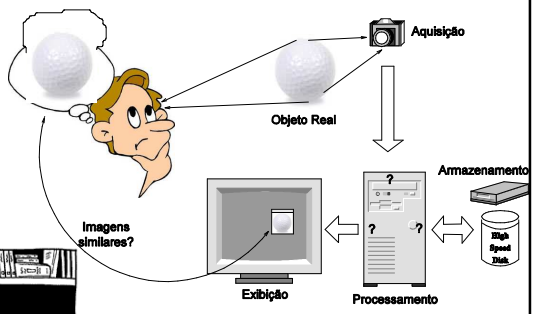
# IA725 – Computação Gráfica I

## Modelos Geométricos

Watt: Capítulo 3
Redbook: Capítulo 2

# Transformações Geométricas

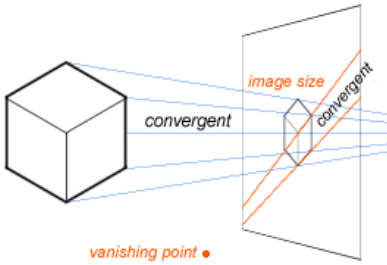
Dois paradigmas para geração de imagens 2D




Conceber as formas em 3D

Conceber as formas em 2D

# Uma Visão Clássica

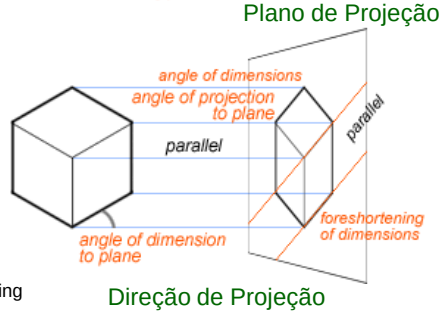



Projeções Perspectivas

Raios projetores convergentes

Centro de Projeção

Plano de Projeção




Projeções Paralelas

x

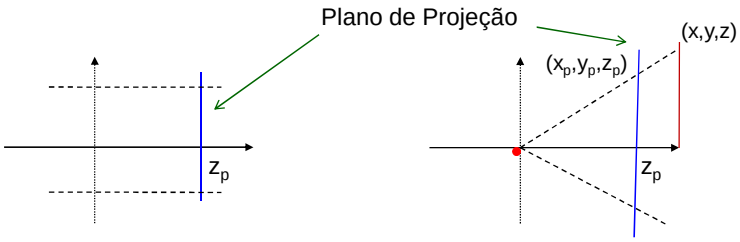
Raios projetores paralelos

Preserva paralelismo

Direção de Projeção



# Projeções
## Casos Triviais



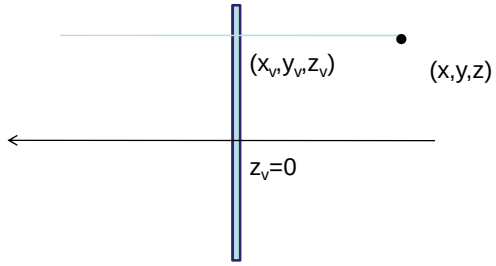

Paralela

Perspectiva

Basta substituir a coordenada z de cada ponto por $z_p$

Coordenadas x, $x_p$, y e $y_p$ coincidem com as "alturas" dos triângulos e as coordenadas z com as "bases" dos triângulos. Problema se reduz a obter relação entre estas coordenadas pela semelhança de triângulos

# Projeções Paralelas

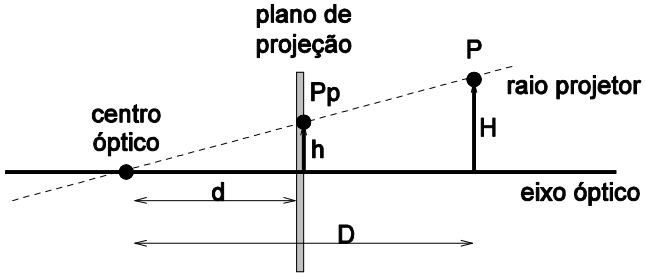



$$x_s = x_v$$
$$y_s = y_v$$
$$z_s = 0$$

$$\begin{pmatrix} x_v \\ y_v \\ z_v \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$



# Projeções Perspectivas

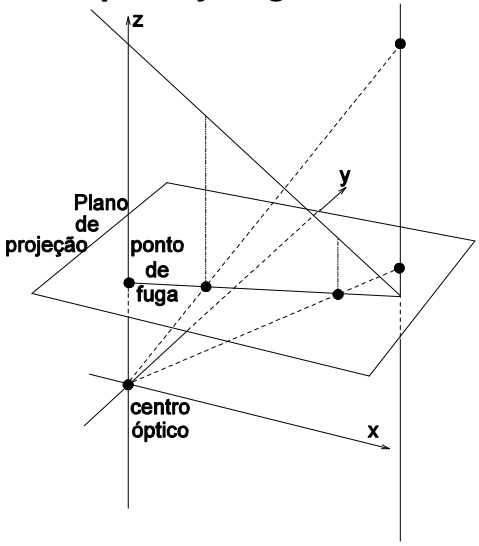



$$x_s = \frac{x_v d}{z_v}$$
$$y_s = \frac{y_v d}{z_v}$$
$$z_s = \frac{z_v d}{z_v} = d$$

$$\begin{pmatrix} x_v \\ y_v \\ z_v \\ z_v/d \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

