Princípio da Extensão Clássico

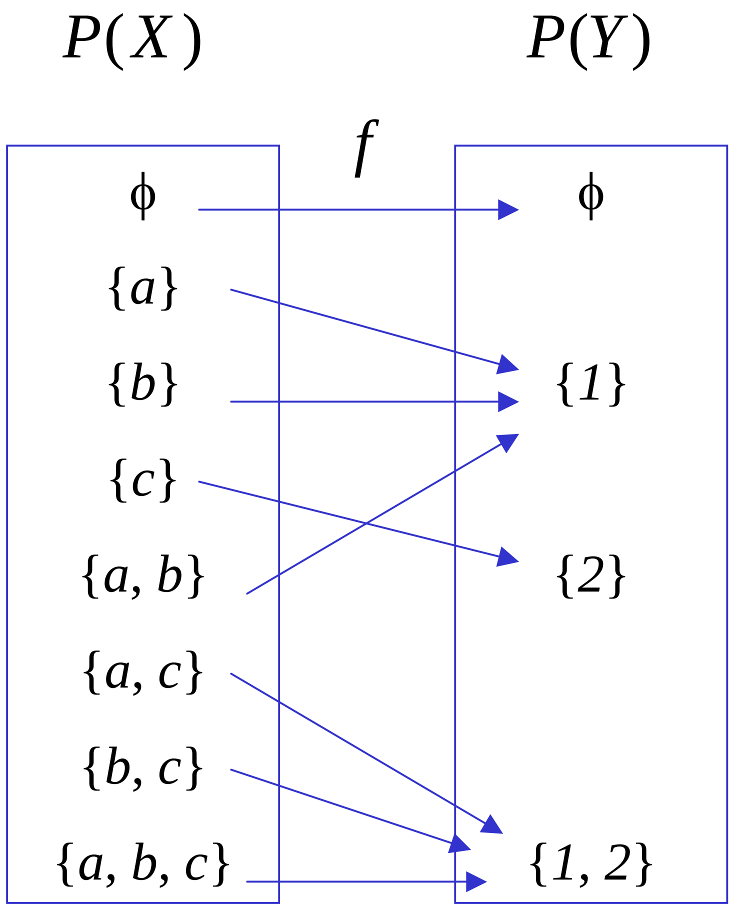
$$f : X \to Y$$

$$f : P(X) \to P(Y)$$

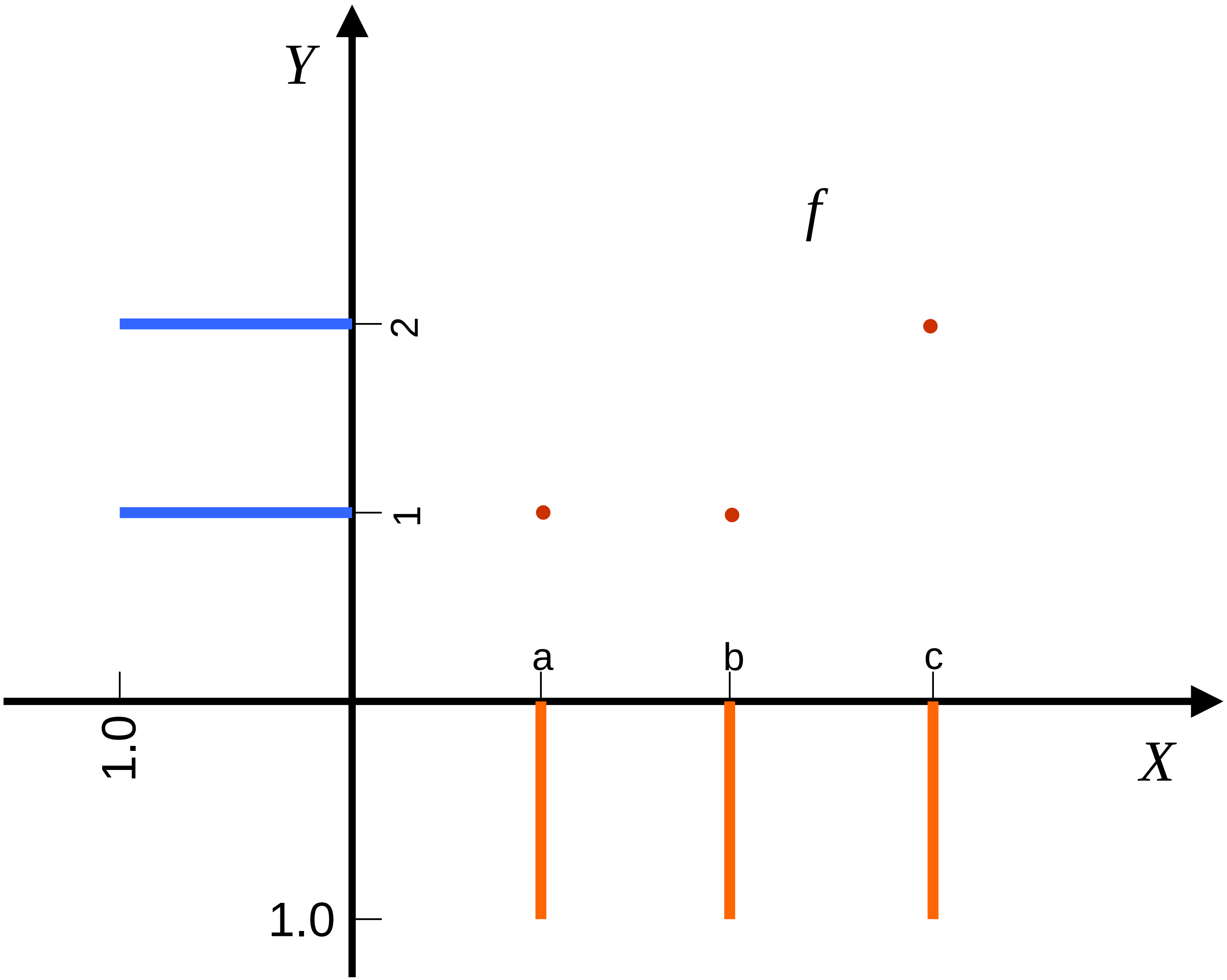
$$f(A) = \{y \mid y = f(x), x \in A\}$$

$X = \{a, b, c\}$ e $Y = \{1, 2\}$

$$f: \; a \to 1$$
$$b \to 1$$
$$c \to 2$$



$$B(y) = [f(A)](y) = \sup_{x|y=f(x)} A(x)$$



