

10 Fuzzy Modeling: Principles and Methodology

*Fuzzy Systems Engineering
Toward Human-Centric Computing*

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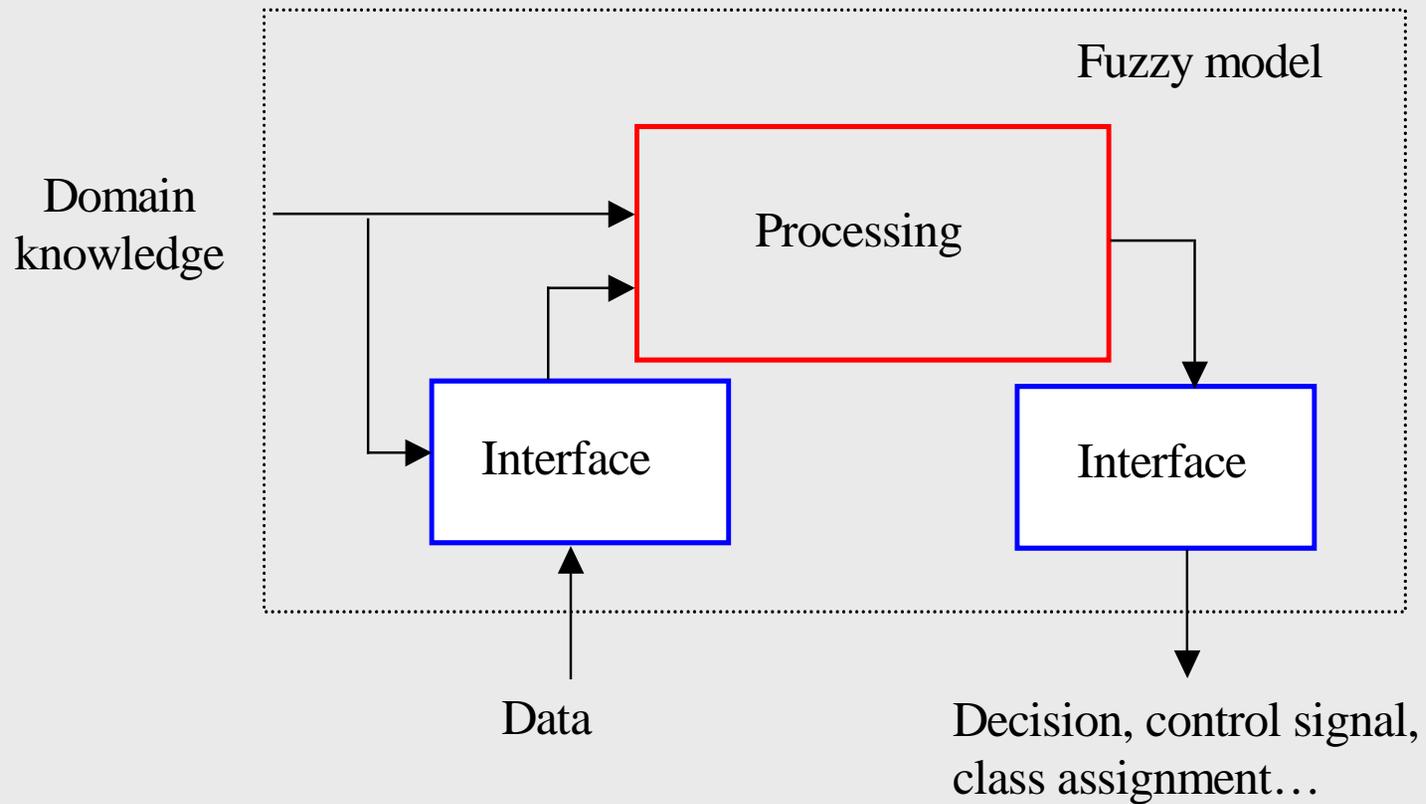
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10.1 The architectural blueprint of fuzzy models

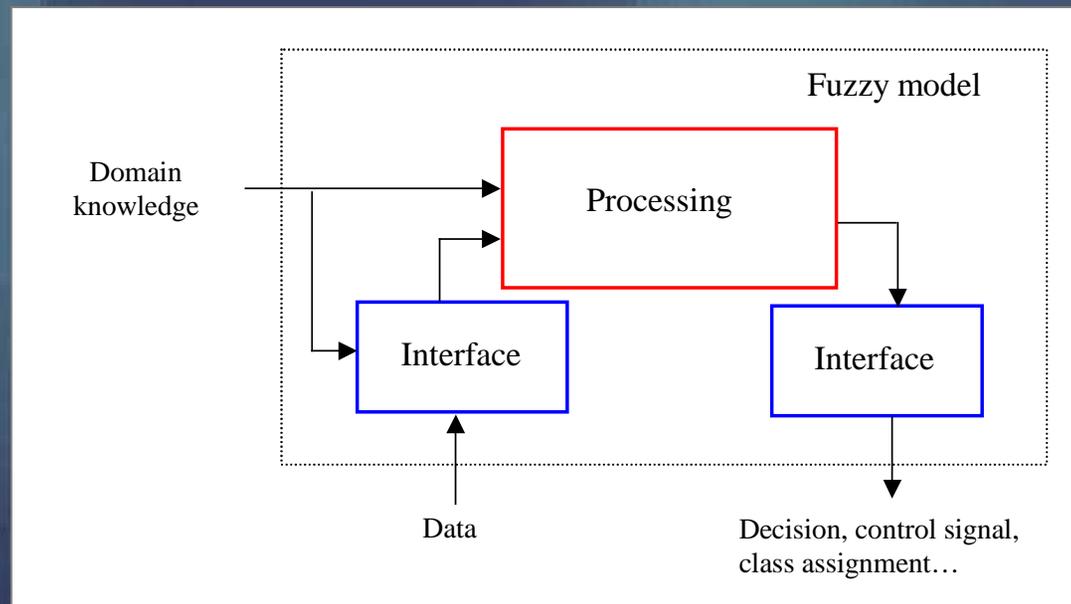
Preamble

- Fuzzy models operate on information granules that are fuzzy sets and fuzzy relations
- Information granules are abstract realizations of concepts used in modeling
- As modeling is realized at higher, more abstract level, fuzzy models give rise to a general architecture in which we highlight three main functional modules, that is
 - input interface
 - processing module
 - output interface

General architecture

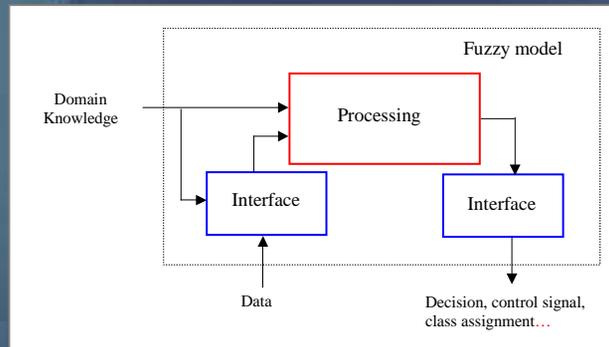


General architecture: functional modules



- **Input interface:** accepts heterogeneous data (information granules and numeric data) and converts them to internal format where processing at the level of fuzzy sets is carried out
- **Processing module:** processing pertinent to information granules
- **Output interface:** converts results of processing information granules into the format acceptable by the modeling environment

Functional modules of fuzzy models: rule-based systems



- Processing module: collection of rules, $i = 1, 2, \dots, N$

If condition₁ is A_i **and** condition₂ is B_i **then** action (decision, conclusion) is D_i

- Input interface:

