10 Fuzzy Modeling: Principles and Methodology

Fuzzy Systems Engineering
Toward Human-Centric Computing

Contents

10.1 The architectural blueprint of fuzzy models

Preamble

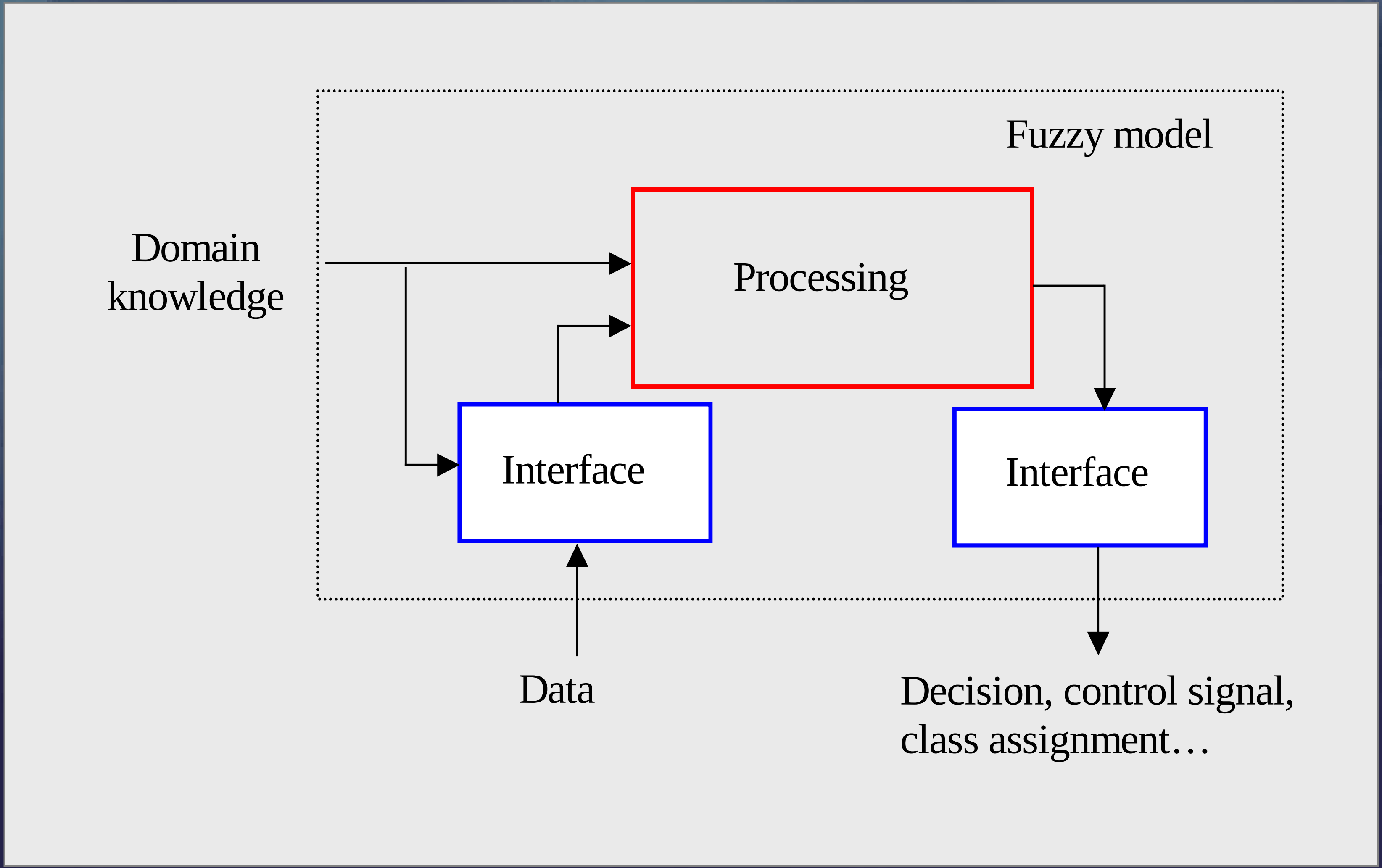
- Fuzzy models operate on information granules that are fuzzy sets and fuzzy relations

- Information granules are abstract realizations of concepts used in modeling

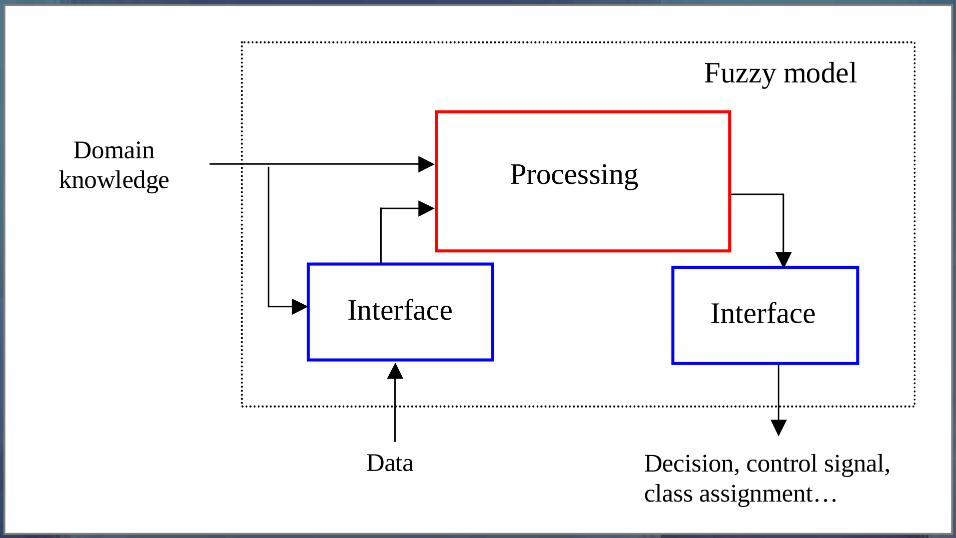
- As modeling is realized at higher, more abstract level, fuzzy models give rise to a general architecture in which we highlight three main functional modules, that is

  – input interface
  – processing module
  – output interface

A general architecture

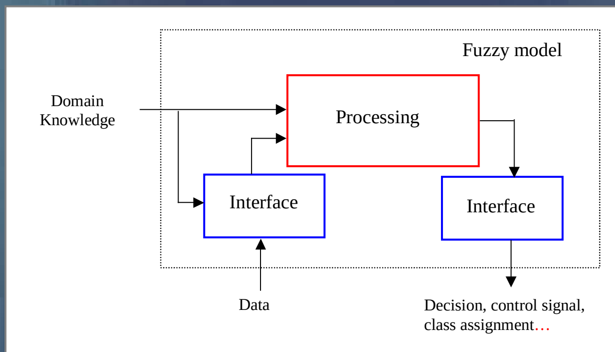


A general architecture: functional modules



- **Input interface:** accepting heterogeneous data (information granules and numeric data) and converting them to internal format where processing at the level of fuzzy sets is carried out
- **Processing module:** processing pertinent to information granules
- **Output interface:** converting results of processing information granules into the format acceptable by the modeling environment

Functional modules of fuzzy models: rule-based systems



- Processing module: collection of rules, $i = 1, 2, \ldots, N$

  If condition$_1$ is A_i ***and*** condition$_2$ is B_i **then** action (decision, conclusion) is D_i

- Input interface:

