A Fuzzy Optimization Model for Single-Period Inventory Problem

H. Behret and C. Kahraman

Abstract— **In this paper, the optimization of single-period inventory problem under uncertainty is analyzed. Due to lack of historical data, the demand is subjectively determined and represented by a fuzzy distribution. Uncertain demand causes an uncertain total cost function. This paper intends to find an analytical method for determining the exact expected value of total cost function for a fuzzy single-period inventory problem. To determine the optimum order quantity that minimizes the fuzzy total cost function we use the expected value of a fuzzy function based on credibility theory. The closed-form solutions to the optimum order quantities and corresponding total cost values are derived. Numerical illustrations are presented to demonstrate the validity of the proposed method and to analyze the effects of model parameters on optimum order quantity and optimum cost value. The proposed methodology is applicable to other inventory models under uncertainty.**

Index Terms— **Credibility theory, Fuzzy optimization, Single-period inventory problem, Fuzzy demand.**

I. Introduction

Single-period inventory problem, also known as newsboy problem, tries to find the product's order quantity that minimizes the expected cost of seller with random demand. In single-period inventory problem, product orders are given before the selling period begins. There is no option for an additional order during the selling period or there will be a penalty cost for this re-order. The assumption of the single-period inventory problem is that if any inventory remains at the end of the period, either a discount is used to sell it or it is disposed of. If the order quantity is smaller than the realized demand, the seller misses some profit [1].

Single-period inventory problems are associated with the inventory of items such as newspapers, fashion goods which become obsolete quickly, seasonal goods where a second order during the season is difficult or spare parts for a single production run of products which are stocked only once [2].

An extensive literature review on a variety of extensions of the single-period inventory problem and related multi-stage, inventory control models can be found in [3] and [4]. Most of the extensions have been made in the probabilistic framework, in which the uncertainty of demand is described by probability distributions. However, in real world, sometimes the probability distributions of the demands for products are difficult to acquire due to lack of information and historical data. In this case, the demands are approximately specified based on the experience and managerial subjective judgments and described linguistically such as "demand is about *d*". In such cases, the fuzzy set theory, introduced by Zadeh [5], is the best form that adapts all the uncertainty set to the model. When subjective evaluations are considered, the possibility theory takes the place of the probability theory [6]. The fuzzy set theory can represent linguistic data which cannot be easily modeled by other methods [7].

In the literature, the fuzzy set theory has been applied to inventory problems to handle the uncertainties related to the demand or cost coefficients. An extended review of the application of the fuzzy set theory in inventory management can be found in [8]. The advantage of using the fuzzy set theory in modeling the inventory problems is its ability to quantify vagueness and imprecision.

Some papers have dealt with single-period inventory problem using the fuzzy set theory. Petrovic et al. [9] developed two fuzzy models to handle uncertainty in the single-period inventory problem under discrete fuzzy demand. In the paper the concept of level-2 fuzzy set, s-fuzzification and the method of arithmetic defuzzification are employed to access an optimum order quantity.

Ishii and Konno [10] introduced a fuzzy newsboy model restricted to shortage cost that is given by an L-shape fuzzy number while the demand is still stochastic. An optimum order quantity is obtained in the sense of fuzzy max (min) order of the profit function.

Li et al. [11] studied the single-period inventory problem in two different cases where in one the demand is probabilistic while the cost components are fuzzy and in the other the costs are deterministic but the demand is fuzzy. They showed that the first model reduces to the classical newsboy problem and in the second model the objective function is concave and hence one can readily compute an optimal solution. They applied ordering fuzzy numbers with respect to their total integral values to maximize the total profit.

In order to minimize the fuzzy total cost, Kao and Hsu [12] constructed a single-period inventory model. They adopted a method for ranking fuzzy numbers to find the optimum order quantity in terms of the cost.

Dutta et al. [13] presented a single-period inventory model in an imprecise and uncertain mixed environment. They introduced demand as a fuzzy random variable and

Manuscript received March 3, 2011; revised April 01, 2011.