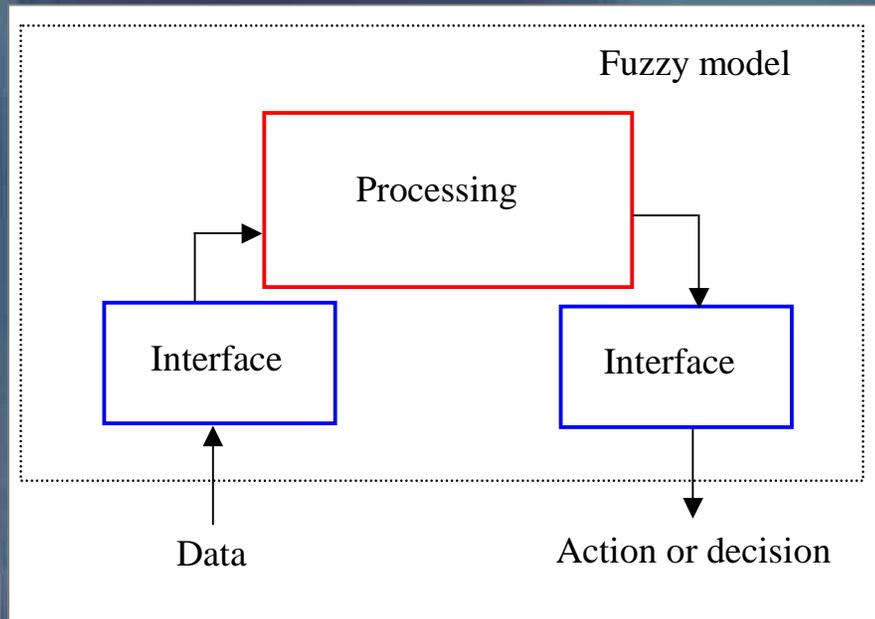
input X : express it in terms of fuzzy sets A_i present in the conditions of rules

- Output interface:

decode the result of processing, say fuzzy set D , in the format required by the modeling environment, say a single numeric entity

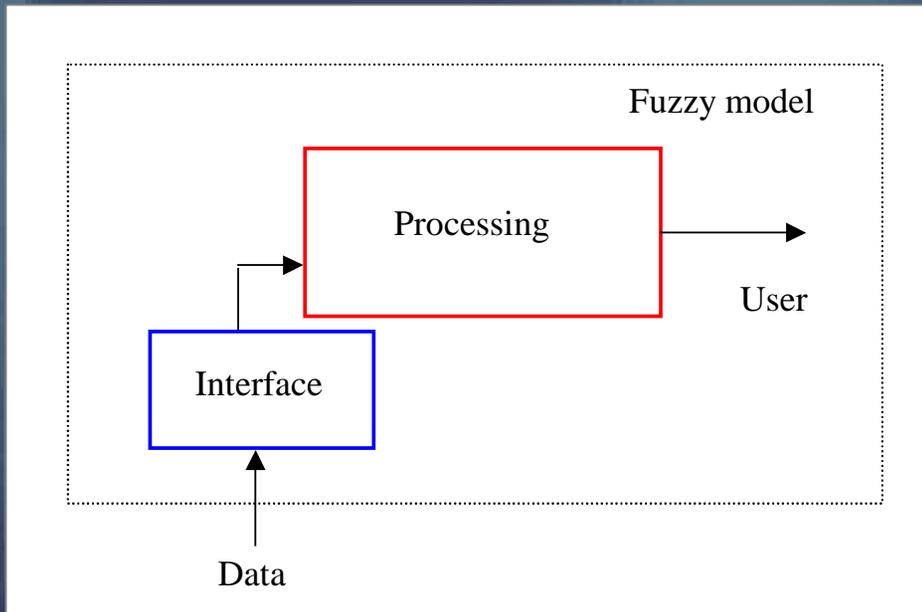
10.2 Key phases of the development and use of fuzzy models

Main modes of use of fuzzy models (a)



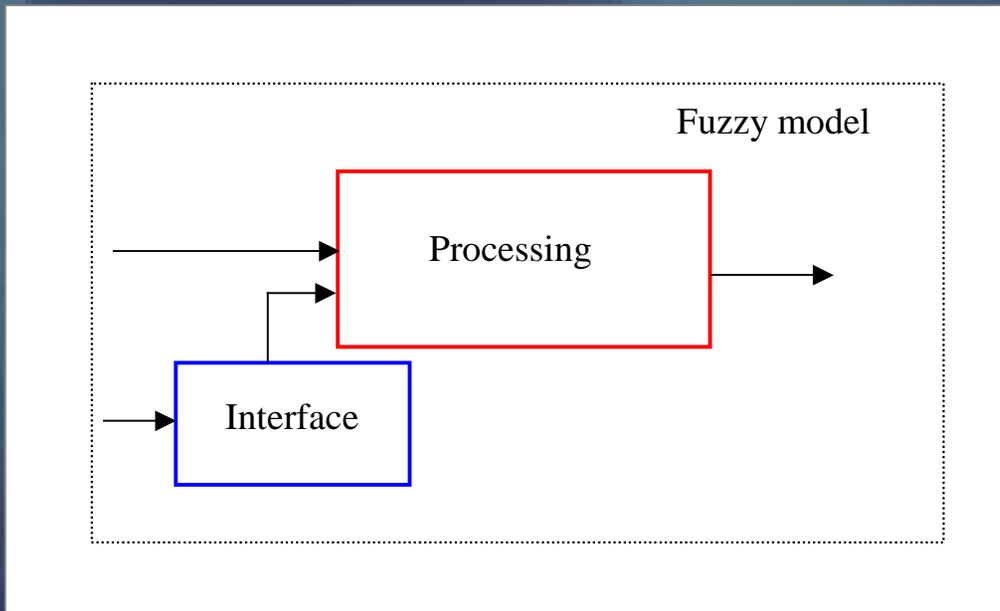
- The use of numeric data and generation of numeric results
- Module reflects a large modeling spectrum
- After development, model is used in purely numerical fashion accepts numbers and produce numbers as nonlinear I/O mappings

Main modes of use of fuzzy models (b)



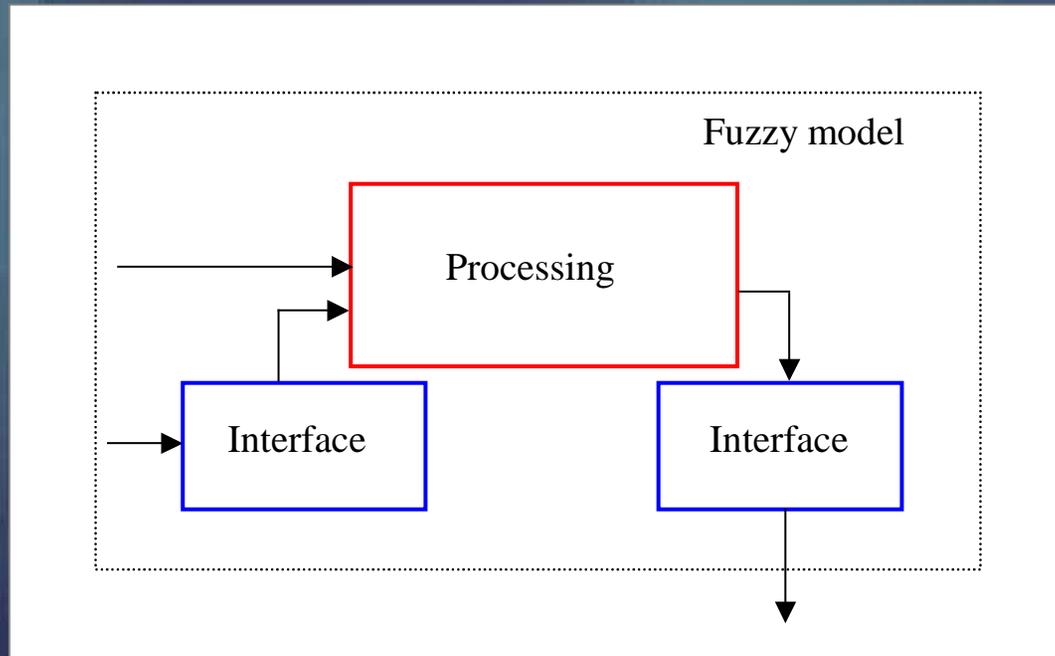
- use of numeric data and granular results (fuzzy sets)
- User centric: more informative and comprehensive than numbers
- User provided with preferences (membership degrees) associated with a collection of possible outcomes

Main modes of use of fuzzy models (c)



- Granular input data and fuzzy sets as outputs
- Scenarios where we encounter collection of linguistic observations
- Examples: expert judgment, unreliable sensor readings, etc.