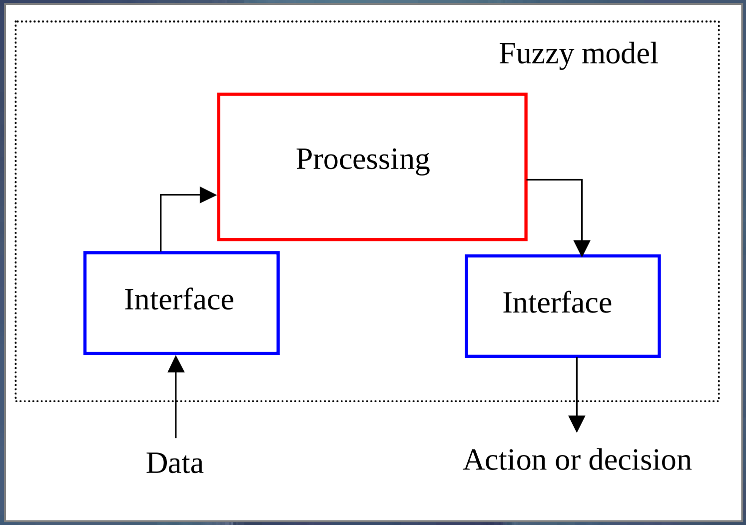
  input X: express it in terms of fuzzy sets A_i present in the conditions of rules

- Output interface:

  decode the result of processing, say fuzzy set D, in the format required by the modeling environment, say a single numeric entity

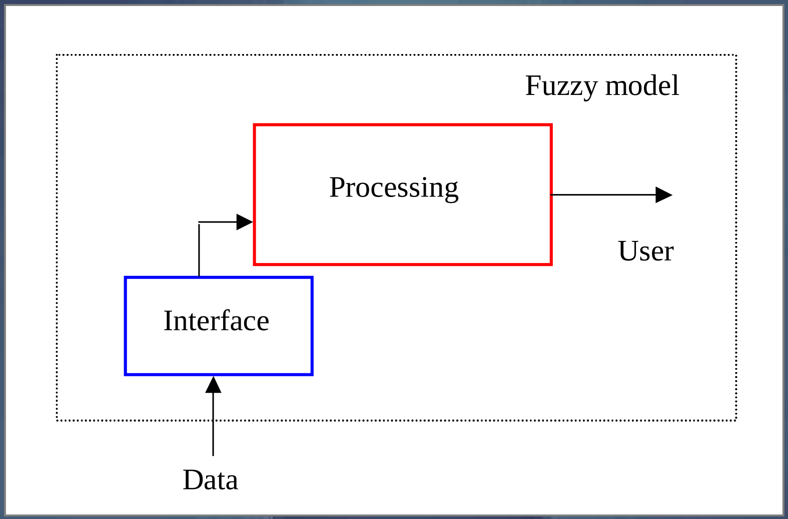
10.2 Key phases of the development and use of fuzzy models

Main modes of use of fuzzy models (a)



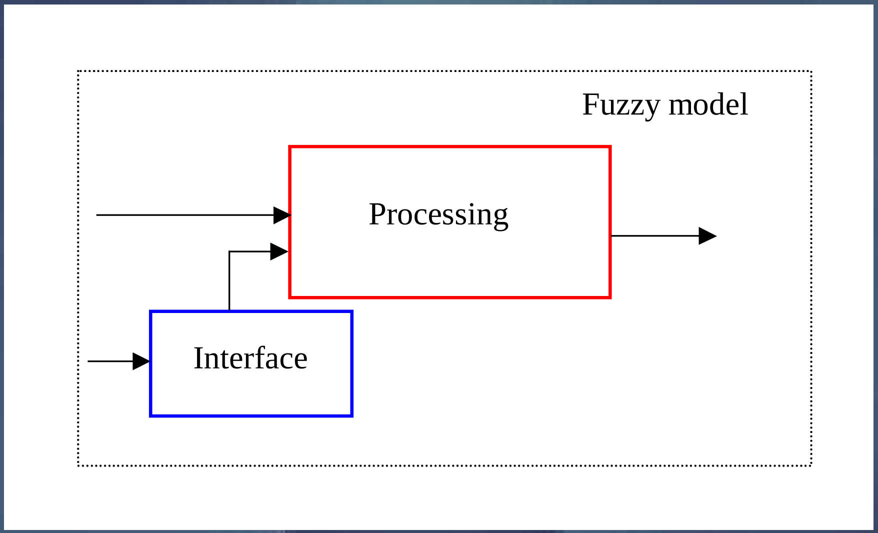
- The use of numeric data and generation of numeric results
- Module reflects a large modeling spectrum
- After development, model is used in purely numerical fashion accepts numbers and produce numbers as nonlinear I/O mappings

Main modes of use of fuzzy models (b)



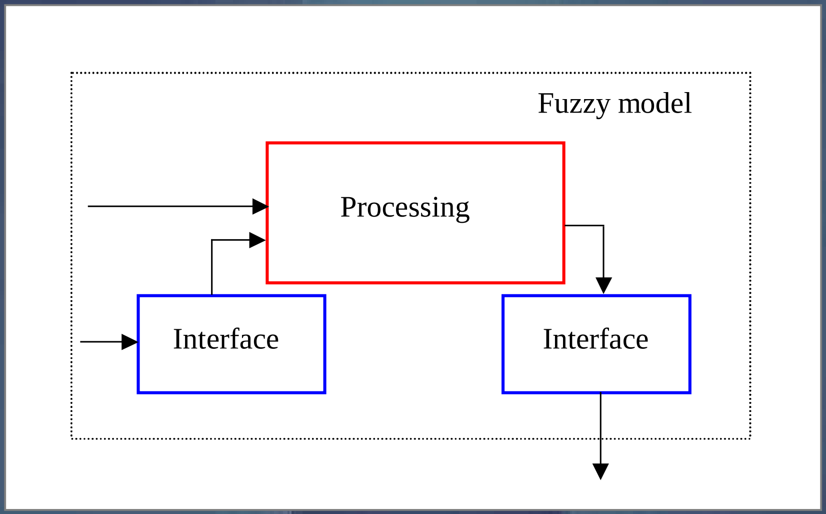
- use of numeric data and granular results (fuzzy sets)
- User centric: more informative and comprehensive than numbers
- User provided with preferences (membership degrees) associated with a collection of possible outcomes

Main modes of use of fuzzy models (c)



- Granular input data and fuzzy sets as outputs
- Scenarios where we encounter collection of linguistic observations
- Examples: expert judgment, unreliable sensor readings, etc.

Main modes of use of fuzzy models (d)



- Use of fuzzy sets as model inputs and outputs
- Granular data forming aggregates of detailed numeric data

10.3 Main categories of fuzzy models: An overview

Main categories of models: An overview

- Diversified landscape of fuzzy models - selected categories:
  - tabular fuzzy models
  - rule-based fuzzy models
  - fuzzy relational models including associative memories
  - fuzzy decision trees
  - fuzzy neural networks
  - fuzzy cognitive maps
  - ….

Main categories of models: Some design considerations

- Expressive power
- Processing capabilities ∈
- Design schemes and ensuing optimization
- Interpretability
- Ability to deal with heterogeneous data
- ….

Tabular fuzzy models

- Table **of relationships** between the variables of the system granulated by some fuzzy sets.

