Fuzzy Mathematical Programming Model for Optimizing Airport Capacity Utilization

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Introduction

In recent years, congestion has become a problem in some elements of the air traffic control system (many airports, some air routes, and air traffic control sectors). Every day, hundreds of flights in the U.S. are delayed for at least 15 minutes. Since all aircraft fly several legs every day, in many cases flights are delayed because the arriving flight is delayed. Major airports usually have multiple runways in operation. Every possible runway-operating configuration has an appropriate capacity value. The calculations of these capacities are based on air traffic control separation rules. Previous research and every day practice showed that landing capacities are the main cause of aircraft delays. Congestion is frequently caused by the fact that in Air Traffic Control System demand for airports and their capacities is not constant (in certain time periods "demand" at an airport or on an air route is greater than capacity). The variations in airport capacities can be very high. Meteorological conditions, as well as traffic mix and air traffic controller's skills have a direct influence on airport capacity. Since meteorological conditions cannot be precisely forecast over a longer period of time, the capacity of an airport can only be approximately determined over the long run (i.e., there is uncertainty in the prediction of airport capacities). In this paper, we propose fuzzy mathematical programming model for optimizing airport capacity utilization. The paper is organized in the following way: statement of the problem is given in section 2, proposed solution of the problem is given in section 3, and section 4 contains conclusions and directions for further research.

2. Statement of the Problem

We consider air terminal operations within time period T during one specific day. The system considered in this paper is comprised of n_{af} arrival fixes and n_{df} departure fixes and a runway system. A simple layout of the terminal area along with the arrival and departure fixes is shown in Figure 1. The arrival fixes serve only the arrival flow, and the departure fixes support only the departure flow. The arrival planes have to pass through pre-assigned arrival fix before landing, and the departing planes also have to pass through pre-assigned departure fix after they leave the runway. The runway system, however, handles both arrival and departure flows. The arrival queues are formed at the arrival fixes. After passing through the arrival fix, the arriving planes are accepted at the runway without any further delays. The departure queues are formed before the runway system and the departure flights can therefore be delayed either at their gates or on the taxiway. The arrival and departure fix capacity indicates the maximum number of flights that can go through a fix in a time unit. Fixes usually have different utilization; i.e., traffic is usually not allocated evenly among fixes. Appropriate coordination among runways and fixes seems to be necessary in order to properly use available runways' and fixes' capacities. The problem considered in this paper can be defined as follows: *For a given runway departure and arrival* *demand, and approximately known airport and fix capacities, calculate the real values of airport arrival and departure capacities to be used in operations and flows through the arrival and departure fixes in order to minimize total aircraft delay over a considered time period.*

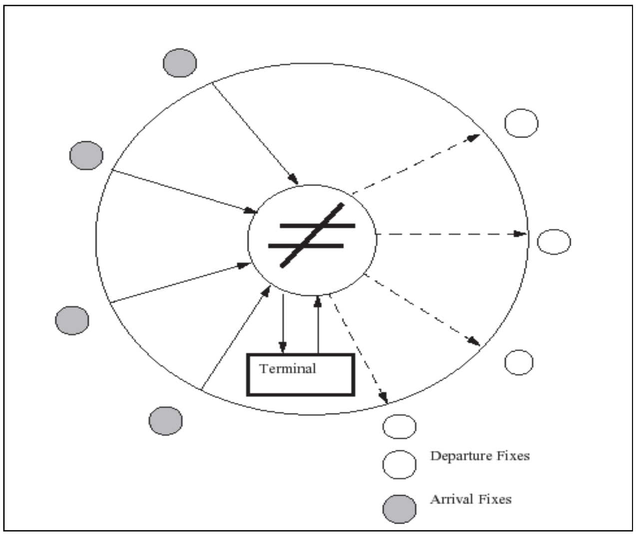

Figure 1 - Terminal Area

The proposed fuzzy mathematical programming model for optimizing airport capacity utilization calculates the optimal values of the airport arrival and departure capacities, and optimal aircraft flows through departure and arrival fixes in 15-minute intervals.

3. Proposed Solution to the Problem

Most airports generally make a trade-off between arrival and departure capacities (Figure 2).