Main modes of use of fuzzy models (d)



- Use of fuzzy sets as model inputs and outputs
- Granular data forming aggregates of detailed numeric data

10.3 Main categories of fuzzy models: An overview

Main categories of models: An overview

- Diversified landscape of fuzzy models - selected categories:
 - tabular fuzzy models
 - rule-based fuzzy models
 - fuzzy relational models including associative memories
 - fuzzy decision trees
 - fuzzy neural networks
 - fuzzy cognitive maps
 -

Main categories of models: Some design considerations

- Expressive power
- Processing capabilities
- Design schemes and ensuing optimization
- Interpretability
- Ability to deal with heterogeneous data
-

Tabular fuzzy models

- Table of **relationships** between the variables of the system granulated by some fuzzy sets.
- Easy to build and interpret
- Limited processing capabilities (not included as a part of the model)

	B_1	B_2	B_3	B_4	B_5
A_1					
A_2			C_3		
A_3					C_1

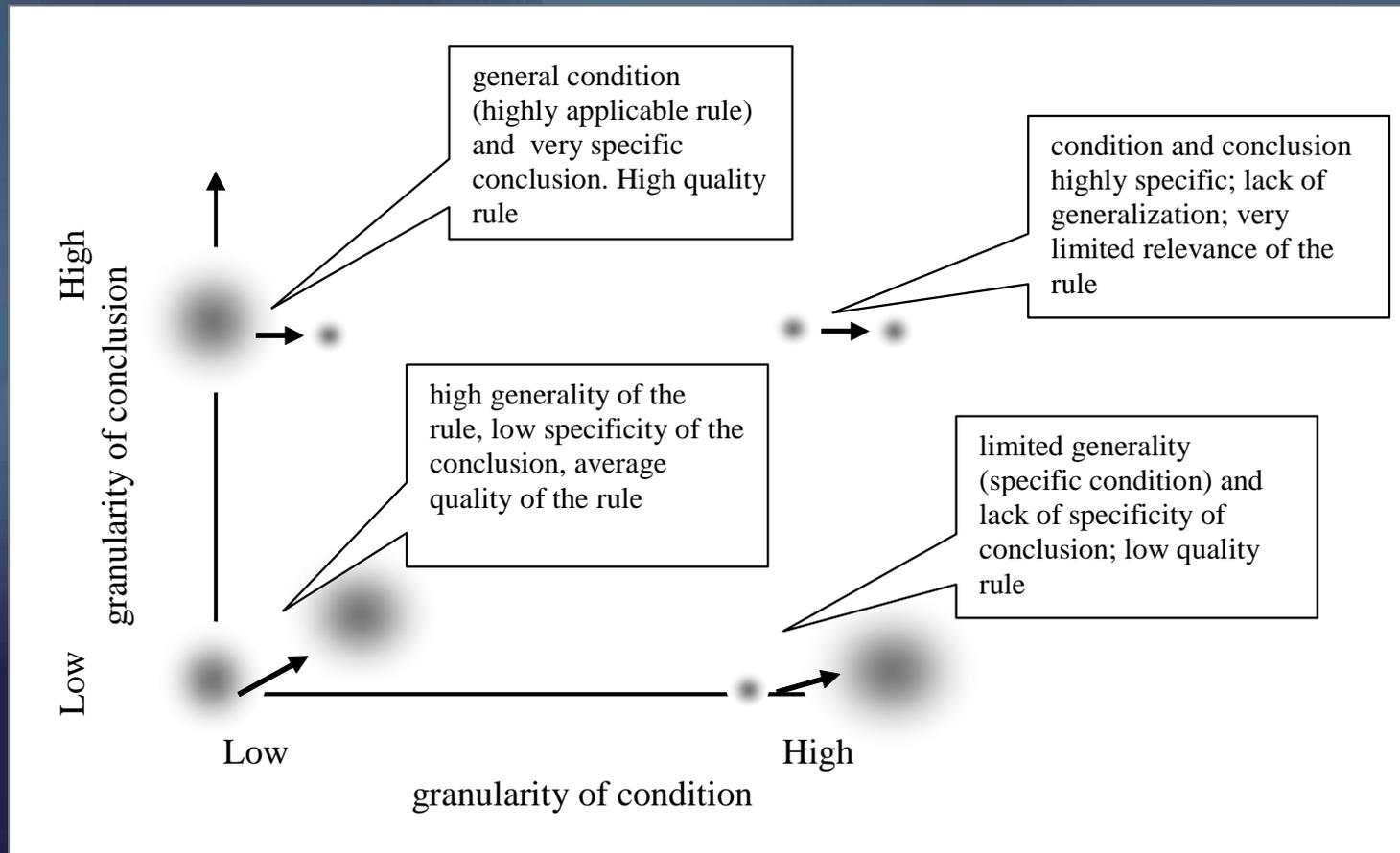
Rule-based fuzzy models

- Highly modular and easily expandable fuzzy models
- Composed of a family of conditional (**If – then**) statements (rules)
- Fuzzy sets occur in their conditions and conclusions
- Standard format

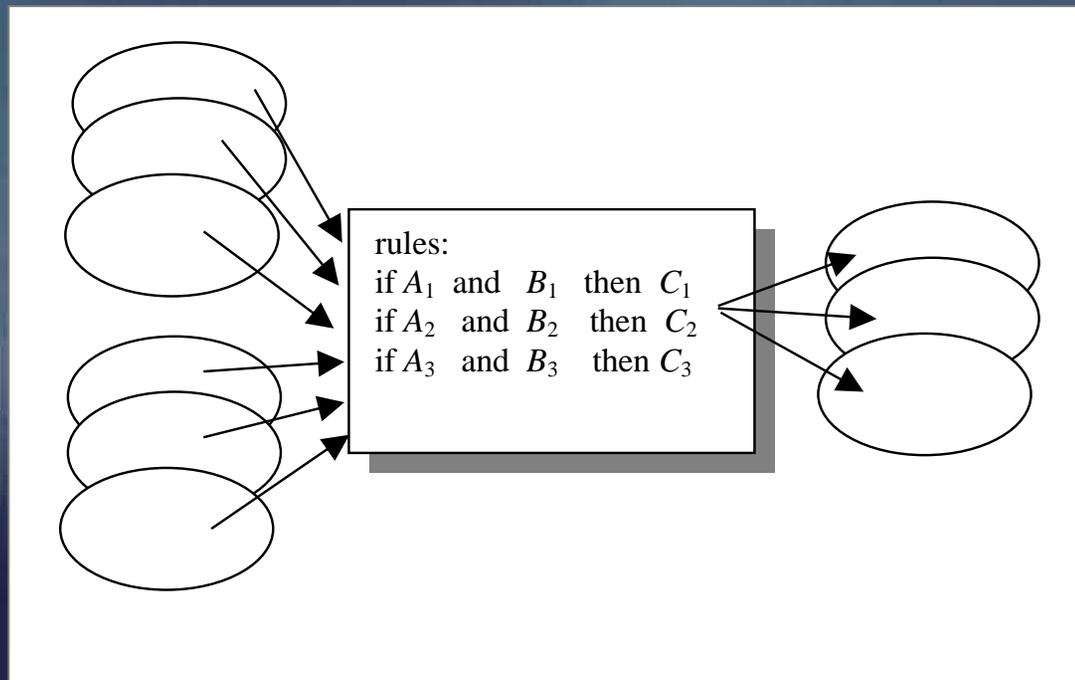
If condition₁ is *A* *and* condition₂ is *B* *and* ... *and* condition_n is *W*
then conclusion is *Z*