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ISBN: 978-988-19251-4-5
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

developed a new methodology to determine the optimum order quantity where the optimum is achieved using a graded mean integration representation. In their second study, Dutta et al. [14] extended the single-period inventory model of profit maximization with a reordering strategy in an imprecise environment. They represented a solution procedure using ordering of fuzzy numbers with respect to their possibilistic mean values.

Ji and Shao [15] extended single-period inventory problem in bi-level context. In another study Shao and Ji [16] extended single-period inventory problem in multi-product case with fuzzy demands under budget constraint. In both studies they adopt credibility theory and they solved their models by a hybrid intelligent algorithm based on genetic algorithm and developed a fuzzy simulation.

Lu [17] studied a fuzzy newsvendor problem to analyze optimum order policy based on probabilistic fuzzy sets with hybrid data. They verified that the fuzzy newsvendor model is one extension of the crisp models.

Chen and Ho [18] proposed an analysis method for the single-period (newsboy) inventory problem with fuzzy demands and incremental quantity discounts. The proposed analysis method is based on ranking fuzzy number and optimization theory.

In this paper, we adopt the concept of credibility measure in the credibility theory proposed by Liu [19]. Among the studies mentioned above, Ji and Shao [15] and Shao and Ji [16] adopted the credibility theory and they solved their models by a hybrid intelligent algorithm based on a genetic algorithm and developed a fuzzy simulation. However, simulation technology and heuristic algorithms generally result in errors and cannot provide an analytical solution.

Our paper intends to find an analytical method for determining the exact expected value of total cost function for a single-period inventory problem under uncertainty. To determine the optimal order quantity that minimizes the fuzzy total cost function we use the expected value of a function of a fuzzy variable with a continuous membership function defined by Xue et al. [20].

The rest of the paper is organized as follows. The preliminary concepts about the fuzzy set theory and the credibility theory are subjected in Section 2. In Section 3, an analytical model for the single-period inventory problem under uncertainty is constructed and the closed-form solutions for this problem are proposed. In the next section, the results are illustrated with numerical examples and finally the conclusion of the study is presented in Section 5.

II. PRELIMINARY CONCEPTS

This study aims at finding an analytical method to determine the exact expected value of total cost function for a single-period inventory problem under uncertainty. The source of the uncertainty in the analyzed problem results from the imprecise demand. In order to find the optimal order quantity that minimizes the fuzzy total cost function we use the expected value of a function of a fuzzy variable with a continuous membership function defined by Xue et al. [20]. In this section, the preliminary concepts about the fuzzy set theory and the credibility theory which will be useful for understanding the proposed model and the solution procedure are explained.

Real life is complex and this complexity arises from uncertainty in the form of ambiguity [21]. The approximate reasoning capability of humans gives the opportunity to understand and analyze complex problems based on imprecise or inexact information. Fuzzy logic provides solutions to complex problems through a similar approach as human reasoning. The major characteristic of fuzzy logic is its ability to accurately reflect the ambiguity in human thinking, subjectivity and knowledge to the model.

Fuzzy sets introduced by Zadeh [5] as a mathematical tool to represent ambiguity and vagueness are a generalization of the classical (crisp) set and it is a class of objects with membership grades defined by a membership function. In a classical set, an element of the universe either belongs to or does not belong to the set while in a fuzzy set, the degree of membership of each element ranges over the unit interval. A fuzzy set can be mathematically represented as $\tilde{A} = \left(x, \mu_{\tilde{A}}(x)\right), \forall x \in X$ where X is the universal set and $\mu_{\tilde{A}}(x)$ is the membership function.

A. Fuzzy Variable

The fuzzy set theory has been applied in many scientific fields. Researchers quantify a fuzzy event using a fuzzy variable (or fuzzy number) or a function of a fuzzy variable [20]. A fuzzy number is a convex normalized fuzzy set whose membership function is piecewise continuous [7].

