

# Princípio da Extensão Clássico

$$f : X \rightarrow Y$$

$$f : P(X) \rightarrow P(Y)$$

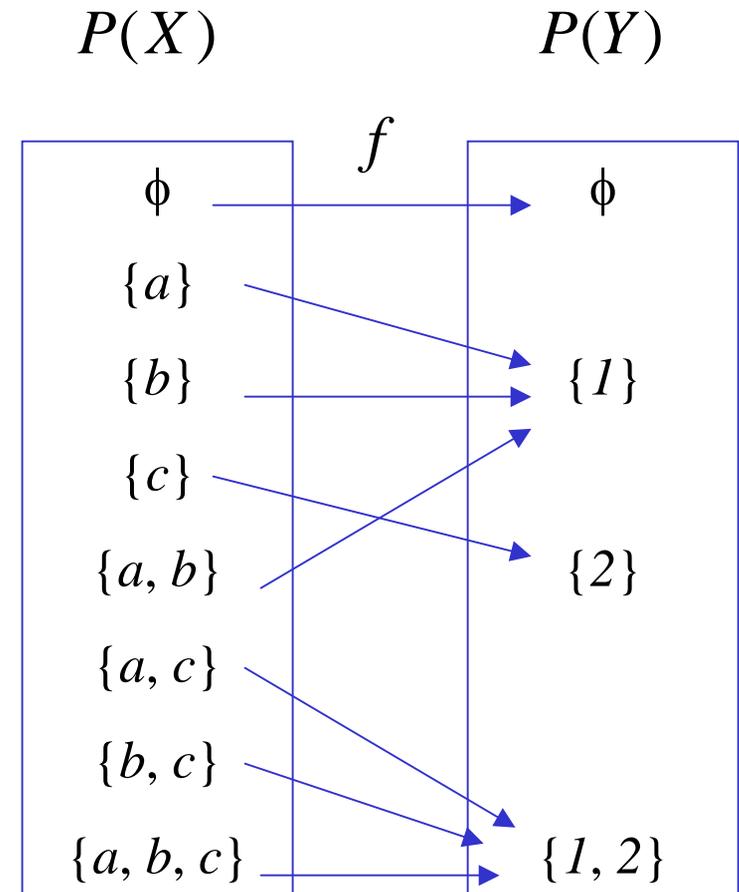
$$f(A) = \{y \mid y = f(x), x \in A\}$$

$$X = \{a, b, c\} \text{ e } Y = \{1, 2\}$$

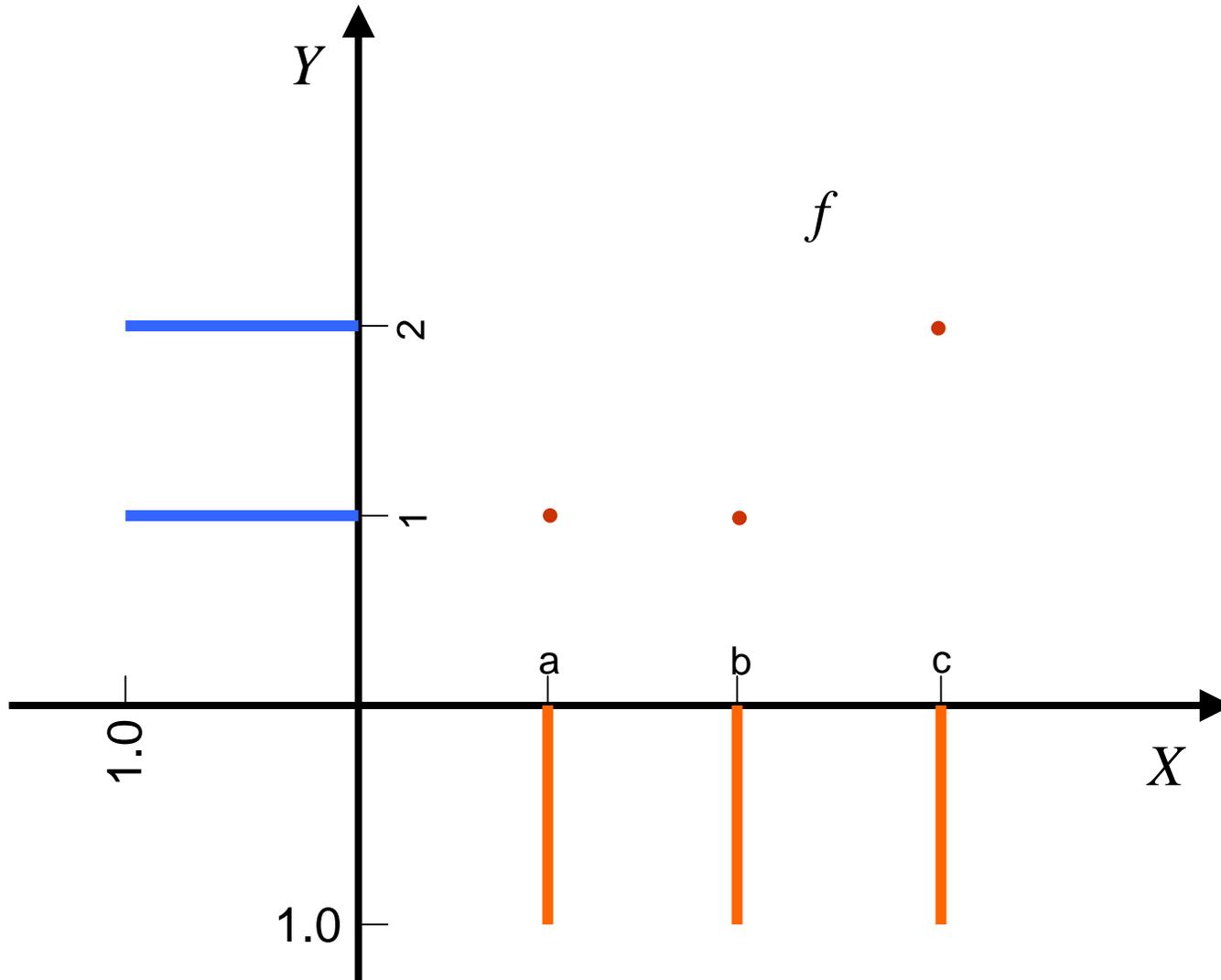
$$f: a \rightarrow 1$$

$$b \rightarrow 1$$

$$c \rightarrow 2$$



$$B(y) = [f(A)](y) = \sup_{x|y=f(x)} A(x)$$



$$f^{-1} : Y \rightarrow X$$

$$f^{-1} : P(Y) \rightarrow P(X)$$

$$f^{-1}(B) = \{x \mid f(x) \in B\}$$