- Easy to build and interpret

- Limited processing capabilities (not included as a part of the model)

	B_1	B_2	B_3	B_4	B_5
A_1					
A_2			C_3		
A_3					C_1

Rule-based fuzzy models

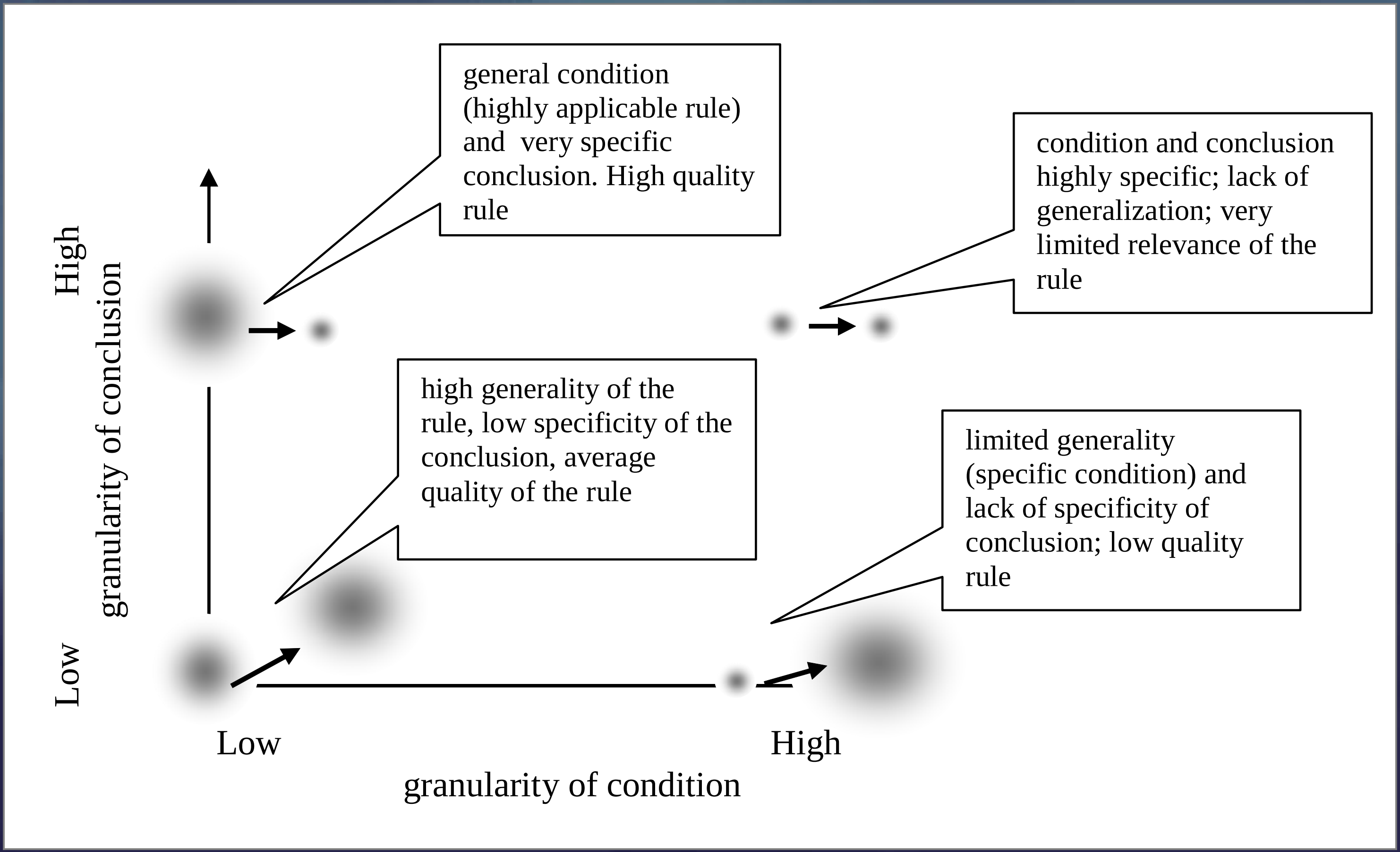
- Highly modular and easily expandable fuzzy models
- Composed of a family of conditional (**If – then)** statements (rules)
- Fuzzy sets occur in their conditions and conclusions
- Standard format

If condition$_1$ is A ***and*** condition$_2$ is B ***and*** … ***and*** condition$_n$ is W

then conclusion is Z

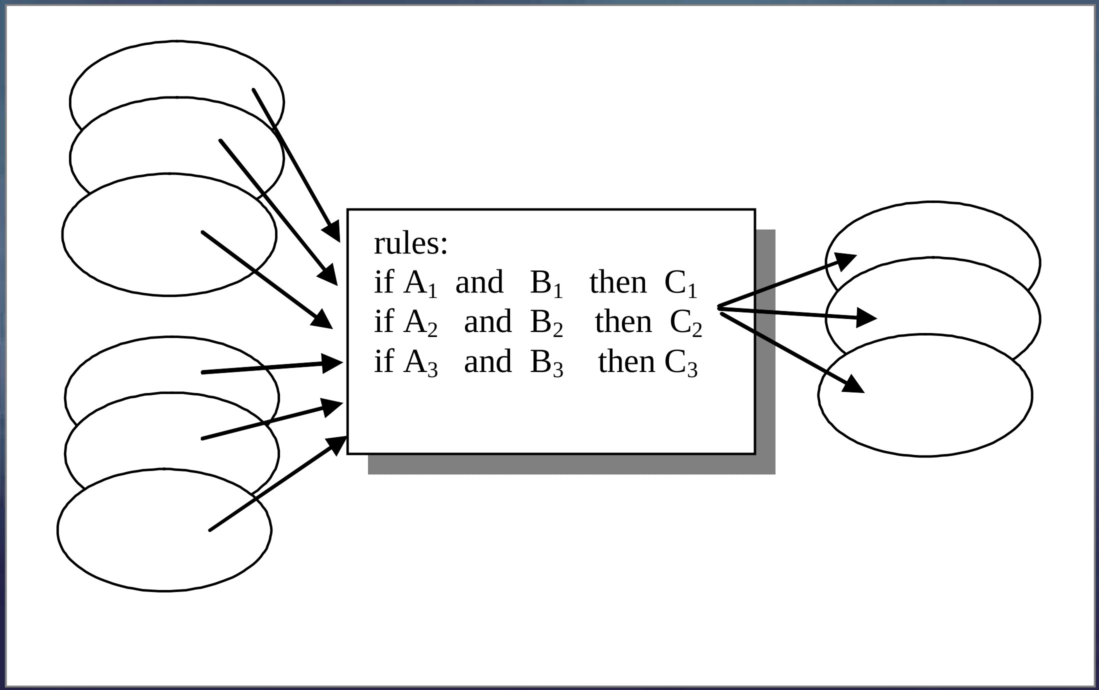
- Conditions ≡ rule antecedent
- Conclusions ≡ rule consequent