$$f^{-1} : Y \rightarrow X$$

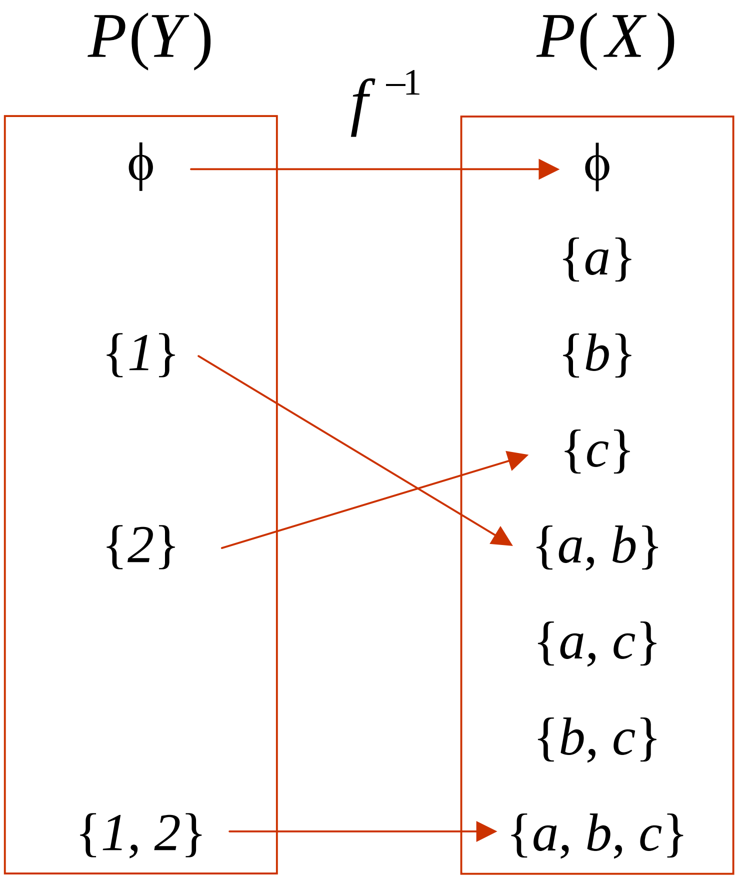
$$f^{-1} : P(Y) \rightarrow P(X)$$

$$f^{-1}(B) = \{x \mid f(x) \in B\}$$

$$[f^{-1}(B)](x) = B(f(x))$$

$X=\{a, b, c\}$ e $Y=\{1, 2\}$

$$f: \begin{array}{l} a \rightarrow 1 \\ b \rightarrow 1 \\ c \rightarrow 2 \end{array}$$