# Projeções Perspectivas

## Interpretação geométrica

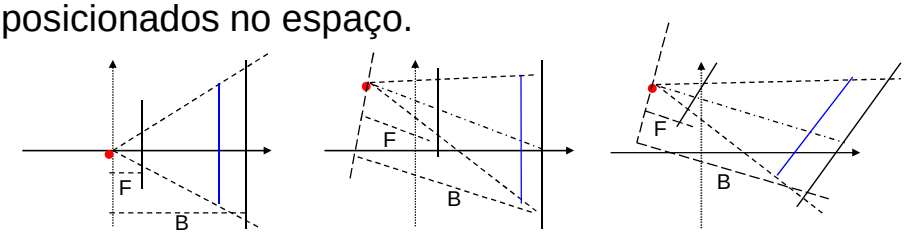





# Projeções

## Diversidade de Casos

- Plano de projeção tem o vetor normal na direção do eixo z e o centro de projeção sobre o eixo z.
- Plano de projeção tem o vetor normal na direção do eixo z e o centro de projeção arbitrariamente posicionado.
- Tanto o plano quanto o centro são arbitrariamente posicionados no espaço.

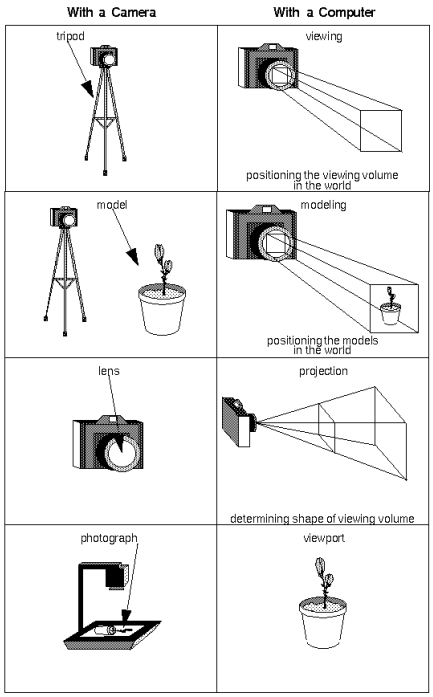


# Distintos Espaços

Referencial da Câmera

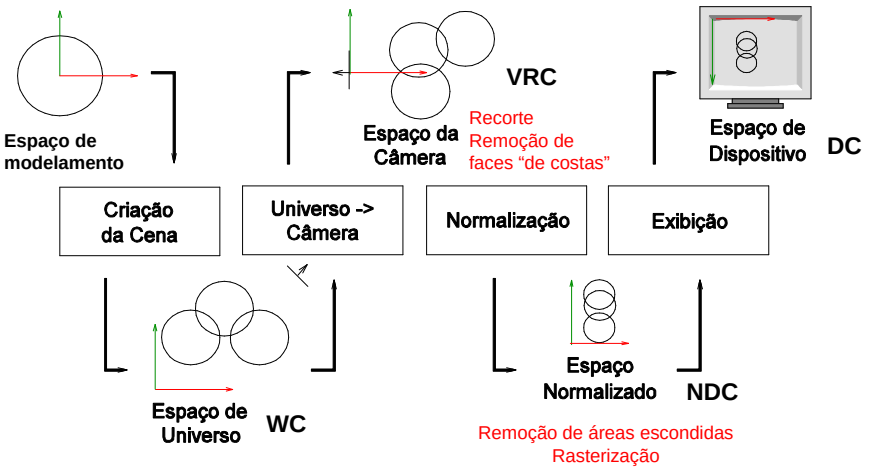
Referencial Normalizado

Referencial da Janela

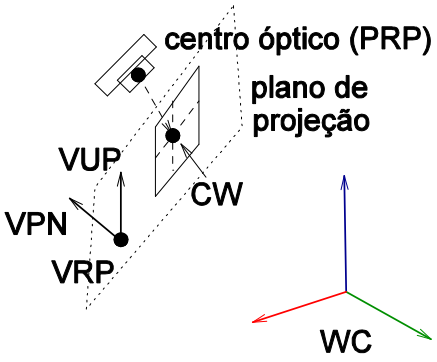


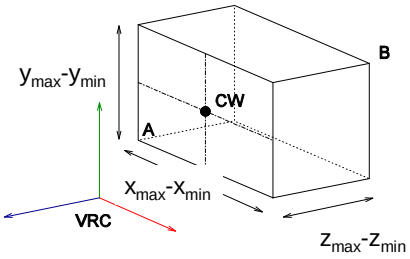



# Fluxo de Projeção



# Como especificar a “base” do espaço da câmera?



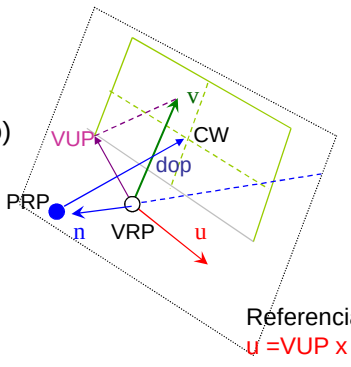

A = ($x_{min}$,$y_{min}$,$z_{min}$)
B=($x_{max}$,$y_{max}$,$z_{max}$)