Suppose $\tilde{X}$ is a generalized fuzzy number (known as *L-R* type fuzzy number), whose membership function $\mu_{\tilde{X}}(x)$ satisfies the following conditions with $0 < w \leq 1$ and $-\infty < l < m < n < u < \infty$,[22];

1) $\mu_{\tilde{X}}(x)$ is a continuous mapping from $(\Re)$, to the closed interval$[0,1]$,
2) $\mu_{\tilde{X}}(x) = 0, -\infty < x < l,$
3) $\mu_{\tilde{X}}(x) = L(x)$, is strictly increasing on $[l, m]$,
4) $\mu_{\tilde{X}}(x) = w,\ m < x < n,$
5) $\mu_{\tilde{X}}(x) = R(x)$, is strictly decreasing on$[n, u]$,
6) $\mu_{\tilde{X}}(x) = 0,\ u < x < u.$

This type of generalized fuzzy number is denoted as $\tilde{X} = (l, m, n, u; w)_{LR}$. When $= 1$, it can be simplified as $\tilde{X} = (l, m, n, u)_{LR}$.

B. Credibility Theory

The possibility theory was proposed by Zadeh [6], and developed by many researchers such as Dubois and Prade [23]. In the possibility theory, there are two measures including possibility and necessary measures. A fuzzy event may fail even though its possibility achieves 1, and hold even though its necessity is 0. Possibility measure is thought as a parallel concept of probability measure. However, as many researchers mentioned before, these two measures have partial differences. Necessity measure is the dual of possibility measure [23]. However, neither possibility measure nor necessity measure has self-duality property. A self-dual measure is absolutely needed in both theory and practice. In order to define a self-dual measure, Liu and Liu [19] introduced the concept of credibility measure in 2002. In this concept credibility measure resembles the similar properties by probability measure. Credibility measure plays the role of probability measure in fuzzy world [24].

The credibility theory developed as a branch of mathematics for studying the behavior of fuzzy phenomena

ISBN: 978-988-19251-4-5
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

[25, 26].

Let ξ be a fuzzy variable with the membership function $\mu(x)$and r be a real number, then the possibility and necessity measure of the fuzzy event $\{\xi \leq r\}$ can be respectively represented as in (1) and (2);

$$Pos\{\xi \leq r\} = supp_{x \leq r}\,\mu(x) \qquad (1)$$

$$Nec\{\xi \leq r\} = 1 - Pos\{\xi > r\} = 1 - \underset{x>r}{supp}\,\mu(x) \qquad (2)$$

The credibility measure which is introduced by Liu and Liu [19] is the average of possibility measure and necessity measure. The credibility measure of the fuzzy event $\{\xi \leq r\}$can be represented as as in (3);

$$Cr\{\xi \leq r\} = \tfrac{1}{2}(Pos\{\xi \leq r\} + Nec\{\xi \leq r\}) \qquad (3)$$

Similarly the credibility measure of the fuzzy event $\{\xi \geq r\}$ can be represented as in (4);

$$Cr\{\xi \geq r\} = \tfrac{1}{2}(Pos\{\xi \geq r\} + Nec\{\xi \geq r\}) \qquad (4)$$

C. *Expected Value of a Function of a Fuzzy Variable*

In the literature the concept of expectation for fuzzy numbers in intervals was defined by Dubois and Prade [27]. The concepts of *expected value* and *the expected value of fuzzy number* were introduced by Heilpern [28], where the expected value of a fuzzy number is defined as the center of the expected interval. Furthermore, several ranking methods for fuzzy numbers are used to find the expected value of a fuzzy number [29], [30] and [31]. In the previous literature getting the analytical expected value of a function of a fuzzy variable has been a complex process for researchers. The expected value is usually computed using simulation techniques or heuristic algorithms which result in errors. However, in the recent literature, Xue et al. [20] introduced an analytical method which uses the properties of the credibility measure, to calculate the expected value of a function of a fuzzy variable.

Let ξ be a fuzzy variable and r be a real number, the expected value of a fuzzy variable $E[\xi]$ can be calculated as in (5) [19];

$$E[\xi] = \int_0^{+\infty} Cr\{\xi \geq r\}dr - \int_{-\infty}^{0} Cr\{\xi \leq r\}dr \qquad (5)$$

provided that at least one of the two integrals is finite.

