d^k - \bar{x}_d)^2 \sum_{k=1}^P (x^k - \bar{x})^2}}$$

# Result



# Forecasting performance average weekly inflow

Error	Models	
	ePL	eTS
RMSE (m <sup>3</sup> /s)	378.71	545.28
MAD (%)	240.55	356.85
MRE (%)	12.54	18.42
RE <sub>max</sub> (%)	75.51	111.22
$\rho$	0.95	0.89
Number of rules	2	2