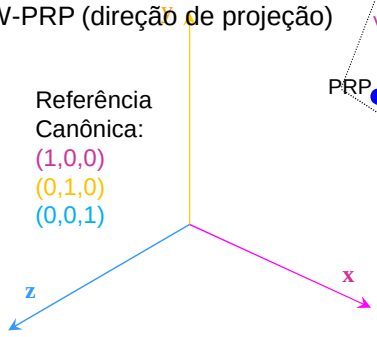
# WC → VRC

## Referencial da Câmera

VUP (up): view up vector
VRP: view reference point
VPN (n): view plane normal
PRP: eye
CW: centro da janela
dop: CW-PRP (direção de projeção)

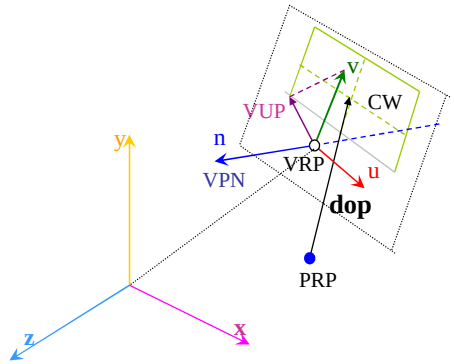



Referência Canônica:
(1,0,0)
(0,1,0)
(0,0,1)




Referencial da Câmera:
u =VUP x n/|VUP x n|
n x u
n = VPN/|VPN|

# WC → VRC




(1) VRP na origem

$$Tr = \begin{bmatrix} 1 & 0 & 0 & -VRP_x \\ 0 & 1 & 0 & -VRP_y \\ 0 & 0 & 1 & -VRP_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(2) (u,v,n) em vetores base ortonormais

$$\vec{n} = \frac{\overrightarrow{VPN}}{\left|\overrightarrow{VPN}\right|}$$

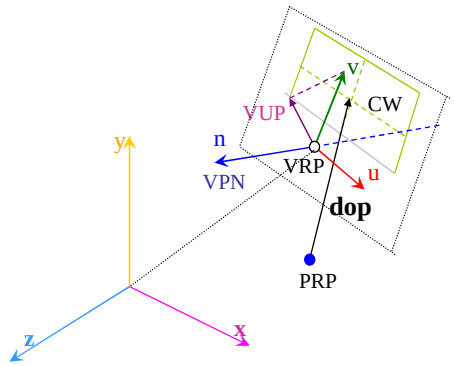
$$\vec{u} = \frac{\overrightarrow{VUP} \times \vec{n}}{\left|\overrightarrow{VUP} \times \vec{n}\right|}$$

$$\vec{v} = \vec{n} \times \vec{u}$$

$$R = \begin{bmatrix} u_x & u_y & u_z & 0 \\ v_x & v_y & v_z & 0 \\ n_x & n_y & n_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$




# WC → VRC




(1) VRP na origem

$$Tr = \begin{bmatrix} 1 & 0 & 0 & -VRP_x \\ 0 & 1 & 0 & -VRP_y \\ 0 & 0 & 1 & -VRP_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(2) (u,v,n) em vetores base ortonormais

$$\vec{n} = \frac{\overrightarrow{VPN}}{\left|\overrightarrow{VPN}\right|}$$

$$\vec{u} = \frac{\overrightarrow{VUP} \times \vec{n}}{\left|\overrightarrow{VUP} \times \vec{n}\right|}$$

$$\vec{v} = \vec{n} \times \vec{u}$$

$$R = \begin{bmatrix} u_x & u_y & u_z & 0 \\ v_x & v_y & v_z & 0 \\ n_x & n_y & n_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

IA725 – 1s2009 - Ting

# WC →VRC
## Projeções Perspectivas

PRP' = R  Tr PRP (em VRC)

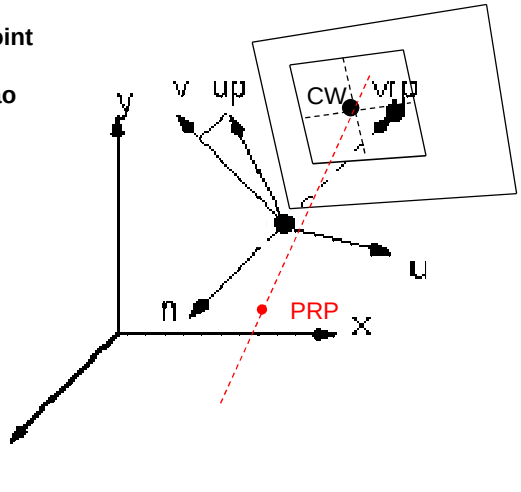
$$Tr_{PRP'} = \begin{bmatrix} 1 & 0 & 0 & -PRP'_x \\ 0 & 1 & 0 & -PRP'_y \\ 0 & 0 & 1 & -PRP'_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



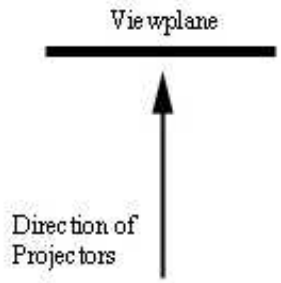
# VRC

**VUP (up): view up vector**
**VRP (center): view reference point**
**VPN (n): view plane normal**
**PRP (eye): projection reference point**
**CW: centro de janela**
**dop (PRP-CW): direção de projeção**

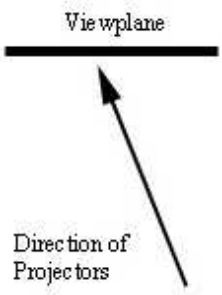
dop  e  VPN  podem ser distintos!