Let ξ be a fuzzy variable with a continuous membership function $\mu_\xi(x)$, and $f: \mathfrak{R} \rightarrow \mathfrak{R}$ is a strictly monotonic function. If the Lebesgue integrals, $\int_0^{+\infty} Cr\{\xi \geq r\}dr$ and $\int_{-\infty}^{0} Cr\{\xi \leq r\}dr$ are finite, then the expected value of a function of a fuzzy variable $E[f(\xi)]$ can be calculated as in (6), [20];

$$E[f(\xi)] = \int_{-\infty}^{+\infty} f(r)dCr\{\xi \leq r\} \qquad (6)$$

Let ξ, be a fuzzy variable with a continuous membership function $\mu(x)$ and $f: \mathfrak{R} \rightarrow \mathfrak{R}$ and $g: \mathfrak{R} \rightarrow \mathfrak{R}$ be two different strictly monotonic functions. If the expected values of ξ,$f(\xi)$ and $g(\xi)$ exist, the following properties can be defined for any numbers p and q , [19];

$$E[pf(\xi) + q] = pE[f(\xi)] + q \qquad (7)$$

$$E[f(\xi) + g(\xi)] = E[f(\xi)] + E[g(\xi)] \qquad (8)$$

Let ξ be a fuzzy variable whose support is $[a, b]$ and $f: \mathfrak{R} \rightarrow \mathfrak{R}$ is a strictly monotonic function. If the Lebesgue integrals, $\int_0^{+\infty} Cr\{\xi \geq r\}dr$ and $\int_{-\infty}^{0} Cr\{\xi \leq r\}dr$ are finite, then the expected value of a function of a fuzzy variable $E[f(\xi)]$ can be calculated as in (9), [20];

$$E[f(\xi)] = \int_a^b f(r)dCr\{\xi \leq r\} \qquad (9)$$

III. PROBLEM FORMULATION AND METHODOLOGY

A. *Expected Value of a Function of a Fuzzy Variable*

Consider a single-period inventory problem where the demand is subjectively believed to be imprecise and represented by a generalized fuzzy number $(\tilde{X})$ with the following membership function given by (10) and Fig.1;

$$\mu_{\tilde{X}}(x) = \begin{cases} L(x), & l \leq x \leq m \\ 1, & m \leq x \leq n \\ R(x), & n \leq x \leq u \\ 0, & other \end{cases} \qquad (10)$$

Here $[m, n]$ are the most likely values of fuzzy number $\tilde{X}$; l and u are the lower and upper values ; $L(x)$and $R(x)$ are the left and right shape functions, respectively.

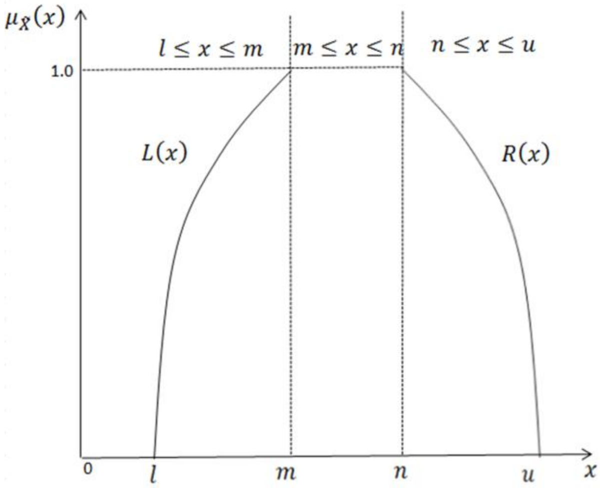

Fig. 1. Membership function of a generalized fuzzy number.

Contrary to demand, inventory cost coefficients are known precisely. A product is produced (or procured) at a cost of v for a single-period, h is the unit inventory holding cost per unit remaining at the end of the period, ($h <$ 0 represents the salvage value per remaining unit), normally the salvage value is less than the unit production cost, i.e. $(v > -h)$ [12]. The selling price per unit is assumed to be equal to the unit inventory shortage cost which is represented by b in the model $(b > v)$. We assume that there is no on-hand inventory at the beginning of the period.