- Conditions \equiv rule antecedent
- Conclusions \equiv rule consequent

Rule-based fuzzy models: Granularity and quality of rules

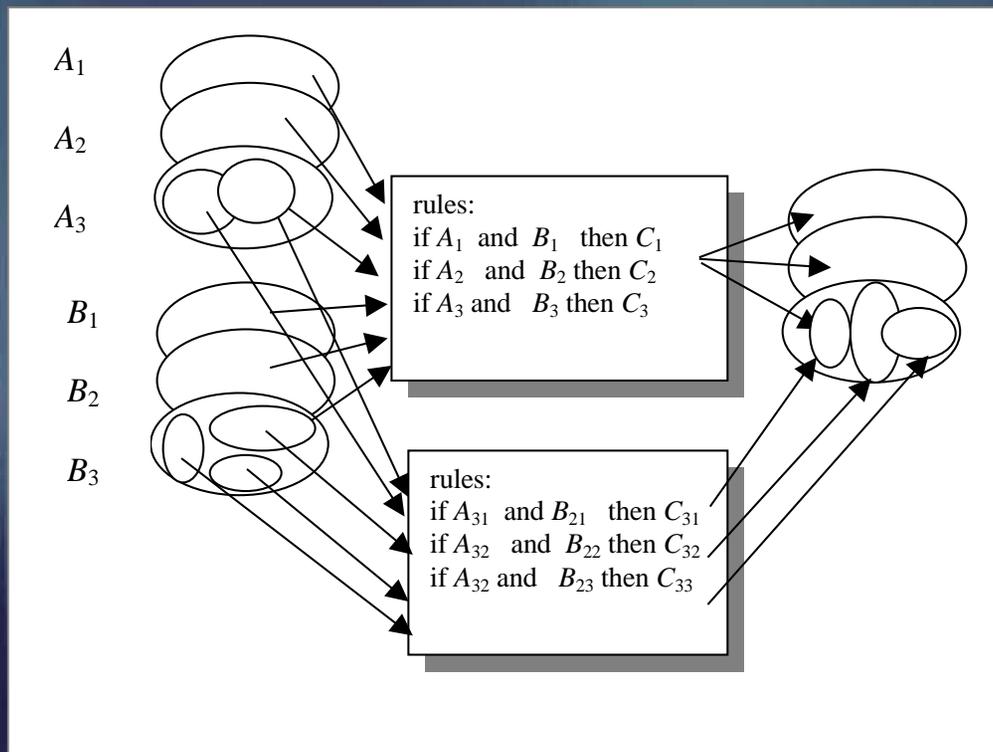


Granularity of information in rule-based systems



Same level of
granularity

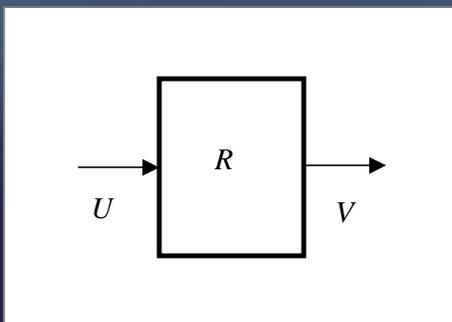
Granularity of information in rule-based systems



Different levels
of granularity

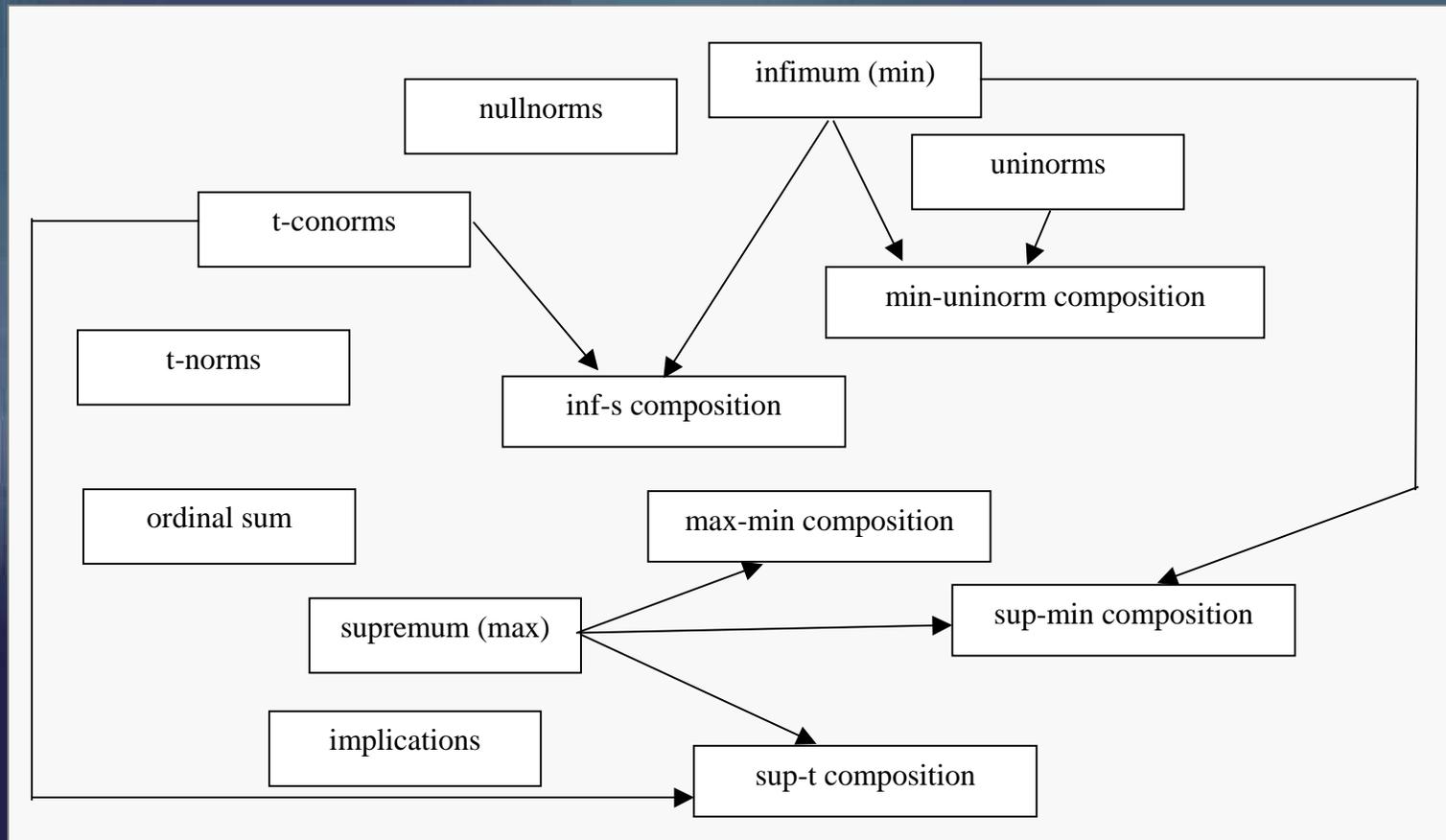
Fuzzy relational models and associative memories

- Relational transformation of fuzzy sets
- Two main modes
 - construction of fuzzy relations-storing
 - inference-recall