# VRC



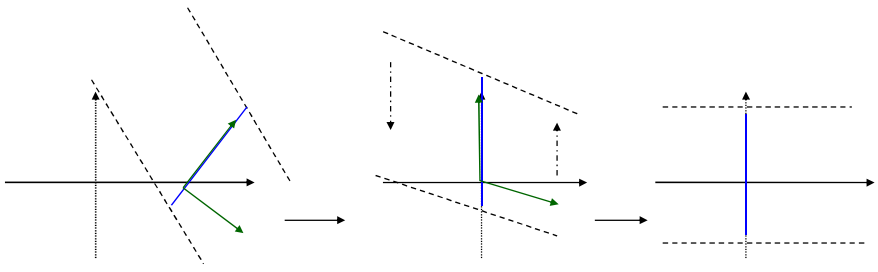

**Ortográficas**




**Oblíquas**



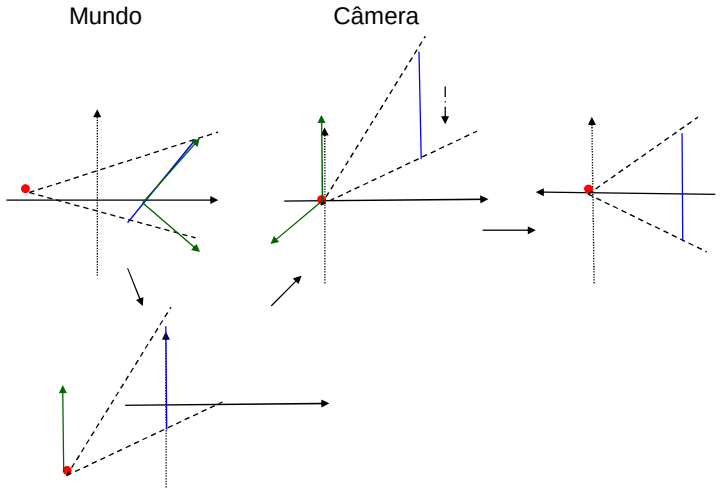
# VRC
## Projeções Paralelas



Oblíqua

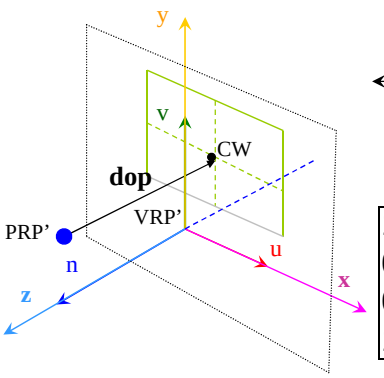
Ortográfica

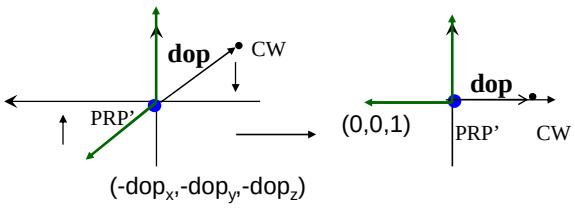
# VRC
## Projeções Perspectivas






# VRC
## Volume “reto”



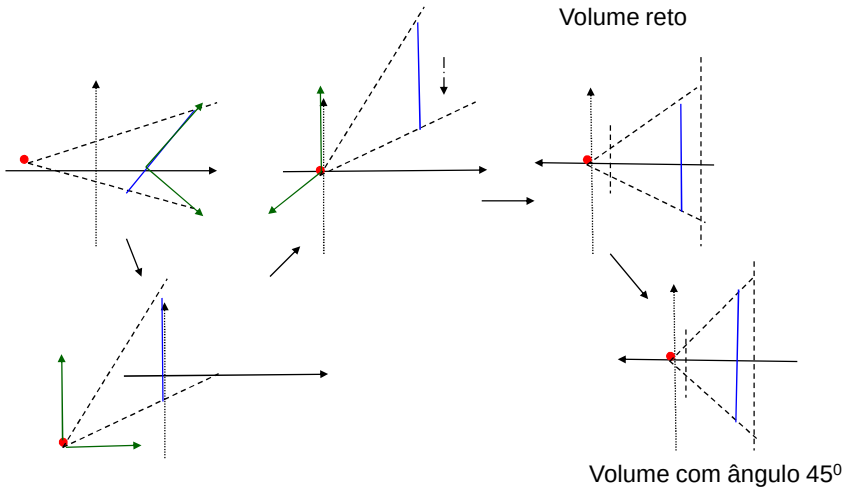




$$\begin{bmatrix} 1 & 0 & -dop_x & 0 \\ 0 & 1 & -dop_y & 0 \\ 0 & 0 & -dop_z & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -dop_z & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

VRP': view reference point em VRC
PRP': projection reference point em VRC
dop: direção de projeção
CW: centro de janela

$$Sh = \begin{bmatrix} 1 & 0 & -(dop_x/dop_z) & 0 \\ 0 & 1 & -(dop_y/dop_z) & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