Uncertain demand will cause an uncertain total cost function. If we order Q units, the total cost function $\widetilde{TC}(Q)$ will be as in (11);

ISBN: 978-988-19251-4-5
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

$$\widetilde{TC}(Q) = \begin{cases} Qv + h(Q - \tilde{X}), & \tilde{X} \leq Q \\ Qv + b(\tilde{X} - Q), & \tilde{X} \geq Q \end{cases} \quad (11)$$

Since the demand is a generalized fuzzy number $(\tilde{X})$, the fuzzy total cost function $\widetilde{TC}(Q)$ will also be a generalized fuzzy number with the same membership function of demand. The problem is to determine the optimal order quantity (Q^*) that minimizes the fuzzy total cost. The membership function of demand is represented as in Fig.1. According to this membership function, it is obvious that the optimal order quantity (Q^*) will remain between l and u, since $(b > v > -h)$ [12].

B. Analytical Model and Solution Methodology

We consider the above single-period inventory problem where the demand and total cost function, which are formulated as in (11), are generalized fuzzy numbers. The fuzzy total cost function, is strictly decreasing in $l \leq x \leq Q$ and strictly increasing in $Q \leq x \leq u$. In order to optimize the single-period inventory problem with fuzzy demand, we use the expected value method which is proposed by Xue et al. [20]. In this context, by using (6) and (8), the expected value of total cost function $E[\widetilde{TC}(Q)]$ of fuzzy demand $\tilde{X}$ will be formulated as in (12);

$$\begin{aligned} E[\widetilde{TC}(Q)] &= \int_l^Q [Qv + h(Q - r)] dCr\{\tilde{X} \leq r\} + \\ &\quad \int_Q^u [Qv + b(r - Q)] dCr\{\tilde{X} \leq r\} \quad (12) \\ &= Qv + hQ \int_l^Q dCr\{\tilde{X} \leq r\} - h \int_l^Q r dCr\{\tilde{X} \leq \\ &\quad r\} + b \int_Q^u r dCr\{\tilde{X} \leq r\} - bQ \int_Q^u dCr\{\tilde{X} \leq r\} \end{aligned}$$

The optimal order quantity (Q^*), will be the value that minimizes the expected value of fuzzy total cost function. To find the minimizing value of Q, we set

$$\frac{dE[\widetilde{TC}(Q)]}{dQ} = 0 \quad (13)$$

Thus, the following equations are obtained,

$$\begin{aligned} \frac{dE[\widetilde{TC}(Q)]}{dQ} &= v + h \int_l^Q dCr\{\tilde{X} \leq r\} - b \int_Q^u dCr\{\tilde{X} \leq r\} \\ 0 &= v + (h + b) Cr\{\tilde{X} \leq Q\} - b, \quad (14) \end{aligned}$$

$$Cr\{\tilde{X} \leq Q\} = \frac{b - v}{h + b} \quad (15)$$

To find the optimum order quantity, we will determine the value of $Cr\{\tilde{X} \leq Q\}$. The credibility of fuzzy demand with the membership function as in (10) will be as in (16);

$$Cr\{\tilde{X} \leq Q\} = \begin{cases} 0, & Q \leq l \\ \frac{L(Q)}{2}, & l \leq Q \leq m \\ \frac{1}{2}, & m \leq Q \leq n \\ 1 - \frac{R(Q)}{2}, & n \leq Q \leq u \\ 1, & Q \geq u \end{cases} \quad (16)$$

There are three cases to be analyzed for the value of Q in discussing the credibility value of fuzzy demand; $Cr\{\tilde{X} \leq Q\}$: $l \leq Q \leq m$, $m \leq Q \leq n$ and $n \leq Q \leq u$.