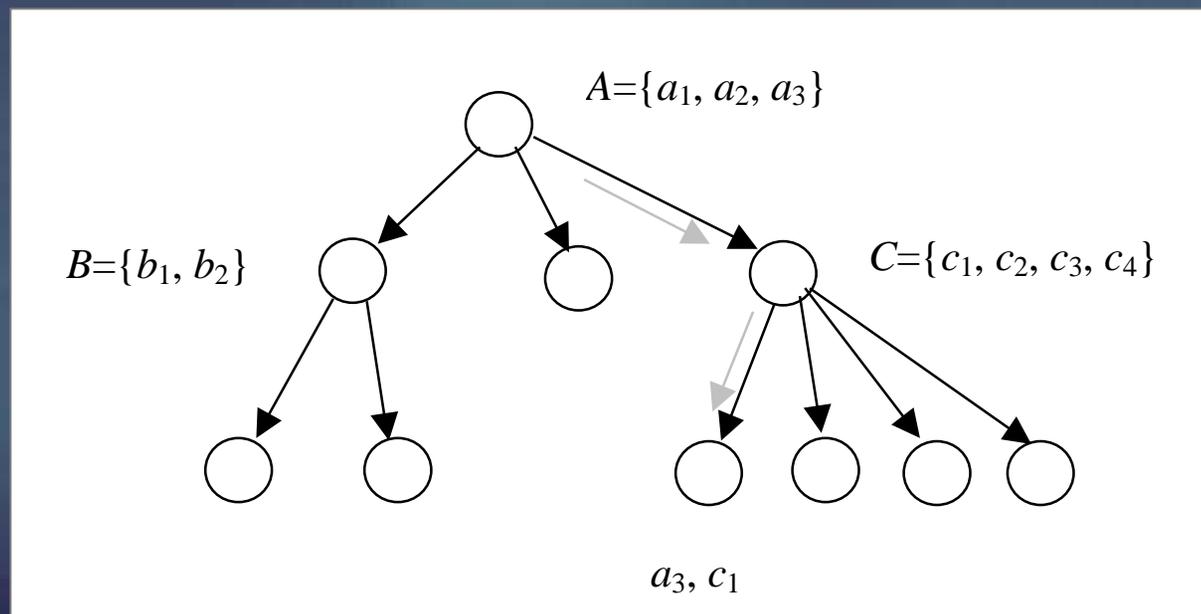
$$R = \bigcup_{k=1}^N (A_k \times B_k)$$
$$V = U \circ R$$

Fuzzy relational structures: A general taxonomy



Fuzzy decision trees

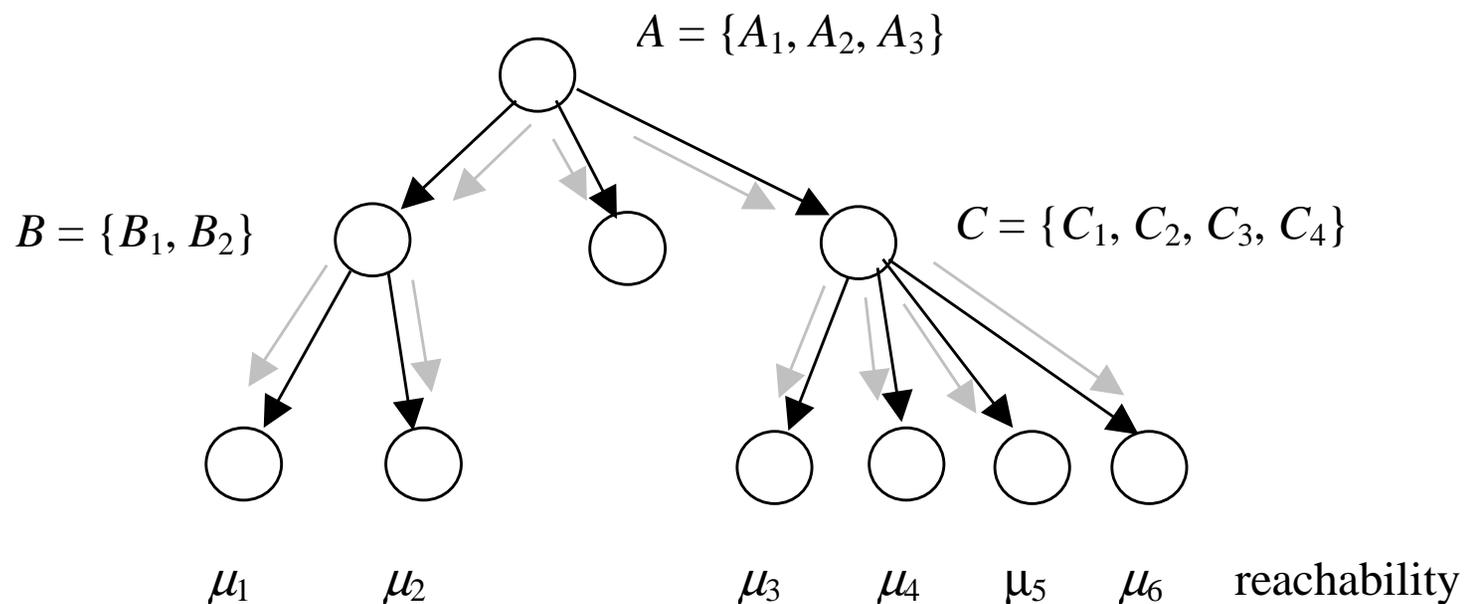
- Generalization of decision trees



- Traversal of tree depending on the values of the attributes: only a single path traversed and a single terminal node reached

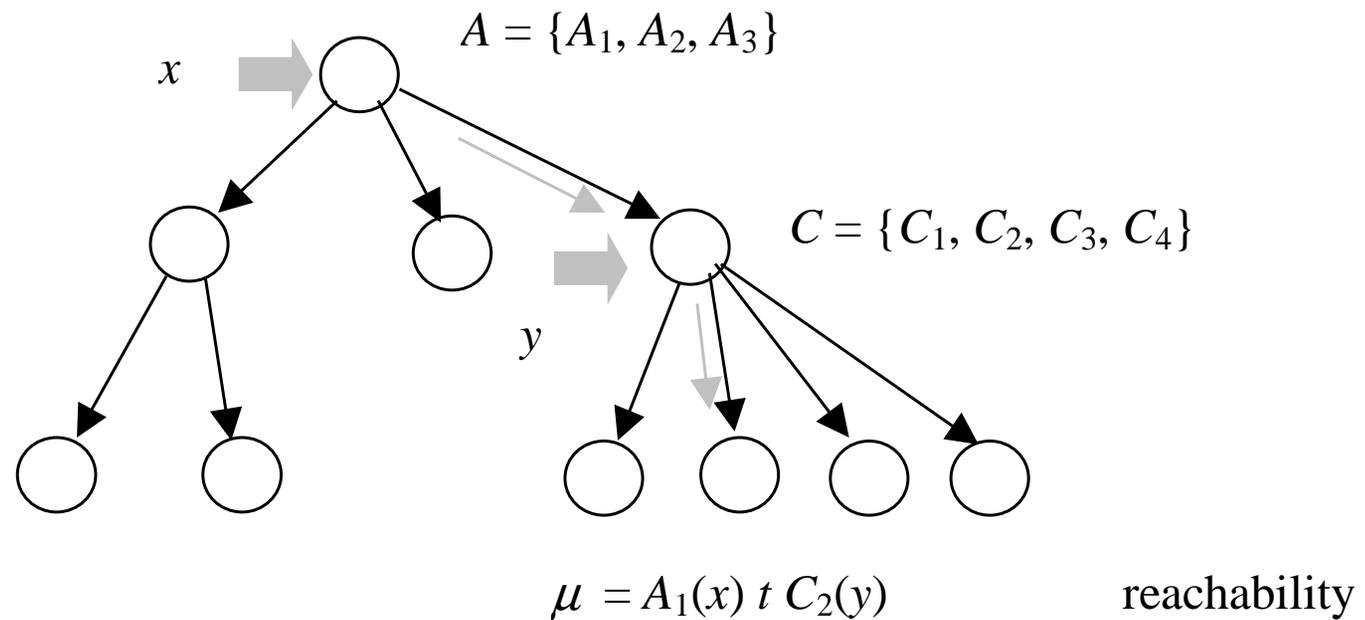
Fuzzy decision trees

- Traversal of a number of paths leading to a number of terminal nodes (reachability levels)