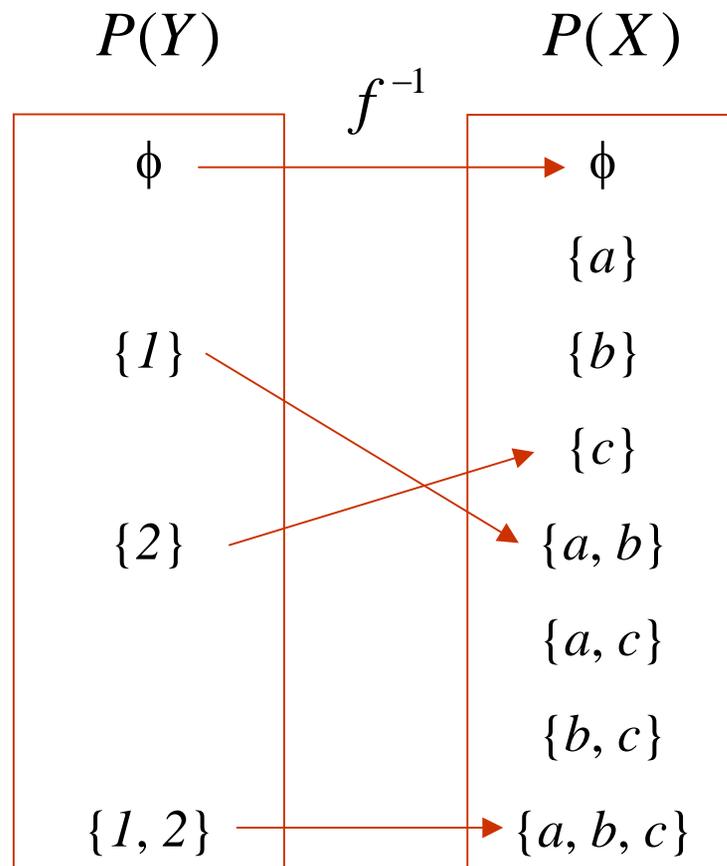
$$[f^{-1}(B)](x) = B(f(x))$$

$$X = \{a, b, c\} \text{ e } Y = \{1, 2\}$$

$$f: a \rightarrow 1$$

$$b \rightarrow 1$$

$$c \rightarrow 2$$

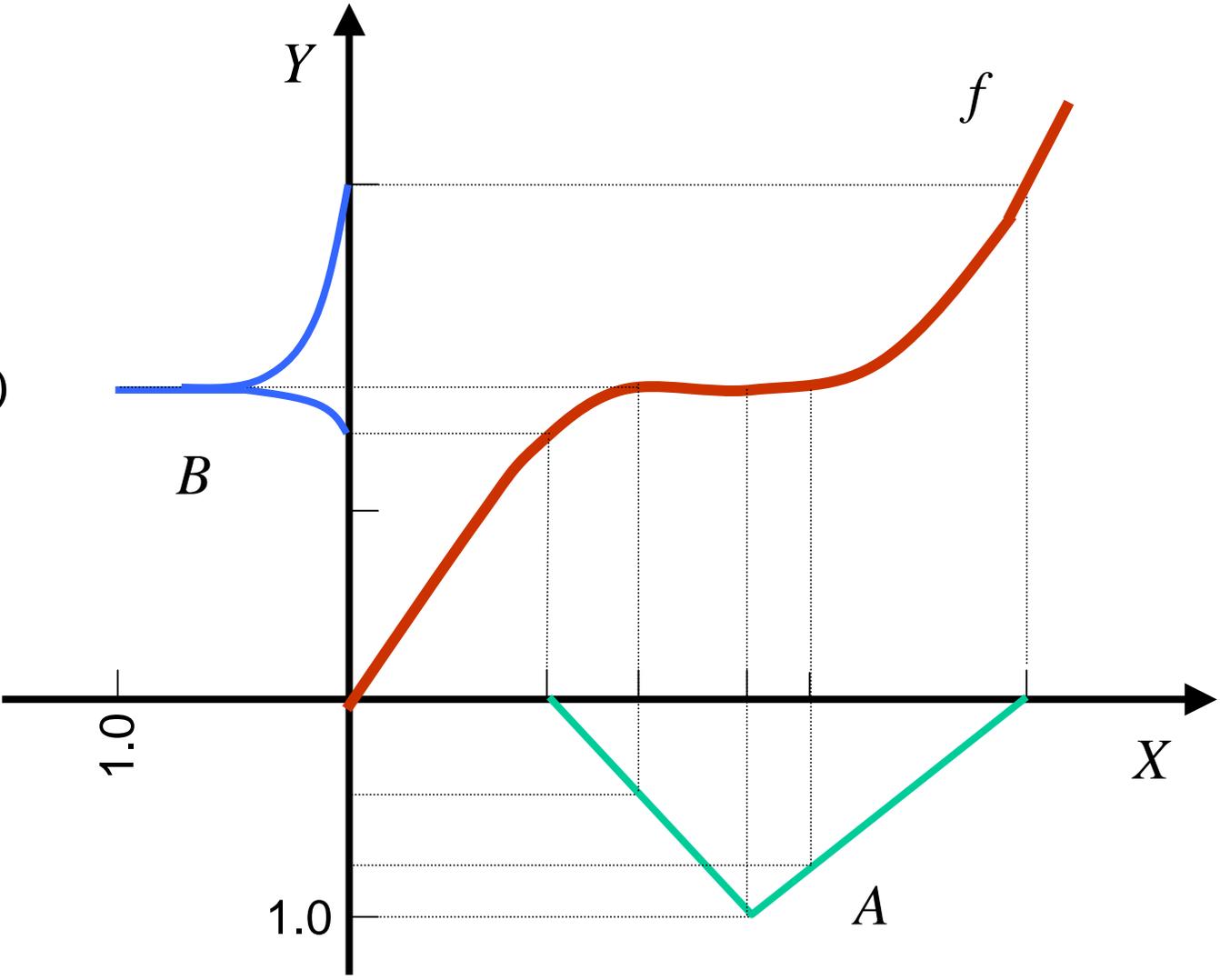


# Princípio da Extensão: Conjuntos Nebulosos

$$f : X \rightarrow Y$$

$$f : F(X) \rightarrow F(Y)$$

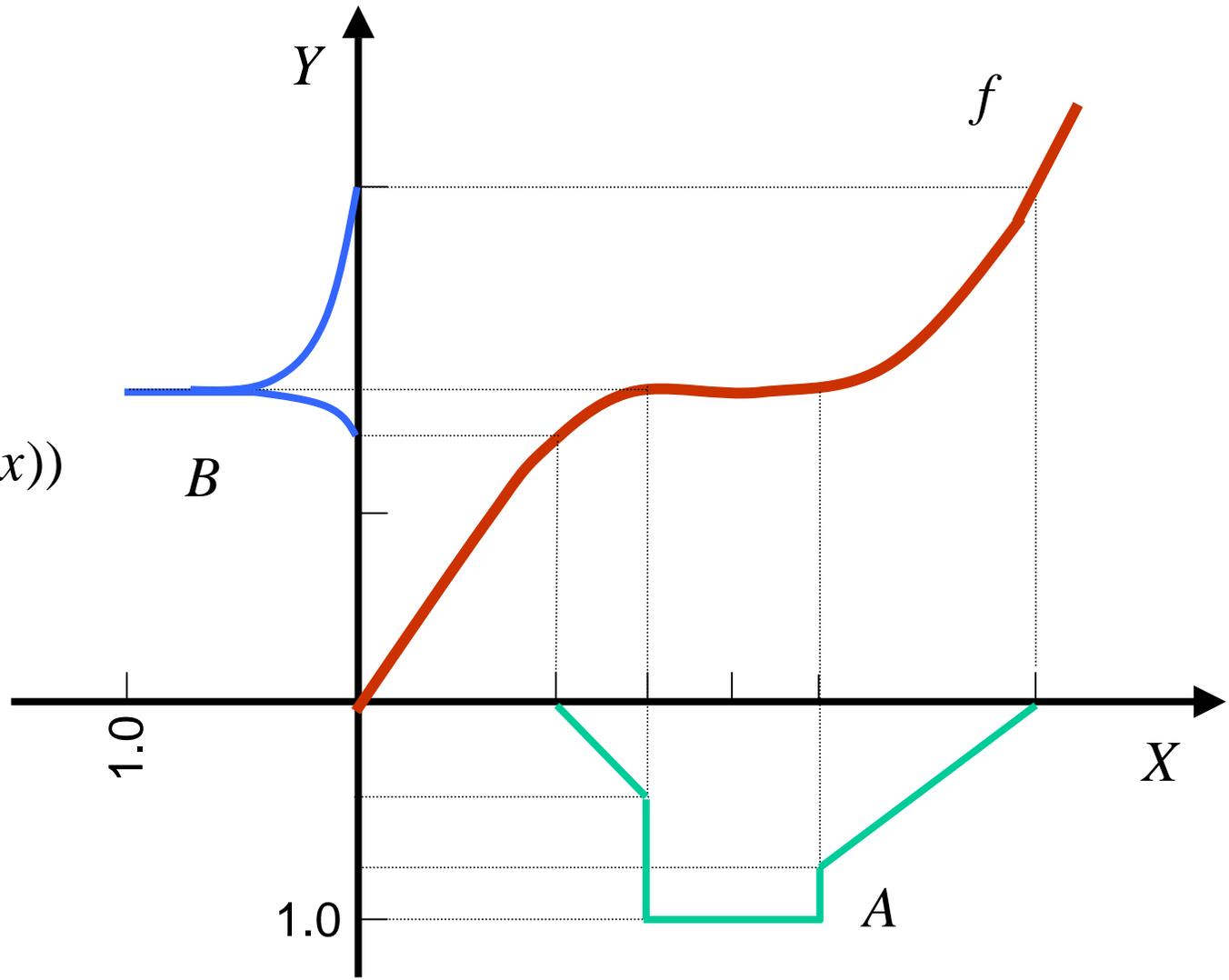
$$B(y) = \sup_{x|y=f(x)} A(x)$$



$$f^{-1} : Y \rightarrow X$$

$$f^{-1} : F(Y) \rightarrow F(X)$$