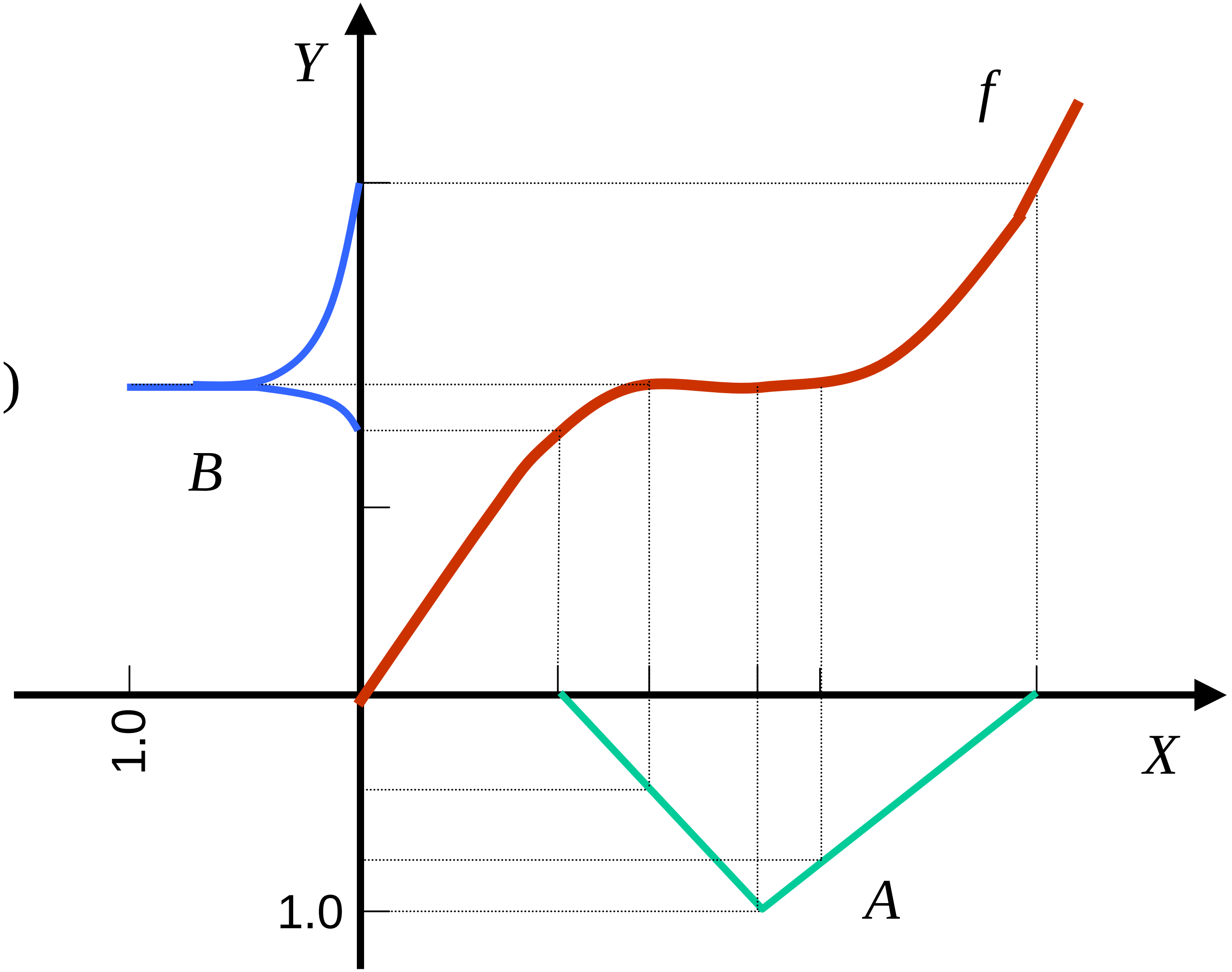


Princípio da Extensão: Conjuntos Nebulosos

$$f : X \rightarrow Y$$

$$f : F(X) \rightarrow F(Y)$$

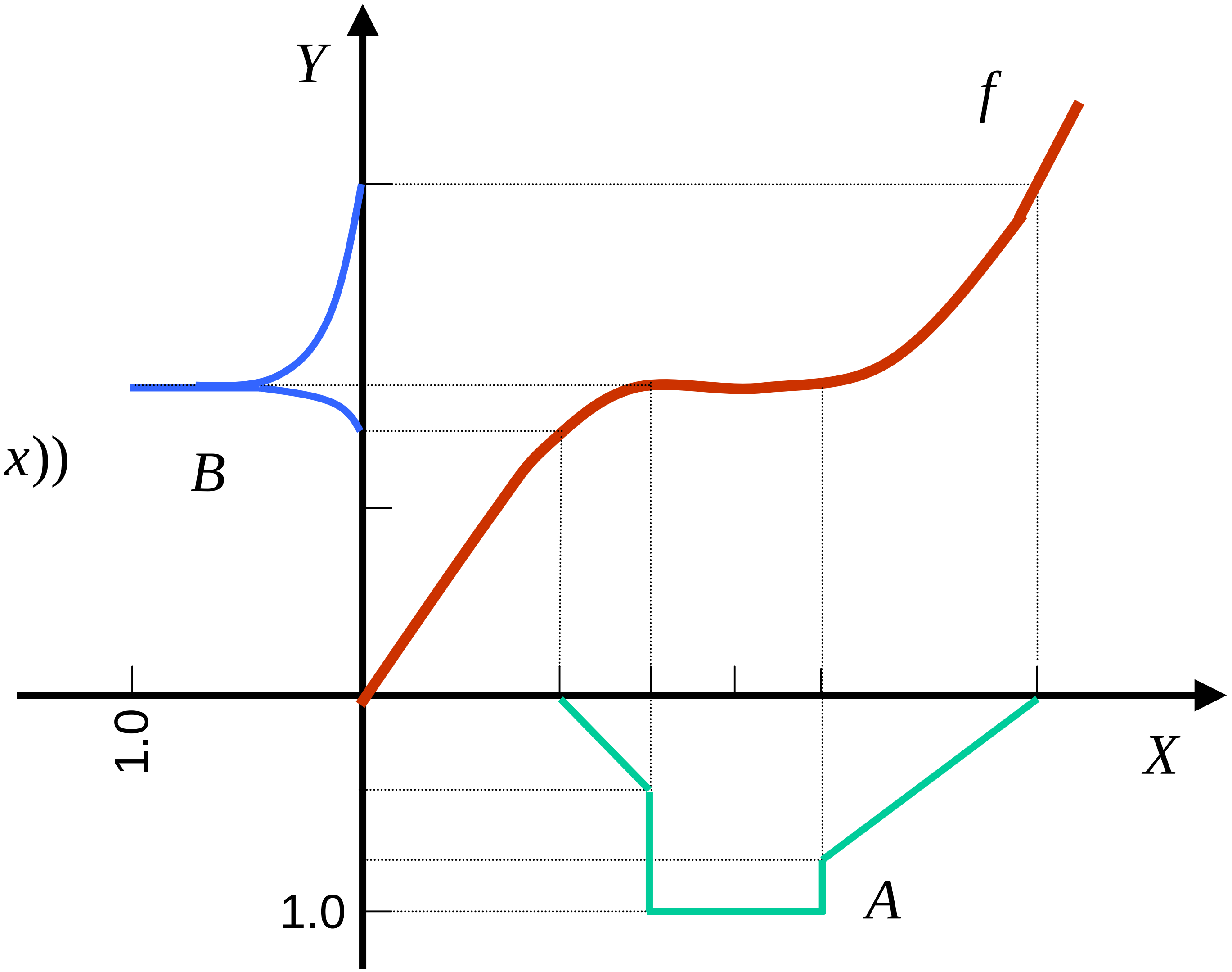
$$B(y) = \sup_{x | y = f(x)} A(x)$$



$$f^{-1}: Y \rightarrow X$$

$$f^{-1}: F(Y) \rightarrow F(X)$$

$$[f^{-1}(B)](x) = B(f(x))$$



Propriedades Induzidas pelo Princípio da Extenxão

$$f : X \to Y \qquad A_i \in F(X) \qquad B_i \in F(Y)$$

$$A_1 \subseteq A_2 \to f(A_1) \subseteq f(A_2)$$

$$f(\bigcup_i A_i) = \bigcup_i f(A_i)$$

$$f(\bigcap_i A_i) \subseteq \bigcap_i f(A_i)$$

$$B_1 \subseteq B_2 \to f^{-1}(B_1)$$

$$f^{-1}(\bigcup_i B_i) = \bigcup_i f^{-1}(B_i)$$

$$f^{-1}(\bigcap_i B_i) = \bigcap_i f^{-1}(B_i)$$

$$f(A) = \phi \quad see \quad A = \phi$$

$$\overline{f^{-1}(B)} = f^{-1}(\overline{B})$$

$$A \subseteq f^{-1}(f(A))$$

$$B \supseteq f(f^{-1}(B))$$

$$A_\alpha = \{x \mid A(x) \geq \alpha\} \qquad A_{\alpha^+} = \{x \mid A(x) > \alpha\}$$

$$[f(A)]_{\alpha^+} = f(A_{\alpha^+})$$

$$[f(A)]_\alpha \supseteq f(A_\alpha)$$

$$f(A) = \bigcup_{\alpha \in [0,1]} f(A_{\alpha^+})$$