Fuzzy decision trees

- Traversal of a number of paths leading to a number of terminal nodes (reachability levels)

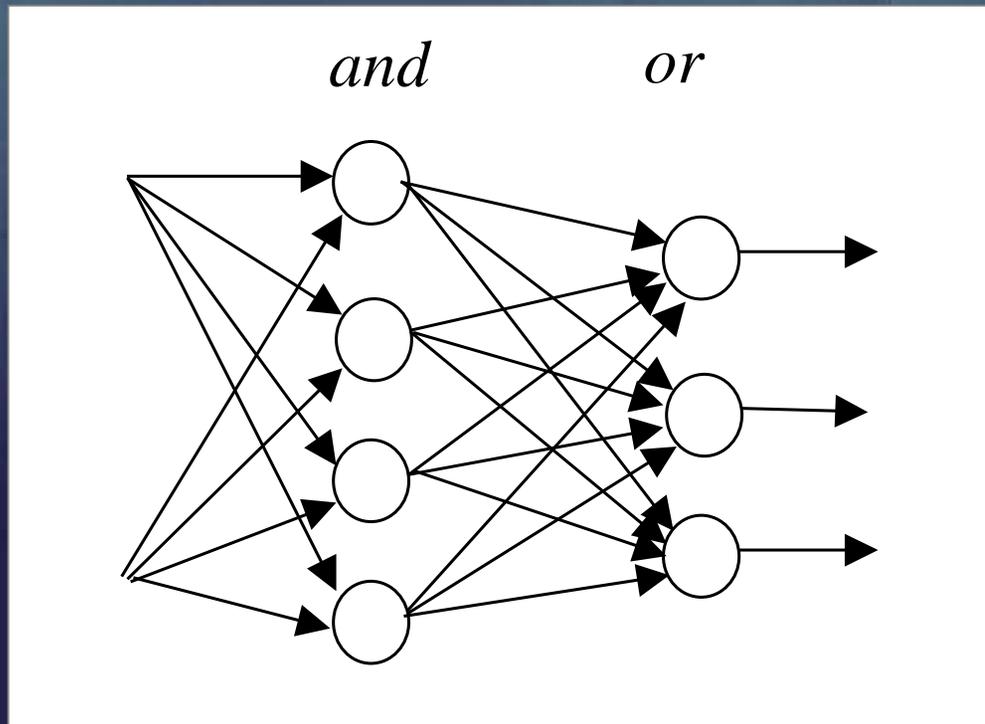


Fuzzy neural networks

- Architectures in which we combine adaptive properties of neural networks with interpretability (transparency) of fuzzy sets
- A suite of fuzzy logic neurons:
 - aggregative neurons (*and*, *or* neurons)
 - referential neurons (dominance, equality, inclusion...)
- Learning mechanisms could be applied to adjustment of connections of neurons
- Each neuron comes with a well-defined semantics; the network could be easily interpreted once the training has been completed

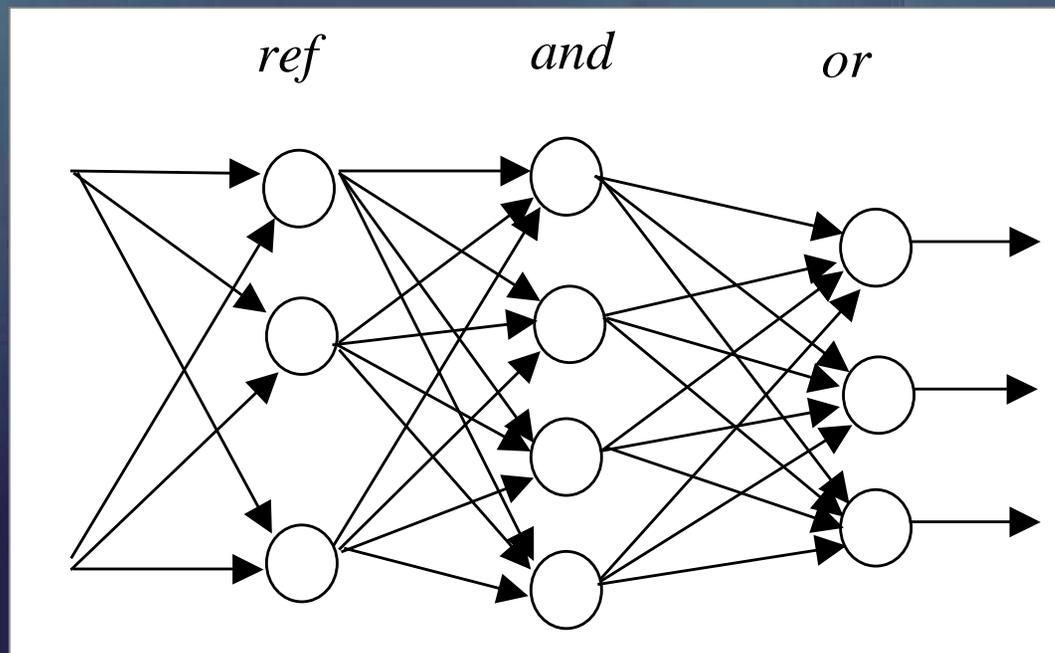
Fuzzy neural networks: Examples of architectures

- Use of *and* and *or* neurons (logic processor)



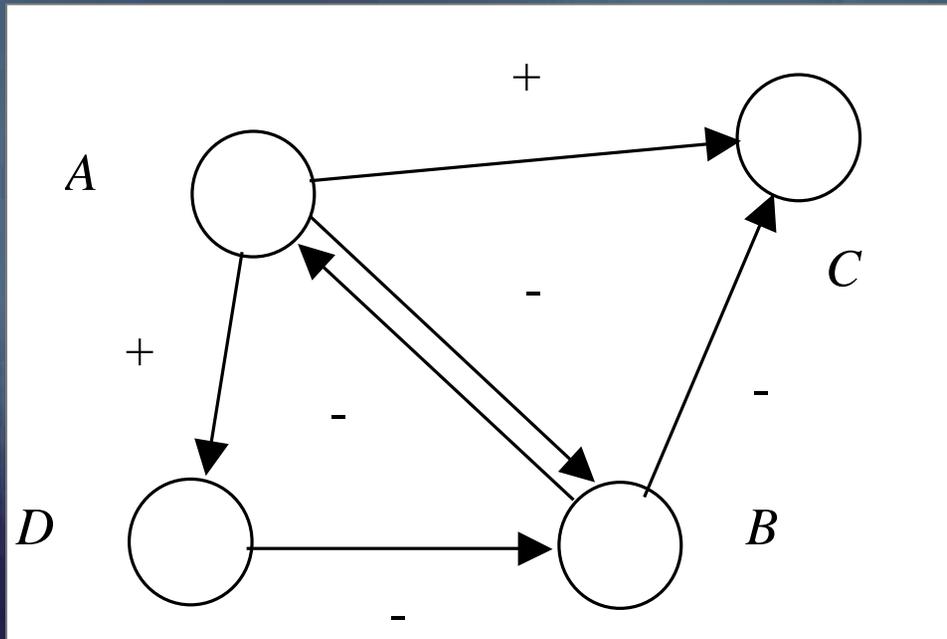
Fuzzy neural networks: Examples of architectures