$$[f^{-1}(B)](x) = B(f(x))$$



# Propriedades Induzidas pelo Princípio da Extensão

$$f : X \rightarrow Y \quad A_i \in F(X) \quad B_i \in F(Y)$$

$$A_1 \subseteq A_2 \rightarrow f(A_1) \subseteq f(A_2)$$

$$f\left(\bigcup_i A_i\right) = \bigcup_i f(A_i)$$

$$f\left(\bigcap_i A_i\right) \subseteq \bigcap_i f(A_i)$$

$$B_1 \subseteq B_2 \rightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$$

$$f^{-1}\left(\bigcup_i B_i\right) = \bigcup_i f^{-1}(B_i)$$

$$f^{-1}\left(\bigcap_i B_i\right) = \bigcap_i f^{-1}(B_i)$$

$$f(A) = \emptyset \quad \text{se e só se} \quad A = \emptyset$$

$$\overline{f^{-1}(B)} = f^{-1}(\overline{B})$$

$$A \subseteq f^{-1}(f(A))$$

$$B \supseteq f(f^{-1}(B))$$

$$A_\alpha = \{x \mid A(x) \geq \alpha\} \quad A_{\alpha^+} = \{x \mid A(x) > \alpha\}$$

$$[f(A)]_{\alpha^+} = f(A_{\alpha^+})$$

$$[f(A)]_\alpha \supseteq f(A_\alpha)$$

$$f(A) = \bigcup_{\alpha \in [0,1]} f(A_{\alpha^+})$$

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