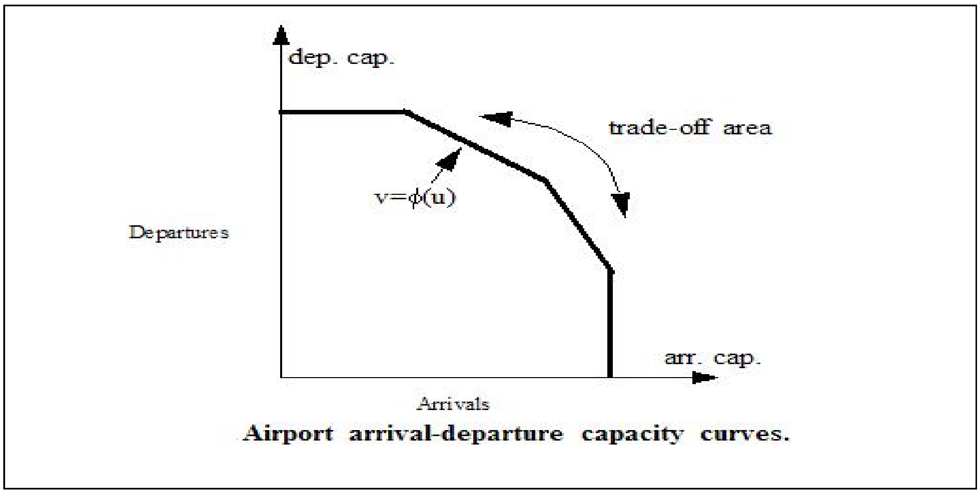

Figure 2 - Airport Capacity Curves

In other words, the number of arrivals and departures actually taking place is somewhere between the maximum number of arrivals and the maximum number of departures. Let us divide time period T into N discrete time intervals whose length is δ_t. We assume that the airport arrival capacity at the i^{th} time interval $\mathbf{U_i}$ is characterized by uncertainty. This capacity can be represented as a triangular fuzzy number $\mathbf{U_i} = (u_{1i}, u_{2i}, u_{3i})$, where u_{1i} is lower (left) boundary of the triangular fuzzy number, u_{2i} is number corresponding to the highest level of presumption, and u_{3i} is upper (right) boundary of the fuzzy number (Figure 3).

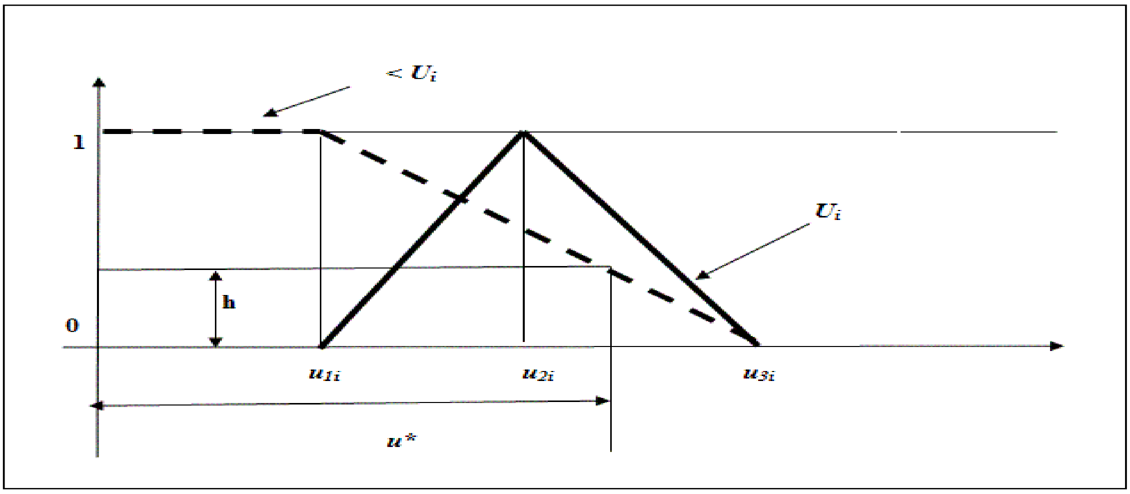

Figure 3 - The Triangular Fuzzy Number $\mathbf{U_i}$ that Represents Arrival Capacity, and the Fuzzy Number "Less than $\mathbf{U_i}$" ($<\mathbf{U_i}$)

Based on experience or intuition, an expert is able to state that, for example, airport capacity is "around 40 aircraft per hour". The airport arrival capacity at the i-th time interval $\mathbf{U_i}$ and the airport departure capacity at the i-th time interval $\mathbf{V_i}$ are interdependent. The airport departure capacity at the i-th time interval $\mathbf{V_i}$ is also represented by a triangular fuzzy number,. $V_i = (v_{1i}, v_{2i}, v_{3i})$ The values v_{1i}, v_{2i}, v_{3i} can be calculated using fuzzy arithmetic rules (Kaufmann and Gupta (1985), Teodorovic and Vukadinovic (1998)), once we know the values of u_{1i}, u_{2i} and u_{3i}. Similarly, we represent the capacity of the j-th arrival fix $F_{A_i}^j = (f_{A_{1i}}^j, f_{A_{2i}}^j, f_{A_{3i}}^j)$, and the capacity of the k-th departure fix $F_{D_i}^k = (f_{D_{1i}}^k, f_{D_{2i}}^k, f_{D_{3i}}^k)$ at the i-th time interval as triangular fuzzy numbers. Let us also introduce the following notation:

- I - a set of time intervals,
- J - a set of arrival fixes,
- K - a set of departure fixes,
- a_i^j - arrival demand through the j-th fix during the i-th time interval,
- d_i^k - departure demand through the k-th fix during the i-th time interval,
- X_i^j - queue at the j-th arrival fix during the i-th time interval,
- Y_i^k - queue at the k-th departure fix during the i-th time interval,

w_i^j - flow through the j-th arrival fix during the i-th time interval,

z_i^k - flow through the k-th departure fix during the i-th time interval.

Let us explain the difference between "crisp" and "fuzzy" constraints using the following example. The following crisp constraint is used in the traditional airport capacity utilization models:

$$\sum_{j=1}^{n_{af}} w_i^j \le u_i \tag{1}$$

This constraint states that the sum of the arrival flows $\sum_{j=1}^{n_{af}} w_i^j$ during the i^{th} time interval must be less than or equal to the airport arrival capacity (u_i). In this case the airport capacity is treated as a deterministic quantity. In an attempt to adequately represent uncertainty, we treat the airport arrival capacity (u_i) as the fuzzy number **Ui.** Figure 3 also shows on the ordinate axis the level of satisfaction h ($0 \le h \le 1$) that we wish to achieve. This level of satisfying the constraint, h, can be achieved when the $\sum_{j=1}^{n_{af}} w_i^j$ is less than or equal to the highest value (u*) of the fuzzy number < $\mathbf{U_i}$ for this level of satisfaction. The highest value u* of the fuzzy number < $\mathbf{U_i}$ for this level of satisfaction can be obtained from the similarity of triangles (Figure 3). Therefore:

$$\sum_{j=1}^{n_{af}} w_i^j \le u_{3i} - h(u_{3i} - u_{1i}) \tag{2}$$

In the same way, we assume that the capacity of the j-th arrival fix and the capacity of the k-th departure fix at the i-th time interval are characterized by uncertainty. We also represent these capacities as triangular fuzzy numbers.

When optimizing airport capacity utilization, we tried to minimize the total aircraft delay during the considered time period while taking care of airport and fix capacities. Objective function $\sum_{i=1}^{N} \gamma_i \left[\sum_{p=1}^{i} \left(\alpha_i \sum_{j=1}^{n_{af}} w_p^j + (1-\alpha_i) \sum_{k=1}^{n_{af}} z_p^k \right) \right]$ represents the total flow through all departure and arrival fixes for all the intervals. The weight given to the arrivals is denoted by α_i, the weight for the departures equals ($1-\alpha_i$), while γ_i represents the weight given to the i-th time interval ($0 \le \gamma_i \le 1$). Total arrival flow through any fix can never exceed the sum of total demand and the initial arrival queue. Similarly, the total arrival flow cannot exceed the arrival capacity of the airport. Similar constraints must also exist for the departure flows. Arrival and departure flows must always be less than or equal to the arrival and departure fix capacities respectively.

Let us introduce "acceptable total flow" into the discussion. In other words, instead of maximizing total flow, we will try to generate acceptable total flow with a level of satisfaction at least equal to h. We define "acceptable total flow" as a triangular fuzzy number **ATF** = (t_1, t_2, t_3).