Rule-based fuzzy models: Granularity and quality of rules



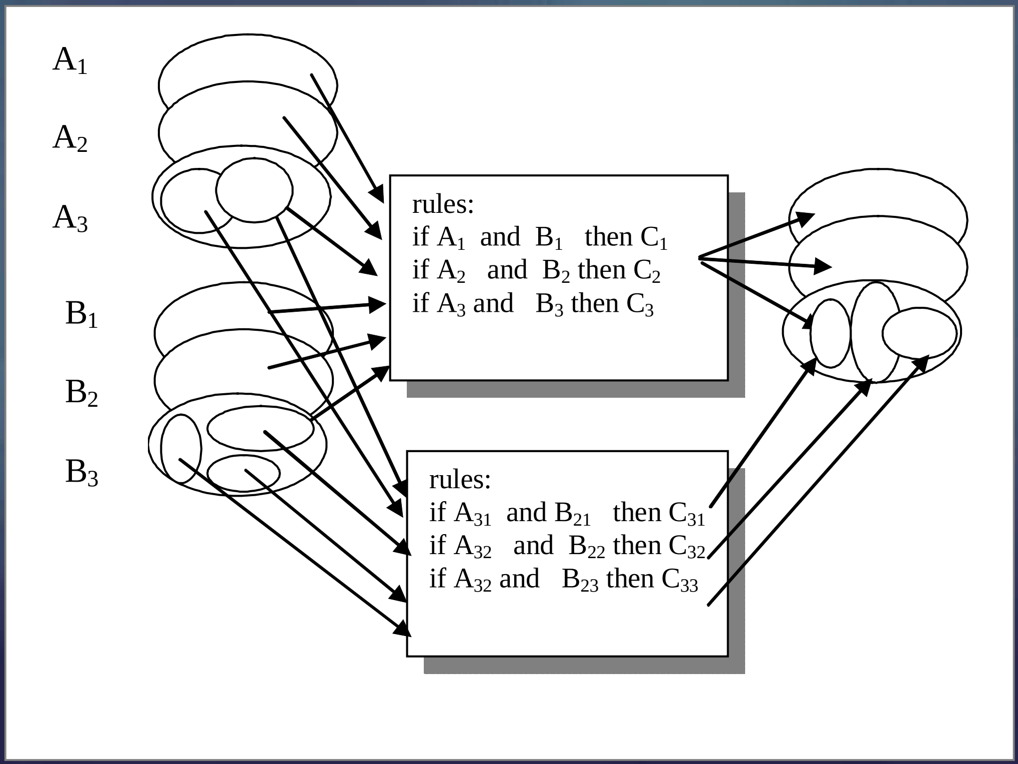
Pedrycz and Gomide, FSE 2007

Granularity of information in rule-based systems



the same level of granularity

Granularity of information in rule-based systems

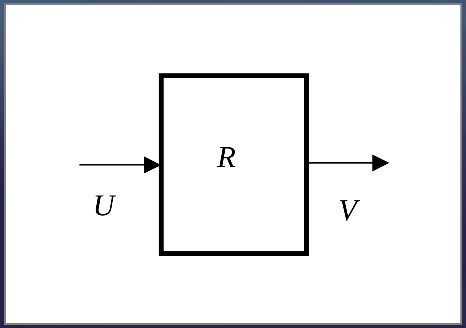


different levels of granularity

Fuzzy relational models and associative memories

- Relational transformation of fuzzy sets

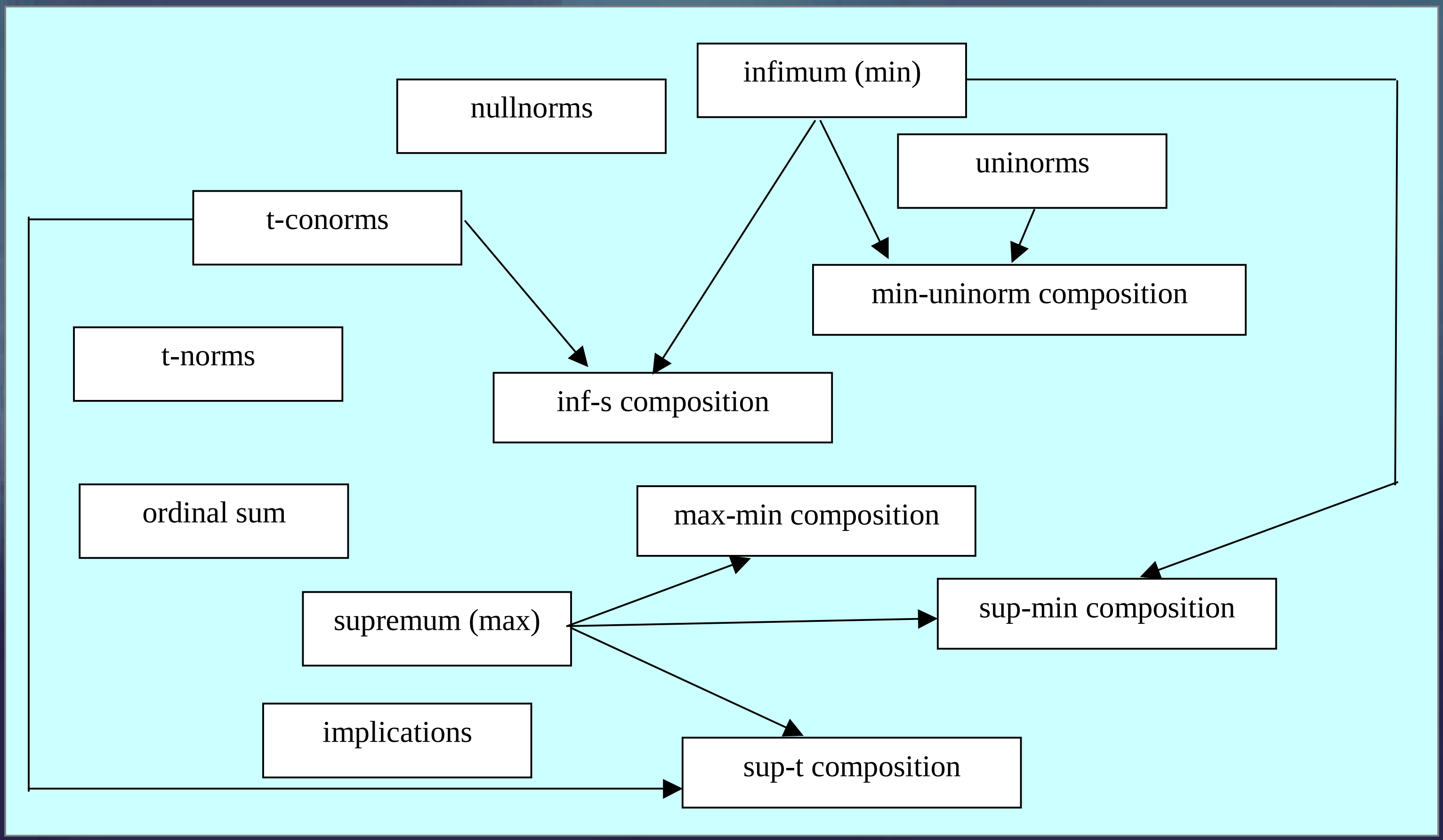
- Two main modes
  - construction of fuzzy relations-storing
  - inference-recall