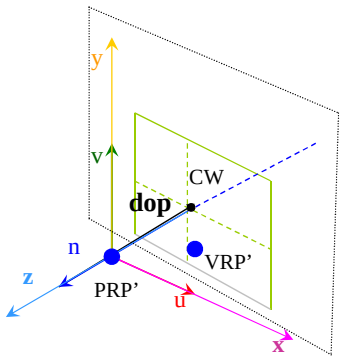
# VRC
## Mudança de Escala

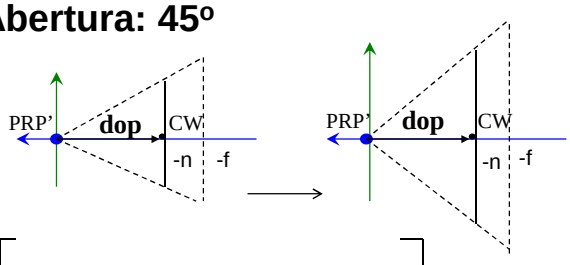





# VRC
## Projeções Perspectivas
## Abertura: 45º

$(u_{min}, u_{max}, v_{min}, v_{max}, F, B)$



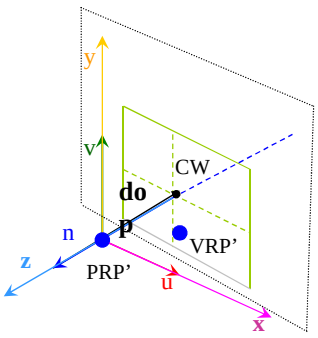



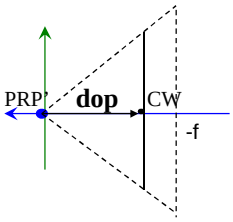

$$\begin{bmatrix} (u_{max}-u_{min})/2 & 0 & 0 \\ 0 & (v_{max}-v_{min})/2 & 0 \\ 0 & 0 & -n \end{bmatrix}$$

$$\begin{bmatrix} n & 0 & 0 \\ 0 & n & 0 \\ 0 & 0 & -n \end{bmatrix}$$

# VRC

$(u_{min}, u_{max}, v_{min}, v_{max}, F, B)$



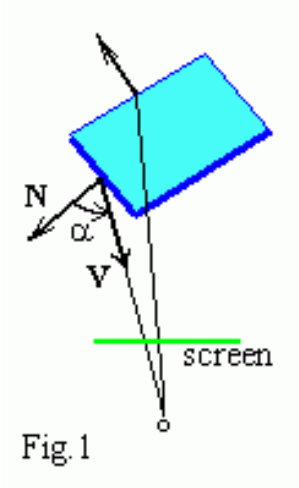




$$S_{per} = \begin{bmatrix} 2n/(u_{max}-u_{min}) & 0 & 0 & 0 \\ 0 & 2n/(v_{max}-v_{min}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



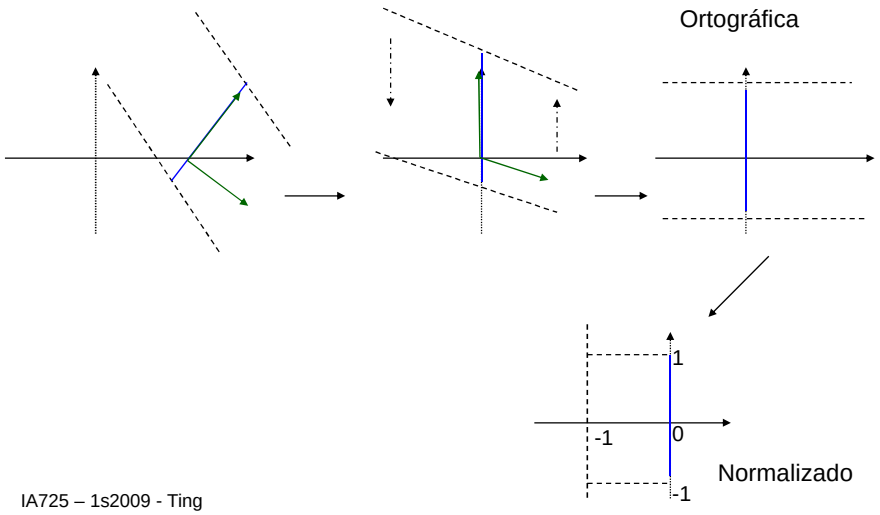
# Descarte de faces

- *Backface Culling*




**N.V** ≤ 0

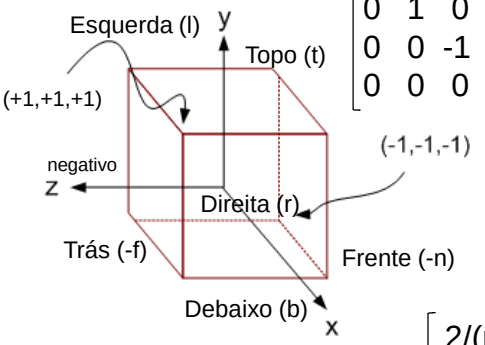
# VRC → NDC
## Projeções Paralelas






# VRC → NDC
## Projeções Paralelas

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2/(r-l) & 0 & 0 & 0 \\ 0 & 2/(t-b) & 0 & 0 \\ 0 & 0 & 2/(n-f) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