Case 1: $l \leq Q \leq m$

$$Cr\{\tilde{X} \leq Q\} = \frac{b - v}{h + b} = \frac{L(Q)}{2} \quad (17)$$

Thus, the following equations are obtained,

$$L(Q) = \frac{2(b - v)}{h + b} \text{ and } Q^* = L^{-1}\left(\frac{2(b - v)}{h + b}\right). \quad (18)$$

Here, the value of $2(b - v)/(h + b)$ must lie between [0,1], thus we can say that under the conditions of $b \geq v$ and $b - v \leq h + v$, Q^* will lie in $[l, m]$. Moreover, the second derivative of $E[\widetilde{TC}(Q)]$ with respect to Q is,

$$\frac{d^2 E[\widetilde{TC}(Q)]}{dQ} = \frac{(h + b)}{2} L'(Q). \quad (19)$$

Since, $L(Q)$ is an increasing function in $[l, m]$, we find that $L'(Q) > 0$. The values of h and b are positive, so that we obtain $\frac{d^2 E[\widetilde{TC}(Q)]}{dQ} > 0$ which implies that Q^* is the optimum value which minimizes $E[\widetilde{TC}(Q)]$. The expected fuzzy total cost value for the optimum order quantity will be as below;

$$\begin{aligned} E[\widetilde{TC}(Q^*)] = &-h \int_l^{L^{-1}\left(\frac{2(b-v)}{h+b}\right)} r d\left(\frac{L(r)}{2}\right) \\ &+ b \int_{L^{-1}\left(\frac{2(b-v)}{h+b}\right)}^m r d\left(\frac{L(r)}{2}\right) \quad (20) \\ &+ b \int_n^u r d\left(1 - \frac{R(r)}{2}\right) \end{aligned}$$

Case 2: $m \leq Q \leq n$

$$Cr\{\tilde{X} \leq Q\} = \frac{b - v}{h + b} = \frac{1}{2} \quad (21)$$

Optimum order quantity (Q^*) is the value that the first derivative of the expected value of fuzzy total cost equals to zero, in this case,

$$\begin{aligned} \frac{dE[\widetilde{TC}(Q)]}{dQ} &= v + (h + b) Cr\{X \leq Q\} - b \\ 0 &= v + (h + b)\frac{1}{2} - b. \quad (22) \end{aligned}$$

If there exists $b - v = h + v$ case, then $E[\widetilde{TC}(Q)]$ will be minimized by any $Q \epsilon [m, n]$. Thereby, under the condition of $b - v = h + v$, we can say that $Q^* \epsilon [m, n]$.

The expected fuzzy total cost value for the optimum order quantity will be as in (23);

$$E[\widetilde{TC}(Q^*)] = -h \int_l^m r d\left(\frac{L(r)}{2}\right) + b \int_n^u r d\left(1 - \frac{R(r)}{2}\right) \quad (23)$$

Case 3: $n \leq Q \leq u$

$$Cr\{\tilde{X} \leq Q\} = \frac{b - v}{h + b} = 1 - \frac{R(Q)}{2} \quad (24)$$

and the following equations are obtained,

$$R(Q) = \frac{2(h + v)}{h + b} \text{ and } Q^* = R^{-1}\left(\frac{2(h + v)}{h + b}\right). \quad (25)$$

Since the value of $2(h + v)/(h + b)$ must be in the range of 0 and 1, under the conditions of $v \geq -h$ and $b - v \geq h + v$, we can say that the value of Q^* will lie in $[n, u]$. Additionally, the second derivative of $E[\widetilde{TC}(Q)]$ with respect to Q will be,

$$\frac{d^2 E[\widetilde{TC}(Q)]}{dQ} = -\frac{(h + b)}{2} R'(Q). \quad (26)$$

Since $R(Q)$ is a decreasing function in the range of l and m, with $L'(Q) > 0$ and the values of h and b are positive, we obtain $\frac{d^2 E[\widetilde{TC}(Q)]}{dQ} > 0$ which implies that Q^* is