$$R = \bigcup_{k=1}^{N} (A_k \times B_k)$$

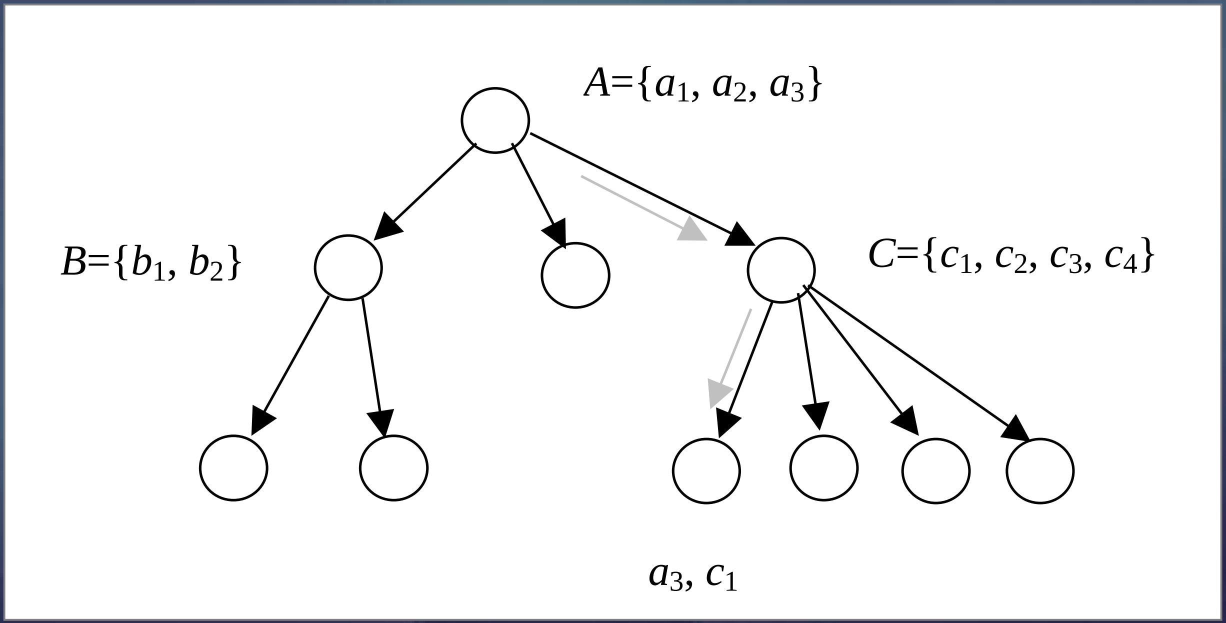
$$V = U \circ R$$

Fuzzy relational structures: A general taxonomy



Fuzzy decision trees

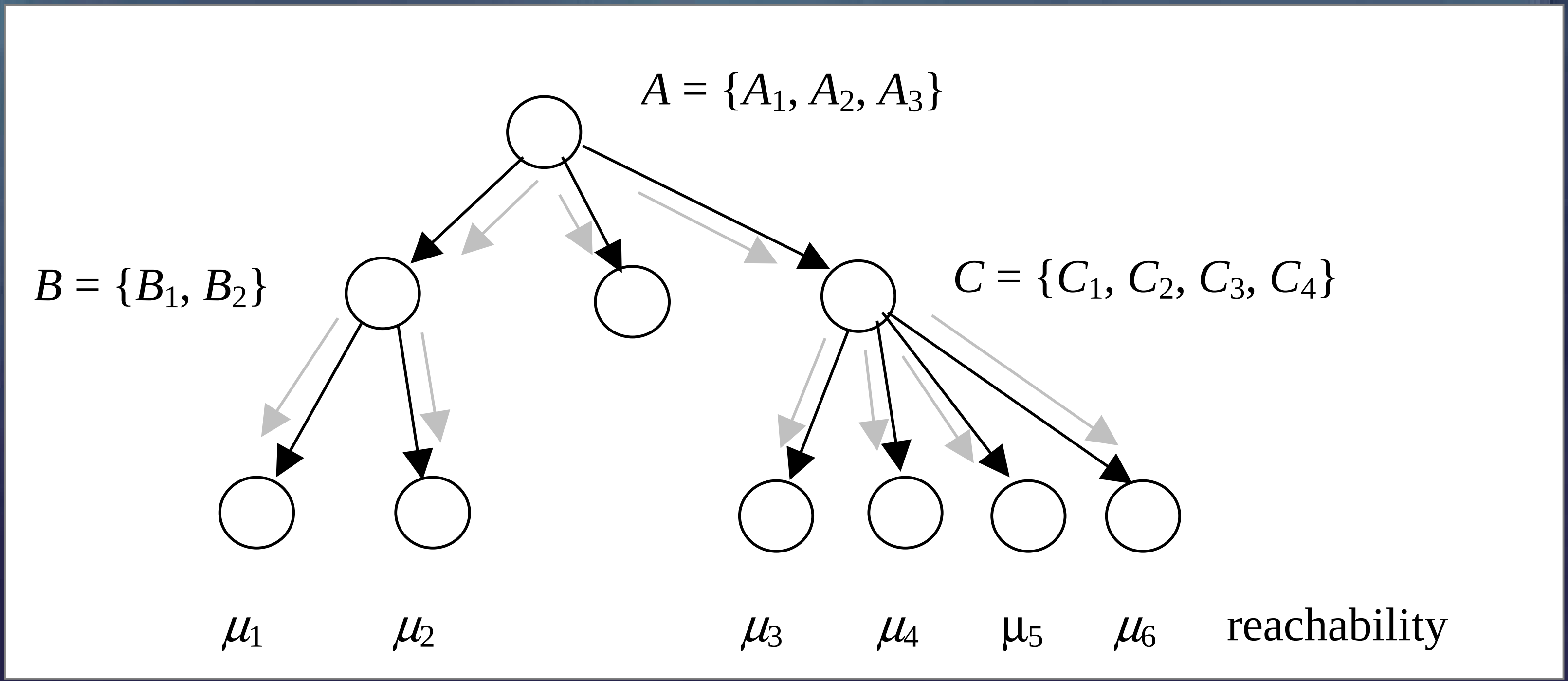
• Generalization of decision trees



• Traversal of tree depending on the values of the attributes: only a single path traversed and a single terminal node reached

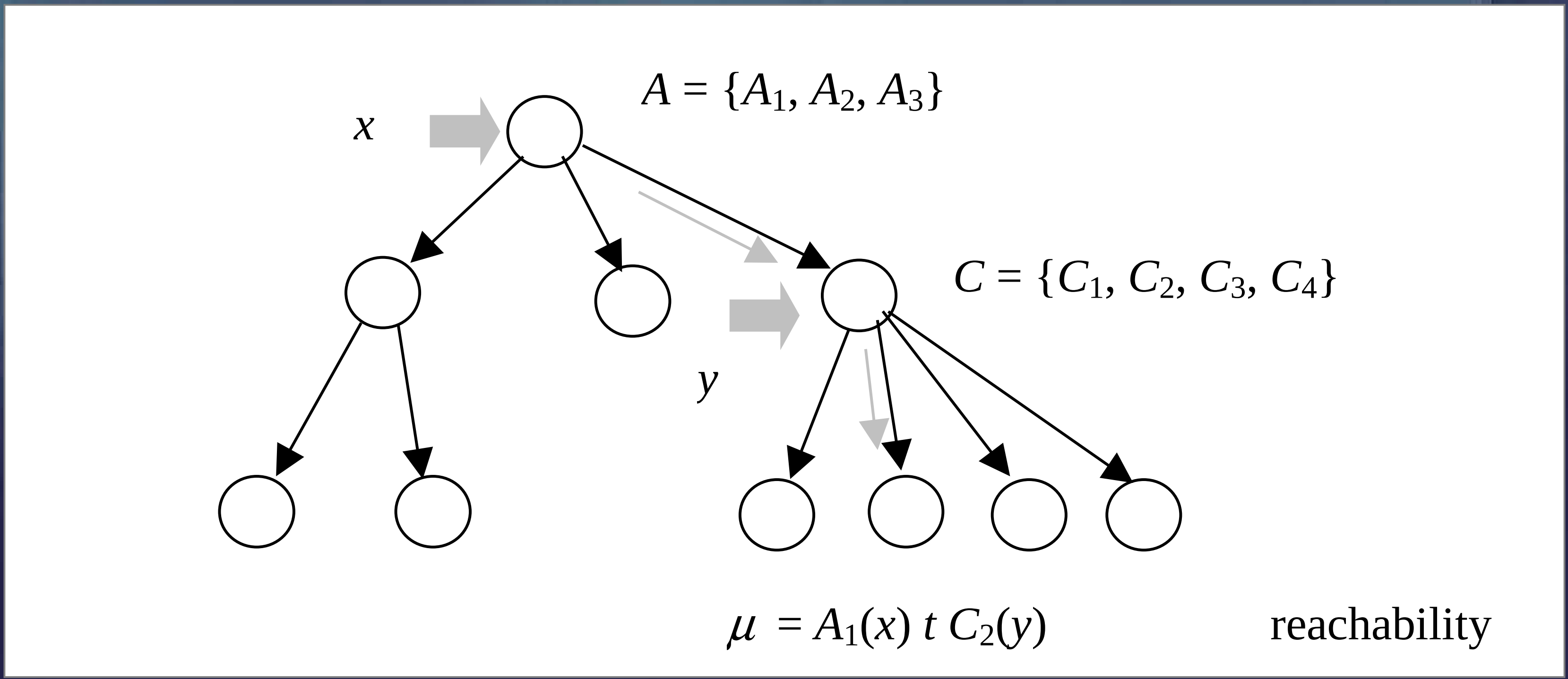
Fuzzy decision trees

- Traversal of a number of paths leading to a number of terminal nodes (reachability levels)



Fuzzy decision trees

• Traversal of a number of paths leading to a number of terminal nodes (reachability levels)