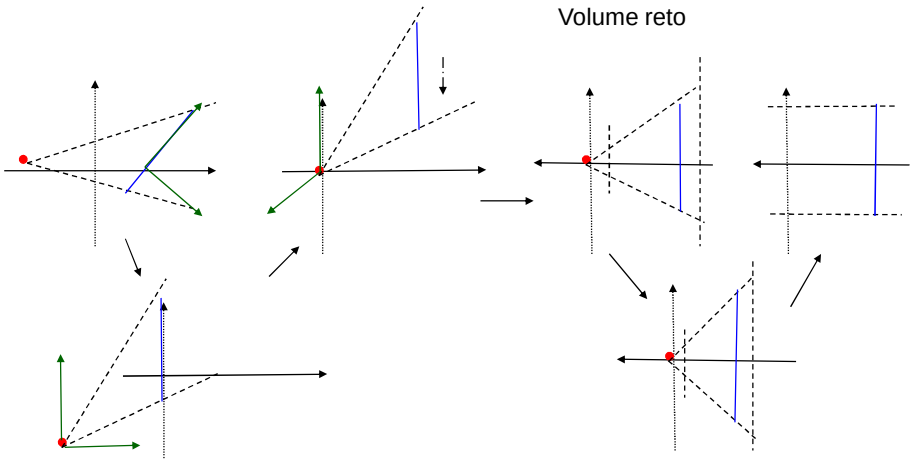

$$\begin{bmatrix} 1 & 0 & 0 & -(r+l)/2 \\ 0 & 1 & 0 & -(t+b)/2 \\ 0 & 0 & 1 & -(-n-f)/2 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

(l,r,b,t,-n,-f) ⟶ (-1,1,-1,1,-1,1)

$$\begin{bmatrix} 2/(r-l) & 0 & 0 & -(r+l)/(r-l) \\ 0 & 2/(t-b) & 0 & -(t+b)/(t-b) \\ 0 & 0 & -2/(f-n) & -(f+n)/(f-n) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

# VRC → NDC

## Projeções Perspectivas

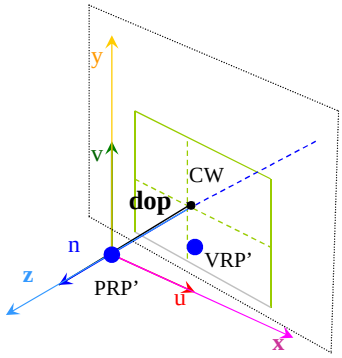


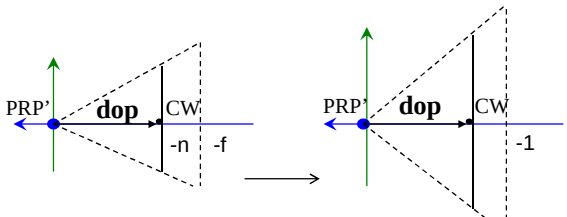



# VRC → NDC

## Espaço Perspectivo Normalizado

$(u_{min}, u_{max}, v_{min}, v_{max}, F, B)$

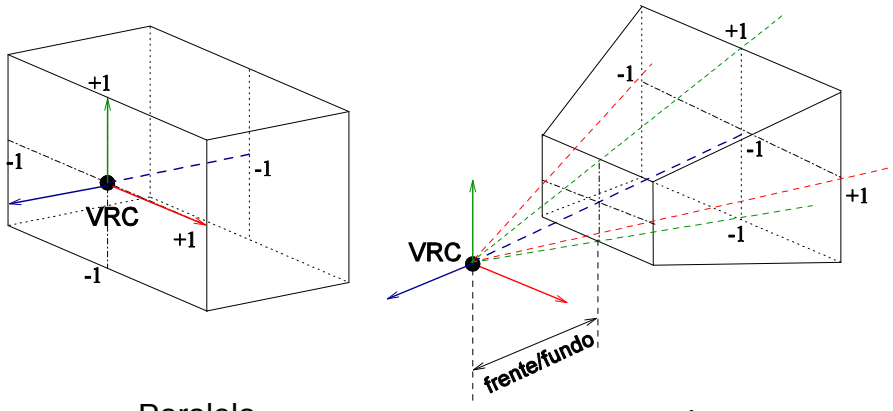



$$\begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & f \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