- Use of *and*, *or* and referential (*ref*) neurons



Network of fuzzy processing units

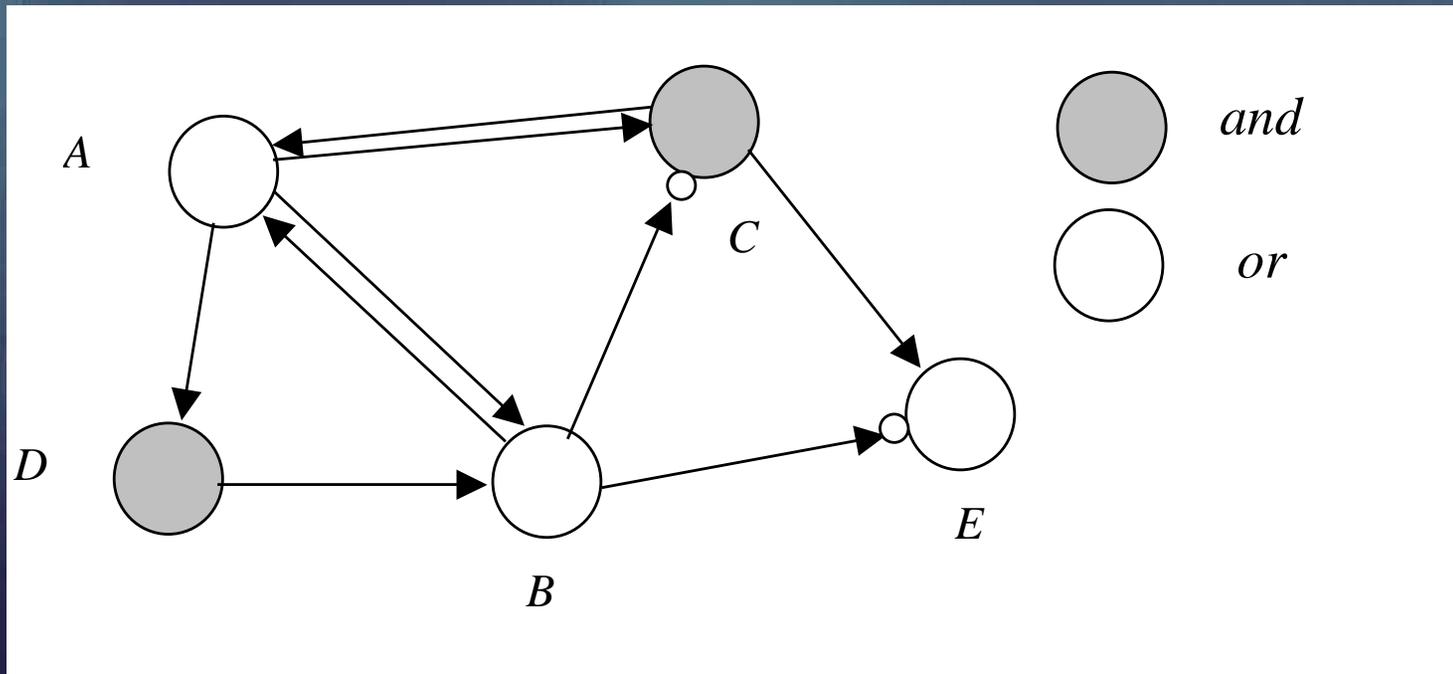
- Representation of concepts and linkages between concepts
- Directed graph: concepts are nodes; linkages are edges



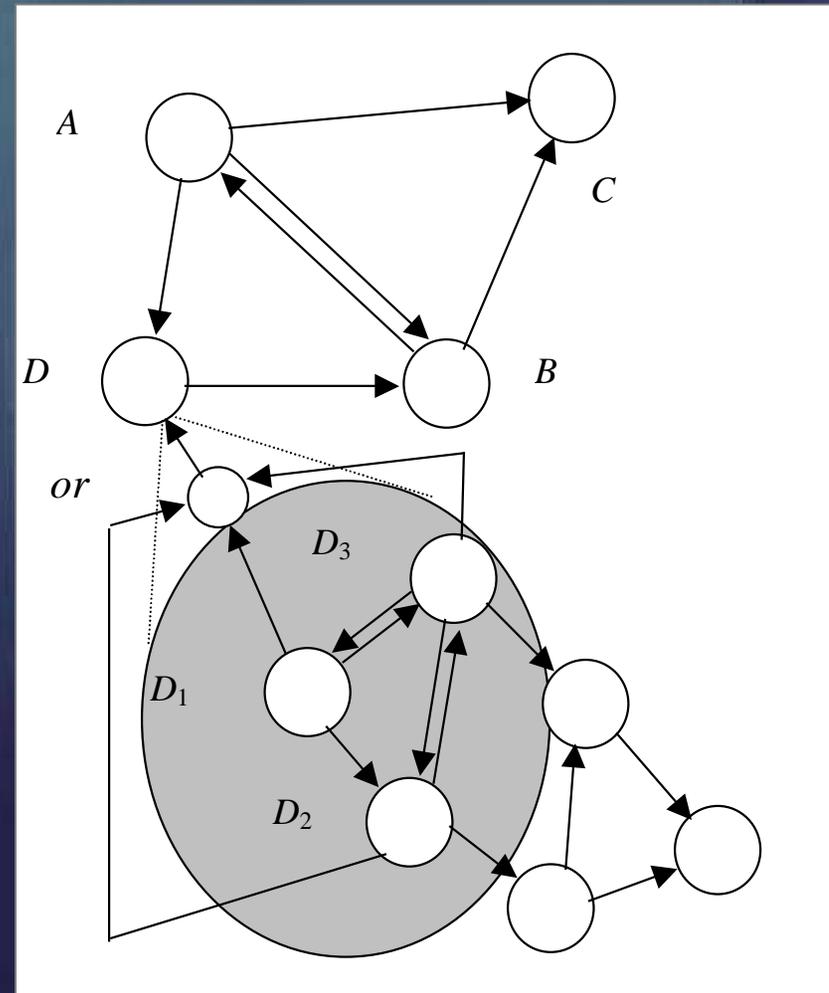
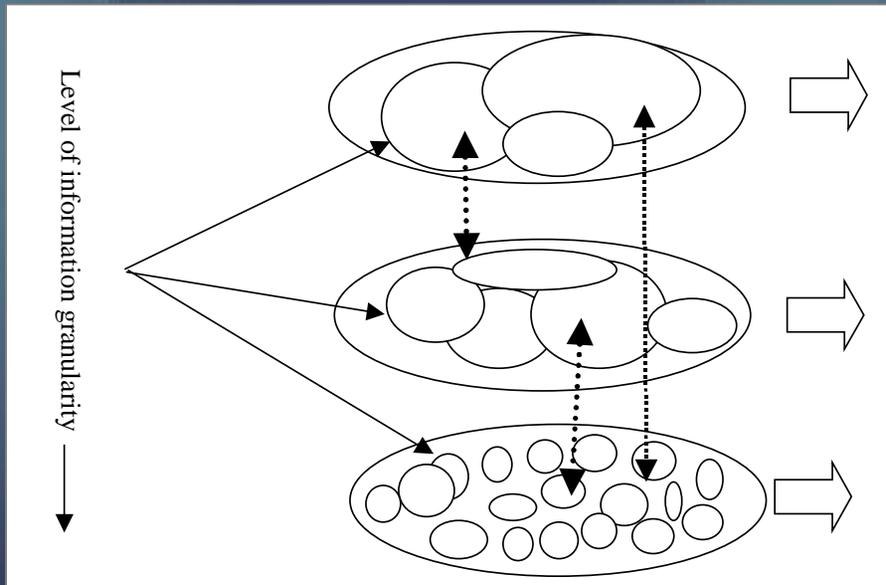
Fuzzy cognitive maps

- $A, B, C,$ and D = concepts.
- Inhibition (-) or excitation (+) between the concepts (nodes)

Fuzzy cognitive maps: extensions



Fuzzy cognitive maps: hierarchy



10.4 Verification and validation Of fuzzy models

Verification and validation of fuzzy models

- **Verification and Validation (V&V)** are concerned with the development of the model and assessment of its usefulness
- **Verification** is concerned with the analysis of the underlying processes of constructing the fuzzy model do we follow sound design principles ?