Fuzzy neural networks

- Architectures in which we combine adaptive properties of neural networks with interpretability (transparency) of fuzzy sets

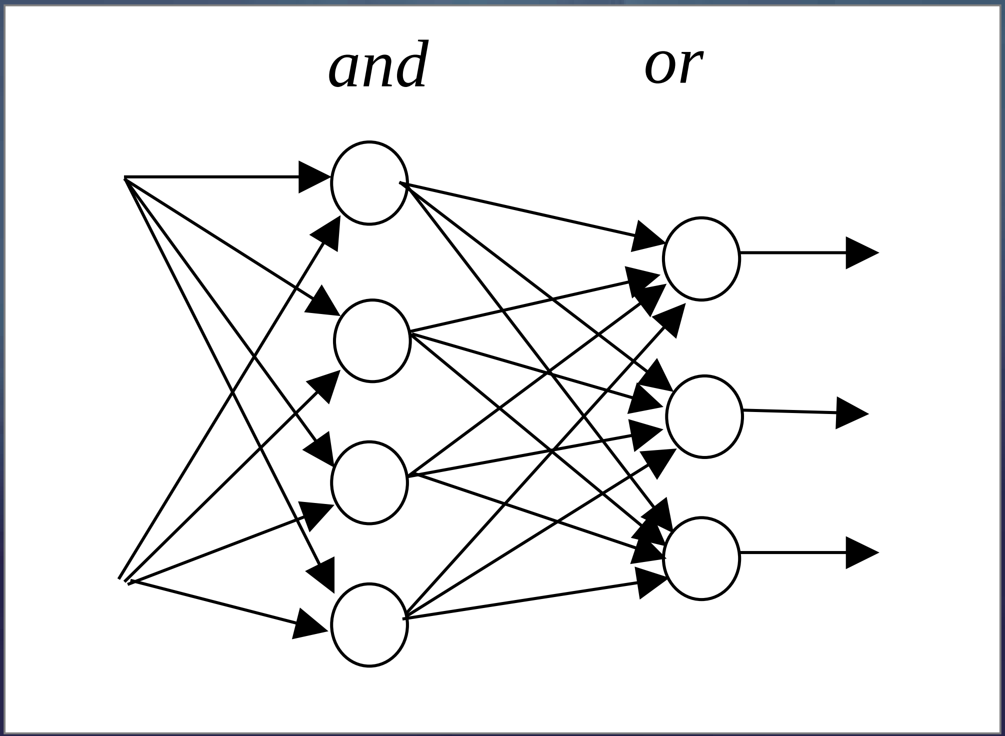
- A suite of fuzzy logic neurons:
  - aggregative neurons (***and***, ***or*** neurons)
  - referential neurons (dominance, equality, inclusion…)

- Learning mechanisms could be applied to adjustment of connections of neurons

- Each neuron comes with a well-defined semantics; the network could be easily interpreted once the training has been completed

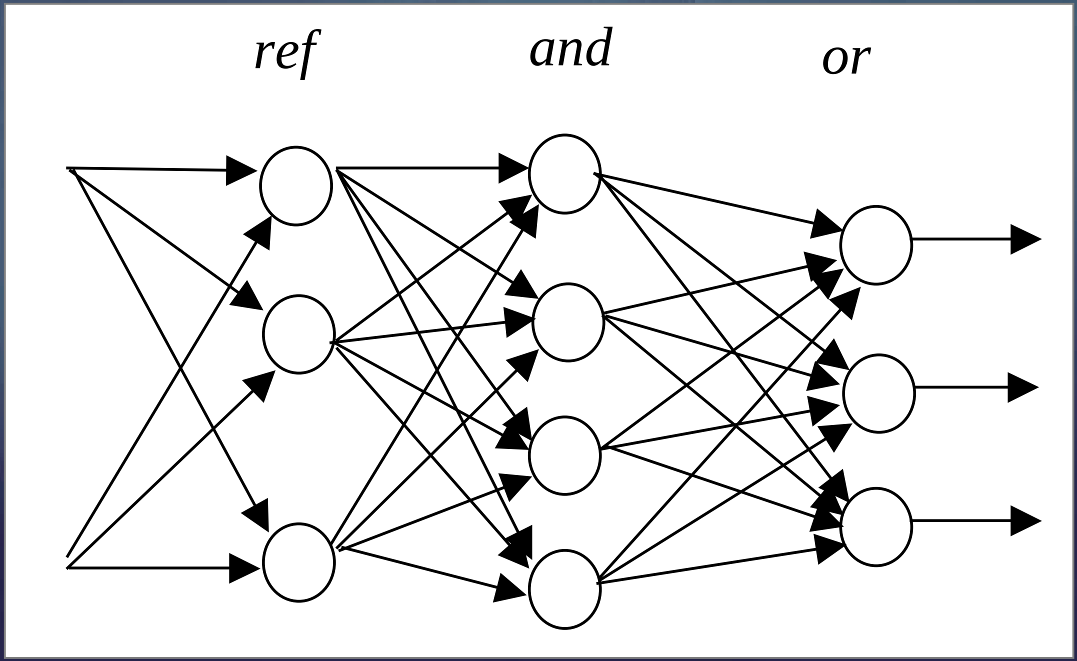
Fuzzy neural networks: Examples of architectures

- Use of ***and*** and ***or*** neurons (logic processor)



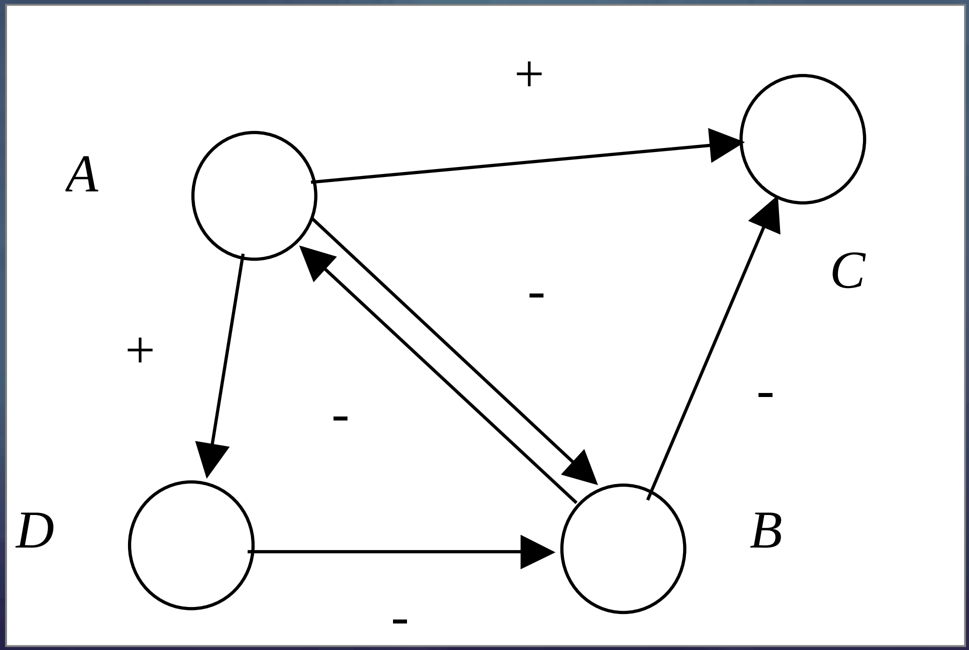
Fuzzy neural networks: Examples of architectures