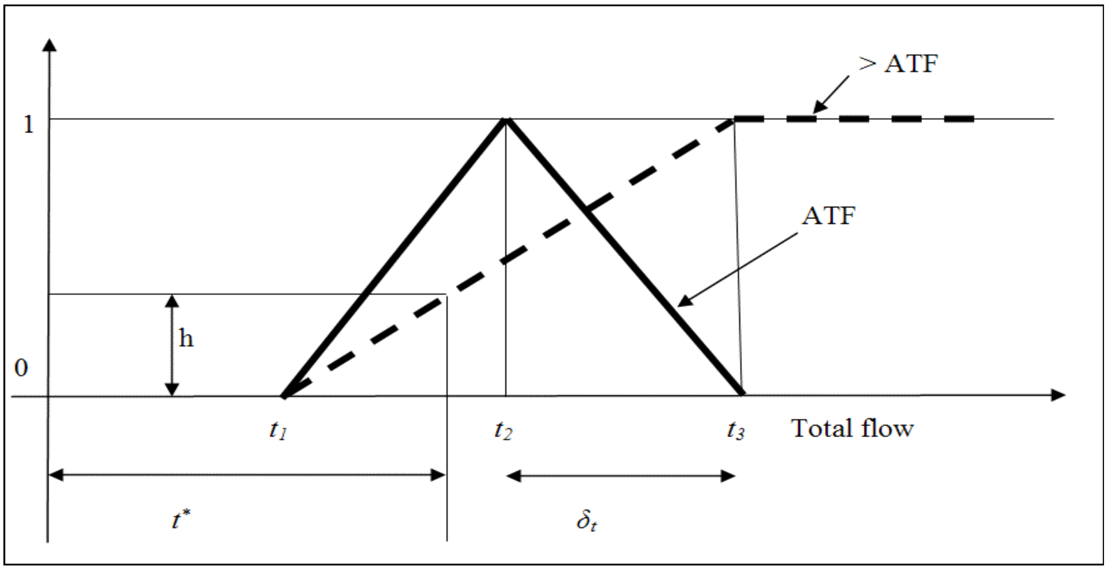

Figure 4 - "Acceptable Total Flow" (**ATF**) and Total Flow Greater than "Acceptable" **(>ATF)**

From Figure 4 we see that a "flow greater than satisfactory" will be achieved with a level of satisfaction at least equal to *h*, if:

$$\sum_{i=1}^{N}\gamma_i\left[\sum_{p=1}^{i}\left(\alpha_i\sum_{j=1}^{n_{af}}w_p^j+(1-\alpha_i)\sum_{k=1}^{n_{df}}z_p^k\right)\right]\geq t_1+h(t_3-t_1) \tag{3}$$

In other words, our objective function has become a constraint, which agrees completely with Bellman and Zadeh (1970) whereby both objective functions and constraints in a fuzzy environment are treated in the same way. Since we have transformed the objective function into a constraint, the question arises of defining a new objective function. We will naturally try to find a solution that maximizes the level of satisfying both the objective function and the constraint, *h*. In other words, in order to determine the optimal solution that satisfies both the objective function and constraint by the maximum possible degree *h*, a fuzzy optimization principle is applied by which *h* is maximized:

Maximize *h*

subject to:

$$\sum_{i=1}^{N}\gamma_i\left[\sum_{p=1}^{i}\left(\alpha_i\sum_{j=1}^{n_{af}}w_p^j+(1-\alpha_i)\sum_{k=1}^{n_{df}}z_p^k\right)\right]\geq t_1+h(t_3-t_1) \tag{4}$$

$$\sum_{p=1}^{i} w_p^j \leq X_1^j + \sum_{p=1}^{i} a_p^j \ , \quad i\in I, j\in J \tag{5}$$

$$\sum_{j=1}^{n_{af}} w_i^j \leq u_{3i}-h(u_{3i}-u_{1i})\,, \qquad i\in I \tag{6}$$

$$\sum_{p=1}^{i} z_p^k \le Y_1^k + \sum_{p=1}^{i} d_p^k \, , \; i \in I, k \in K \tag{7}$$

$$\sum_{k=1}^{n_{df}} z_i^k \le v_{3i} - h(v_{3i} - v_{1i}) \, , \; i \in I \tag{8}$$

$$w_i^j \le f_{A_{3i}}^j - h(f_{A_{3i}}^j - f_{A_{1i}}^j) \, , \; i \in I, j \in J \tag{9}$$

$$z_i^k \le f_{D_{3i}}^k - h(f_{D_{3i}}^k - f_{D_{1i}}^k) \, , \; i \in I, k \in K \tag{10}$$

The obtained flows in the fuzzy optimization model correspond to a certain level of satisfaction of h. By shifting the “acceptable total flow” to the left, the level of satisfaction could be increased. In other words, the achieved level of satisfaction h highly depends on the “acceptable total flow” set up by the decision maker. If we are prepared to accept the fact that all constraints are not completely satisfied, we can considerably increase the total flow. Every pair (h, t_2) corresponds to a certain traffic flow pattern. In this manner, a large number of different traffic flow patterns are generated for the decision maker. The Chicago O’Hare Airport is used in this paper to demonstrate model performance.

4. Conclusion

In this paper, fuzzy mathematical programming model for optimizing airport capacity utilization has been developed. The developed model is tested on the case of the Chicago O’Hare Airport. A sample congested 3 hr period is considered for analysis. The obtained preliminary results are very promising.

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