“Are we building the product *right*?”

- **Validation** is concerned with ensuring that the model (product) meets the requirements of the customer

“Are we building the *right* product?”

Verification of fuzzy models

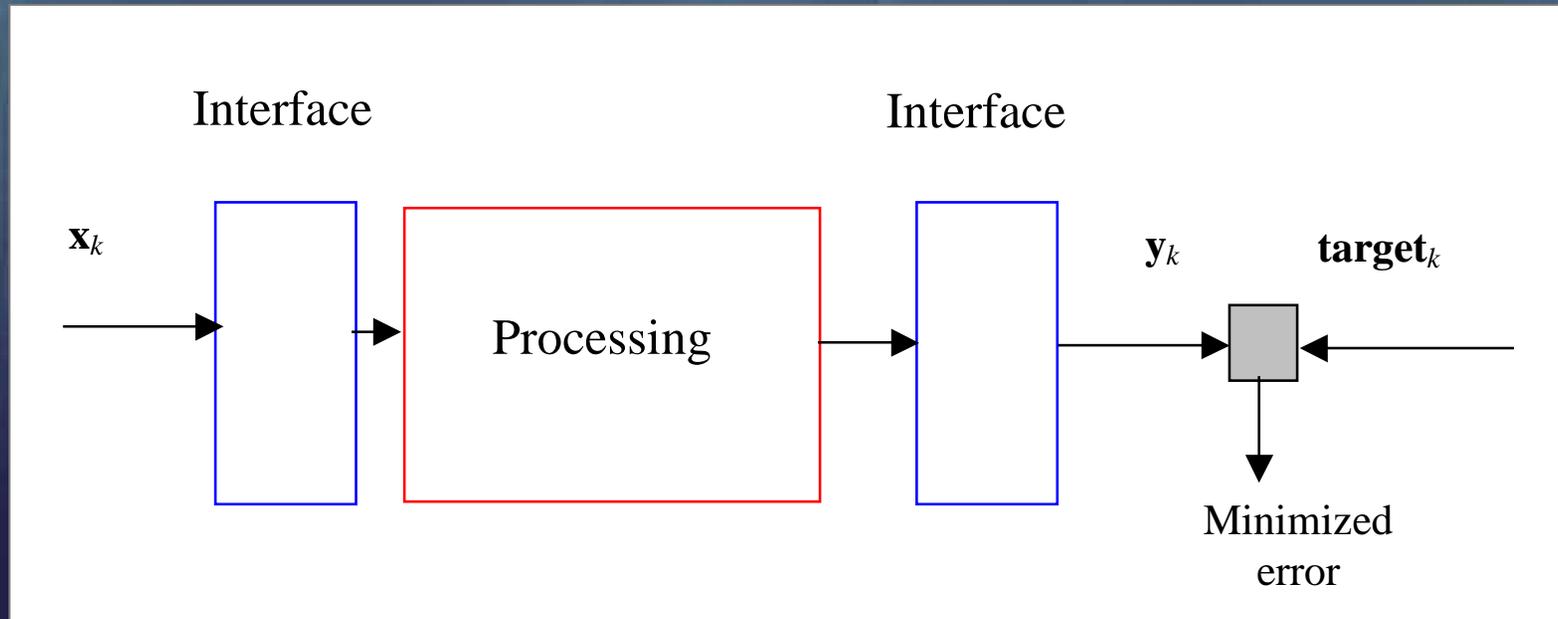
- Sound design principles
 - iterative development process
 - assessment of accuracy
 - generalization capabilities
 - complexity of the model (Occam's principle)
 - high level of autonomy of the model

Fuzzy models: accuracy

- Two ways of expressing accuracy
 - numeric level
 - internal level

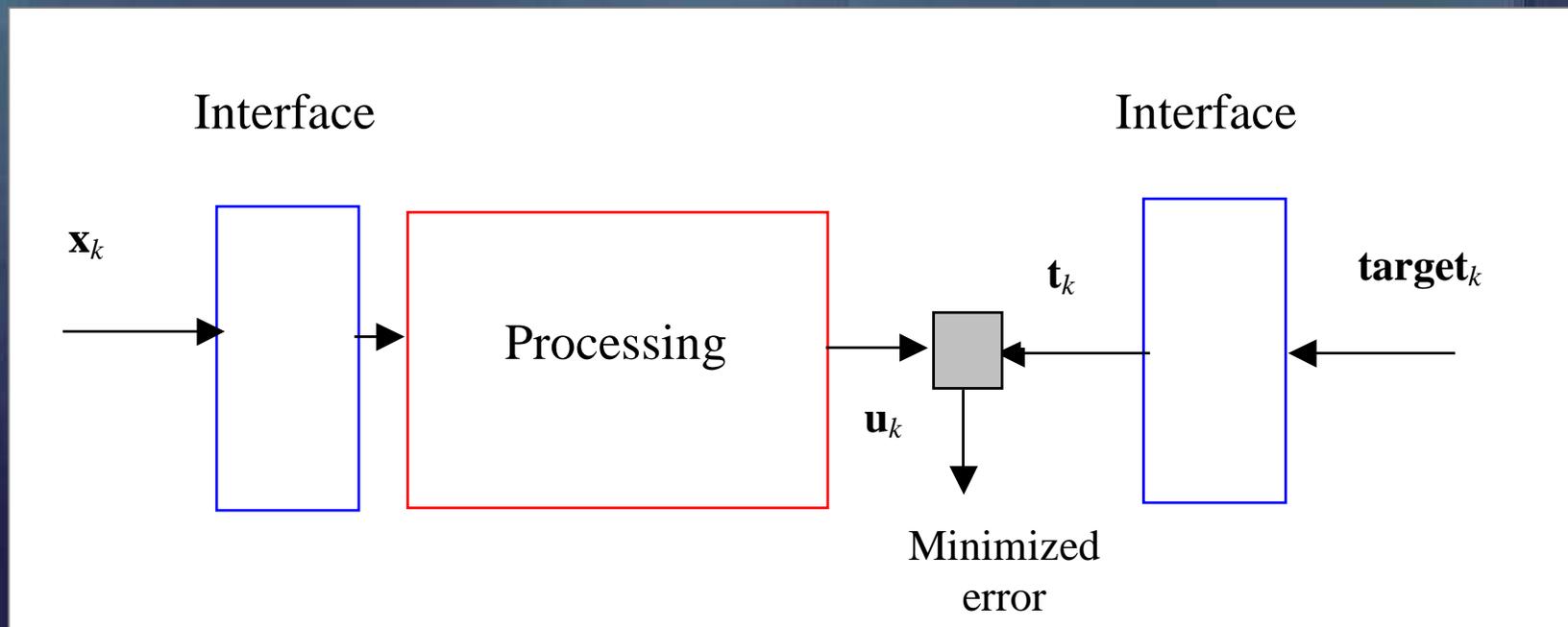
Fuzzy models: accuracy

- Numeric level of expressing accuracy



Fuzzy models: accuracy

- Accuracy expressed at the level of fuzzy sets



Training, validation, and testing data

- To avoid potential bias in assessment of accuracy, data are split into
 - training
 - validation
 - testing subsets
- Training - testing
 - typically 60-40% split
 - 10 fold cross-validation (90-10% split)
 - leave one out strategy

Validation of fuzzy models

- Are we building the *right* model?
- More difficult to quantify:
 - transparency of fuzzy models
 - stability of the fuzzy model
-
- Very often validation criteria are in conflict