- Use of **and**, ***or*** and referential (***ref***) neurons



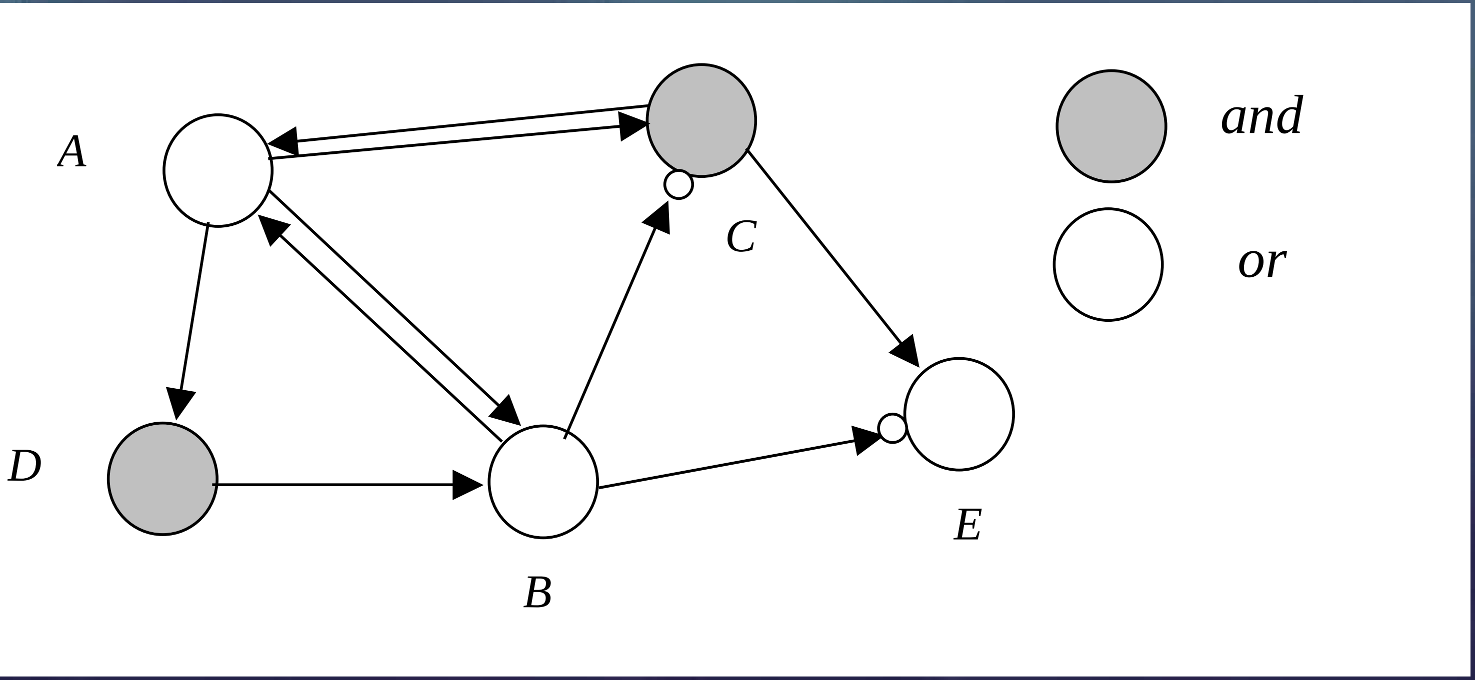
Fuzzy cognitive maps

- Representation of concepts and linkages between concepts
- Directed graph: concepts are nodes; linkages are edges

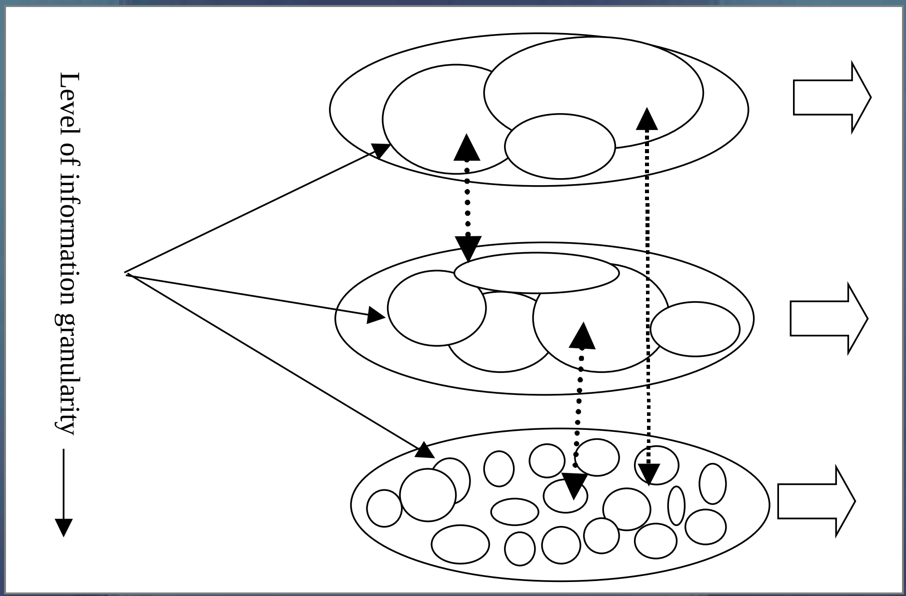


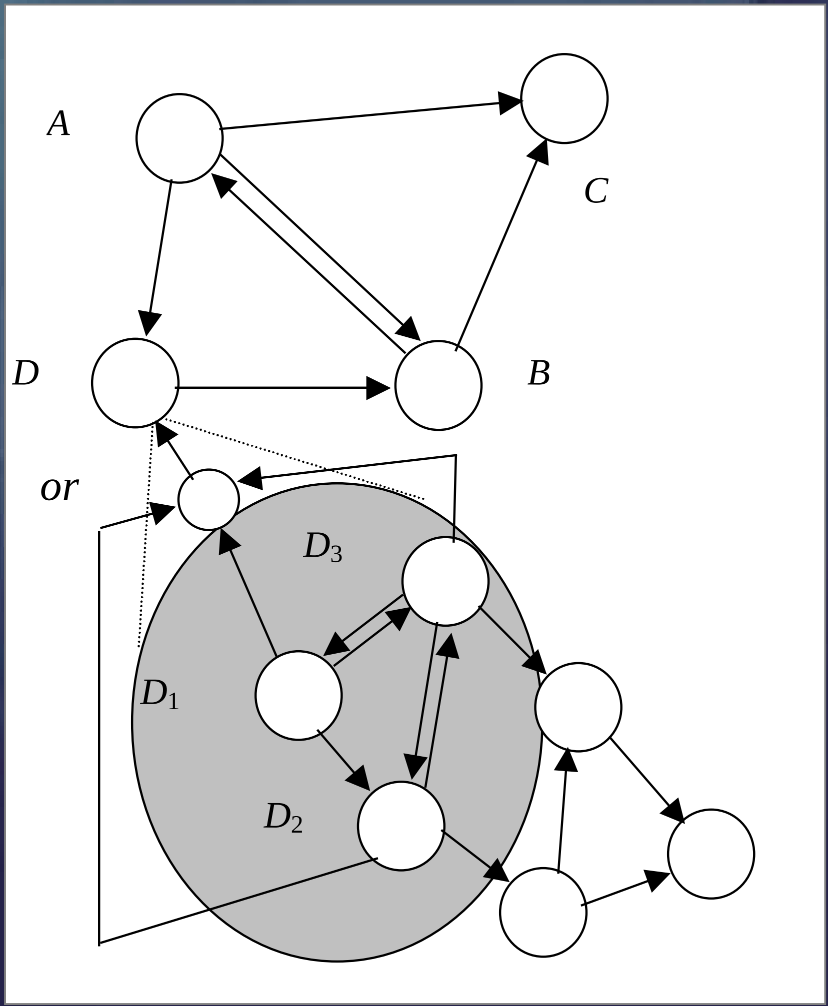
- *A*, *B*, *C*, and *D* = concepts.
- Inhibition (-) or excitation (+) between the concepts (nodes)

Fuzzy cognitive maps: extensions



Fuzzy cognitive maps: hierarchy





10.4 Verification and validation Of fuzzy models

Verification and validation of fuzzy models

- **Verification and Validation** (V&V) are concerned with the development of the model and assessment of its usefulness

- **Verification** is concerned with the analysis of the underlying processes of constructing the fuzzy model do we follow sound design principles ?
  "Are we building the product *right*?"