# Modelo de Espaço Normalizado



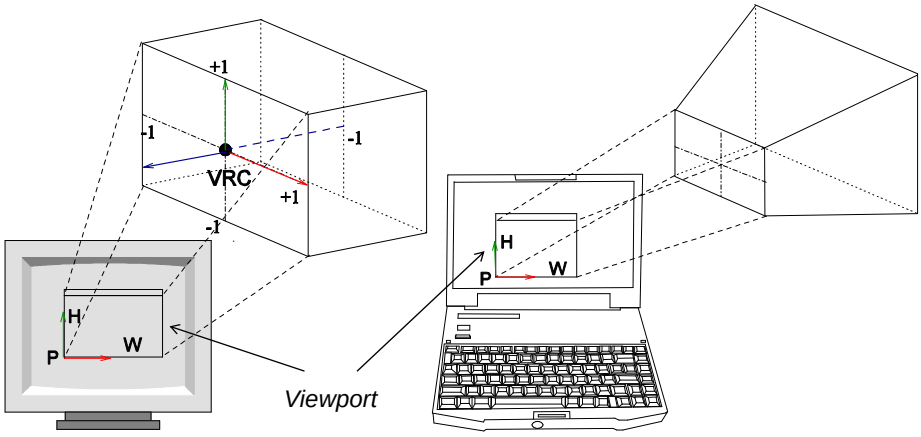

Paralelo

Perspectivo



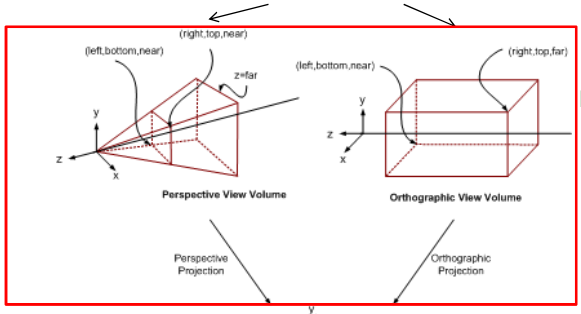
# NDC → Dispositivo

## Uma Alternativa



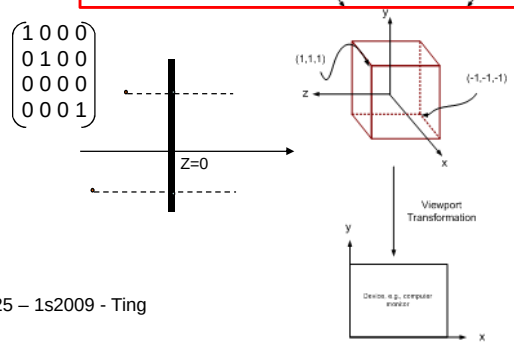
# Universo (WC)

Espaço é deformado para ficar "reto"




Espaço da Câmera "Reto" (VRC)

Câmera → Normalizado



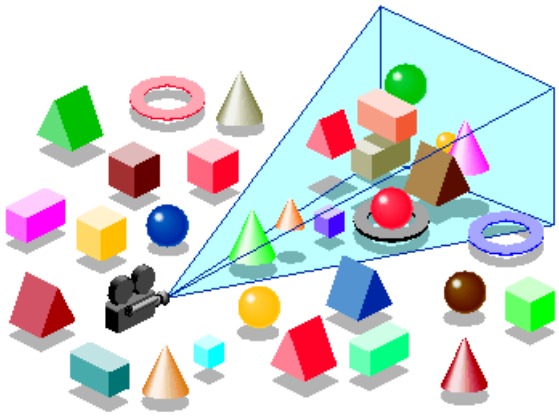

Espaço Normalizado




Espaço da Janela (*Viewport*)