ISBN: 978-988-19251-4-5
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

the optimum value which minimizes $E[\widetilde{TC}(Q)]$. The expected fuzzy total cost value for the optimum order quantity will be as in (27);

$$E[\widetilde{TC}(Q^*)] = -h\int_l^m rd\left(\frac{L(r)}{2}\right) -h\int_n^{R^{-1}\left(\frac{2(h+v)}{h+b}\right)} rd\left(1-\frac{R(r)}{2}\right) +b\int_{R^{-1}\left(\frac{2(h+v)}{h+b}\right)}^{u} rd\left(1-\frac{R(r)}{2}\right) \quad (27)$$

In the literature, trapezoidal fuzzy numbers are used commonly in the applications. Let the demand for the single-period inventory problem to be represented by a trapezoidal fuzzy number with the following membership function,

$$\mu_{\tilde{X}}(x) = \begin{cases} \frac{x-l}{m-l}, & l \le x \le m \\ 1, & m \le x \le n \\ \frac{u-x}{u-n}, & n \le x \le u \\ 0, & other \end{cases} \quad (28)$$

In this case, optimum order quantity will be calculated as in (29);

$$Q^* = \begin{cases} l + \left[\frac{2(b-v)}{h+b}\right](m-l), & b-v \le h+v \\ [m,n], & b-v = h+v \\ u - \left[\frac{2(h+v)}{h+b}\right](u-n), & b-v \ge h+v \end{cases} \quad (29)$$

The above equations, give the closed-form solutions for single-period inventory model under fuzzy demand. Optimum order quantity and the corresponding optimum cost values for single-period inventory problem can be calculated easily by using the proposed approach. Moreover, closed-form solutions give the opportunity to analyze the effects of model parameters on optimum order quantity and optimum cost value.

In the proposed model, when we consider the unit inventory shortage cost as the cost of missed profit, the value of $b-v$ represents the unit profit gained from selling one unit of the item. On the other hand, the value of $h+v$ represents the loss incurred for each unsold item when the unit production cost is v. When the unit profit gained from selling one unit is smaller than the loss incurred for each item left unsold, the inventory policy of the management should consider reducing the leftover units conservatively. In contrast, if the profit gained is larger than the loss incurred, then the inventory policy should be aggressive to meet the possible demand. When the profit equals the loss incurred, the order quantity should be equal to the most likely demand [12].

IV. NUMERICAL ILLUSTRATION

Consider a single-period inventory problem where the demand is a trapezoidal fuzzy number represented as $\tilde{X} = (400,1000,1200,1800)$. The membership function of fuzzy demand is given by the following equation and Fig.2.

$$\mu_{\tilde{X}}(x) = \begin{cases} \frac{x-400}{1000-400}, & 400 \le x \le 1000 \\ 1, & 1000 \le x \le 1200 \\ \frac{1800-x}{1800-1200}, & 1200 \le x \le 1800 \\ 0, & other \end{cases}$$

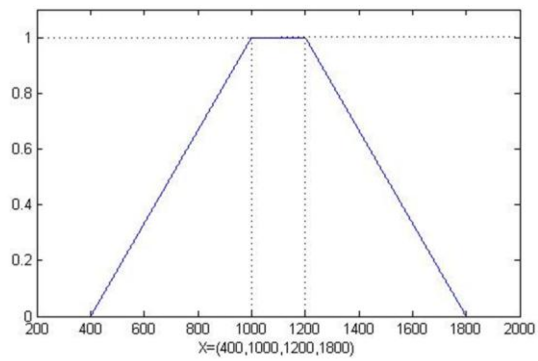

Fig. 2. Trapezoidal membership function of fuzzy demand.

Suppose, the unit production cost is $v = \$30$ for a single-period and inventory holding cost and shortage cost values are respectively, $h = \$10$ and $b = \$50$. Since, the following equation holds; $\$50 - \$30 \le \$10 + \$30$, the optimum order quantity will be in the interval $[400,1000]$. In this case Q^* will be calculated by using (29) as below;

$$Q^* = l + \left[\frac{2(b-v)}{h+b}\right](m-l) = 800$$

And the corresponding cost value for $Q^* = 800$ will be calculated by using (20) as $E[\widetilde{TC}(800)] = \$43{,}000$.