- **Validation** is concerned with ensuring that the model (product) meets the requirements of the customer
  "Are we building the *right* product?"

Verification of fuzzy models

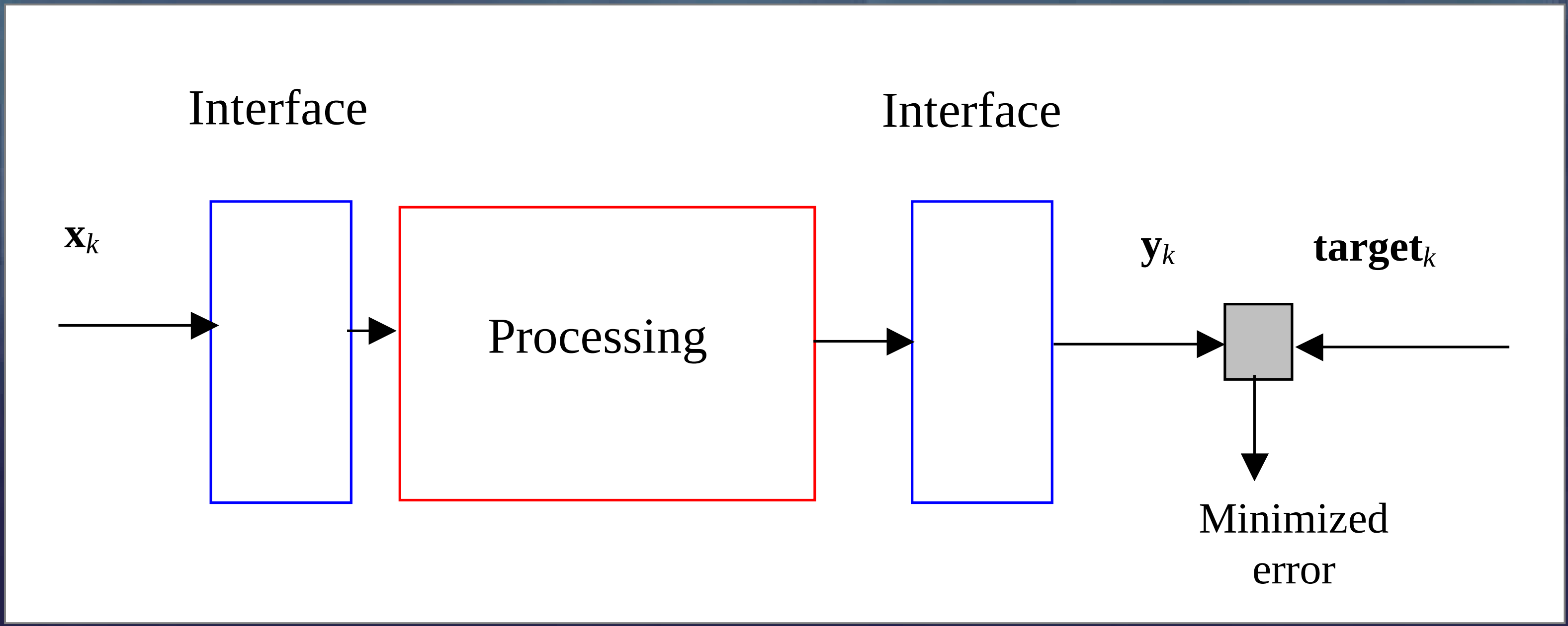
- Sound design principles
  - iterative development process
  - assessment of accuracy
  - generalization capabilities
  - complexity of the model (Occam's principle)
  - high level of autonomy of the model

Fuzzy models: accuracy

- Two ways of expressing accuracy
  - numeric level
  - internal level

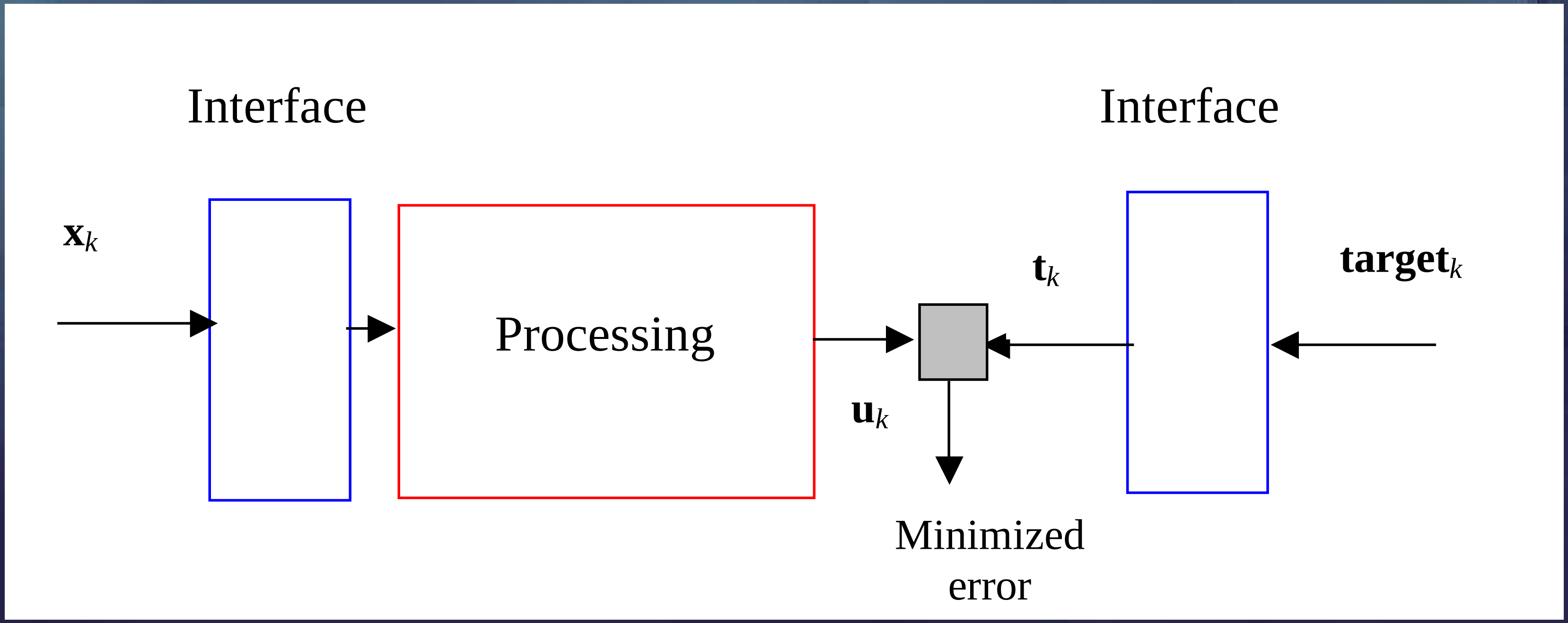
Fuzzy models: accuracy

- Numeric level of expressing accuracy



Fuzzy models: accuracy

- Accuracy expressed at the level of fuzzy sets



Training, validation, and testing data

- To avoid potential bias in assessment of accuracy, data are split into
  - training
  - validation
  - testing subsets
- Training - testing
  - typically 60-40% split
  - 10 fold cross-validation (90-10% split)
  - leave one out strategy

Validation of fuzzy models

- Are we building the ***right*** model?

- More difficult to quantify:

  – transparency of fuzzy models
  – stability of the fuzzy model

….

- Very often validation criteria are in conflict