If we decrease the unit production cost to $v = \$20$, then the second case will remain valid where $\$50 - \$20 = \$10 + \$20$ and the values in the interval of $[1000,1200]$ will all be optimal ($Q^* \in [1000,1200]$). Corresponding cost value for $Q^* \in [1000,1200]$ will be equal to \$34,000, $\forall Q^* \in [1000,1200]$.If we decrease unit production cost a little more to $v = \$10$, then the following equation will hold $\$50 - \$10 \ge \$10 + \$10$ and the optimum order quantity will be in the interval of $[1200,1800]$. For this case, the optimum order quantity and the corresponding cost value will be calculated by using (29) and (27), respectively as $Q^* = 1400$ and $E[\widetilde{TC}(1400)] = \$21{,}000$.

In the above analysis, we notice that the optimum order quantities and the corresponding cost values change according to the unit production cost values. As the unit cost values decrease, corresponding optimum cost values also decrease. In the first case, for $v = \$30$, since the unit profit gained from selling one unit is less than the loss incurred for each item left unsold, the inventory policy acted towards reducing the leftover units by setting the optimum order quantity value as $Q^* = 800$ (which is a value amongst left shape function of demand). In the second case, the unit production cost is reduced to $v = \$20$, for which the profit equals the loss incurred. In this case, the order quantity equals to the most likely demand values ($Q^* \in [1000,1200]$). Finally, in the third case, the unit production cost decreased a little more to the value of $v = \$10$, where the profit gained became larger than the loss incurred. Here the inventory policy acted towards meeting the possible demand by setting the optimum order quantity value as $Q^* = 1400$ (which is a value amongst right shape function of demand).

By using the closed-form solutions for single-period inventory model under fuzzy demand, we can analyze the effects of demand fuzziness on optimum order quantity and optimum cost value. For this purpose, we change the

ISBN: 978-988-19251-4-5
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

fuzziness of demand and observe its effects as in Table 1.

Comparing the values in Table 1, we observe that by decreasing the demand fuzziness, the optimum order quantity increases and total cost value decreases. The reason is that, the less demand uncertainty causes less inventory overage and inventory underage costs which lead to less total cost.

TABLE I
THE EFFECTS OF DEMAND FUZZINESS
($v = \$30, h = \$10$ AND $b = \$50$)

Demand ($\tilde{X}$)	Q^*	$E[\widetilde{TC}(Q^*)]$
(200,1000,1200,2000)	734	\$45,666
(400,1000,1200,1800)	800	\$43,000
(600,1000,1200,1600)	867	\$40,333

V. CONCLUSION

The single-period inventory problem deals with finding the product's order quantity which minimizes the expected cost of seller with random demand. However, in real world, sometimes the probability distribution of the demand for products is difficult to acquire due to lack of information and historical data. This study focuses on possibilistic situations, where the demand is described by a membership function and uncertain demand causes an uncertain total cost function.

This paper proposes an analytical method to obtain the exact expected value of total cost function which is composed of inventory holding, inventory shortage and unit production costs for a single-period inventory problem under uncertainty. To determine the optimum order quantity that minimizes the fuzzy total cost function, the expected value of a fuzzy function based on the credibility theory is employed. By this method, closed-form solutions to the optimum order quantities and corresponding total cost values are derived. The advantages of the closed-form solutions obtained are those: they eliminate the need for enumeration over alternative values and give the opportunity to analyze the effects of model parameters on optimum order quantity and optimum cost value. The proposed methodology, used for optimization based on the credibility theory can be applied to the solution of other complex real world problems where this complexity arises from uncertainty in the form of ambiguity.

The single-period inventory model analyzed in this paper considers only a single type of product. The model can be extended to a multi-product case and the solution procedure can be applied as a further research of this study. Another issue of interest is the examination of the proposed model with imprecise inventory cost coefficients. The analysis of single-period inventory problem with other sources of uncertainty besides imprecise demand is another area of further research.

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