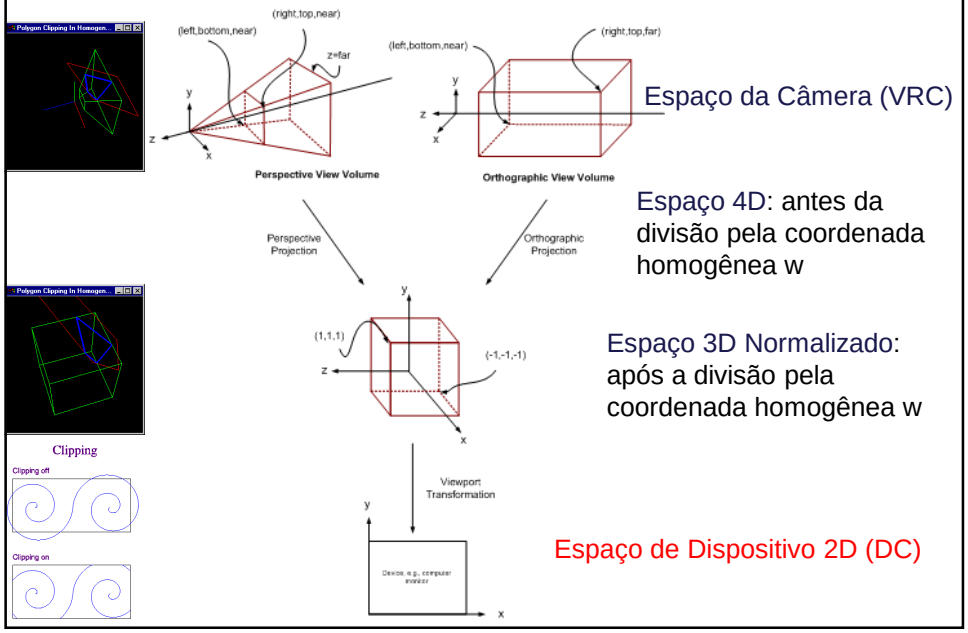
IA725 – 1s2009 - Ting

# Recorte



IA725 – 1s2009 - Ting

# Em qual espaço?



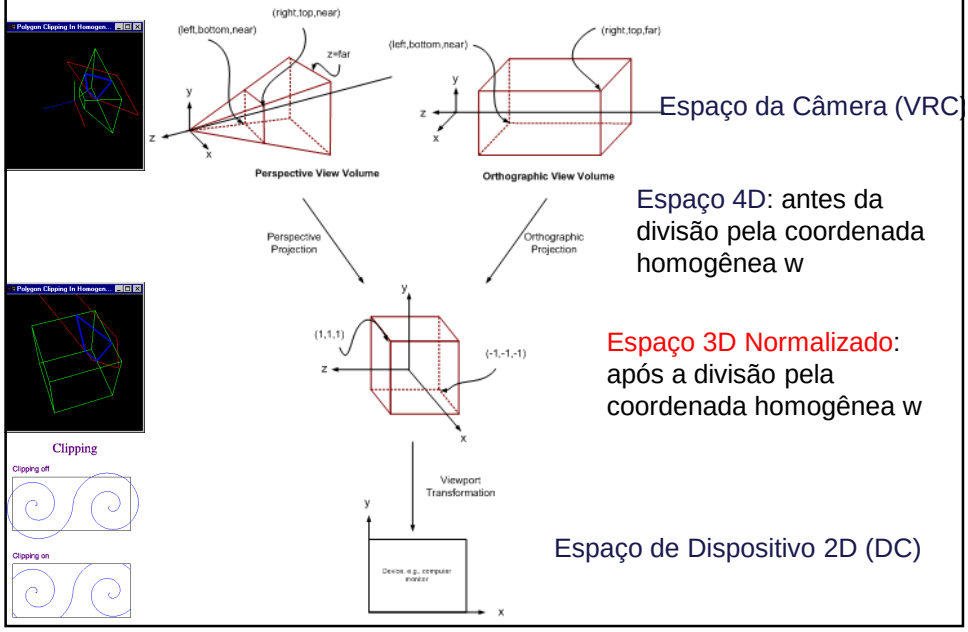

Espaço da Câmera (VRC)

Espaço 4D: antes da divisão pela coordenada homogênea w

Espaço 3D Normalizado: após a divisão pela coordenada homogênea w

Espaço de Dispositivo 2D (DC)

# Em qual espaço?




Espaço da Câmera (VRC)

Espaço 4D: antes da divisão pela coordenada homogênea w

Espaço 3D Normalizado: após a divisão pela coordenada homogênea w

Espaço de Dispositivo 2D (DC)

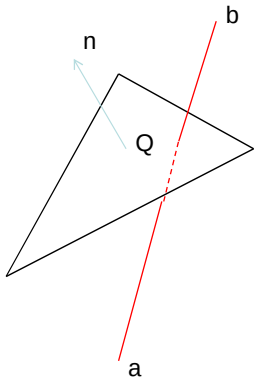
# Espaço da Câmera
## Interseção segmento-plano

$P(t) = a + t\,(b-a)$

$f(P(t)) = n\,(P(t) - Q) = 0$

$n\,((a + t\,(b-a) - Q) = 0$

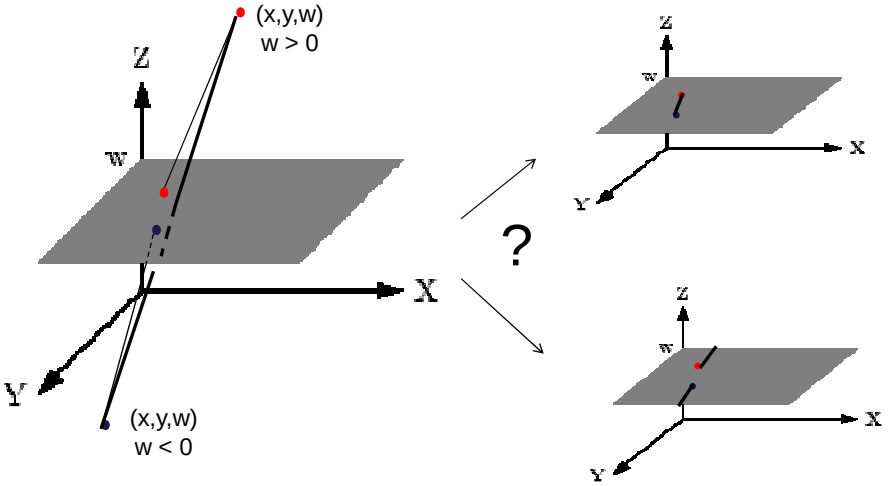
$$t = \frac{n\,(a - Q)}{n\,(a-b)}$$



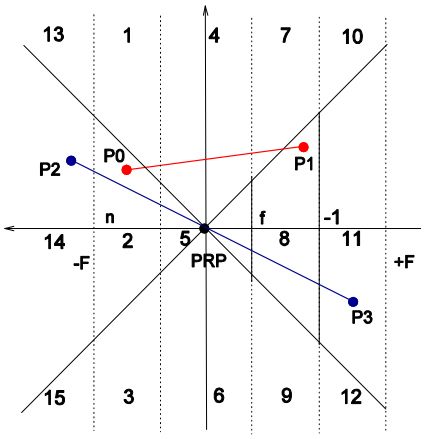

Usar estratégia similar a da Weiler-Atherton para obter formas fechadas!

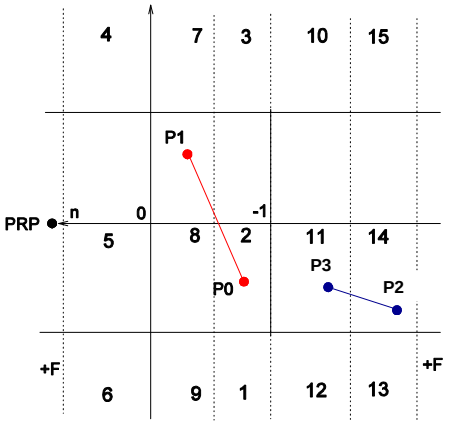


# Recorte em 4D ($w \leq 0$)
## Após a divisão por w?



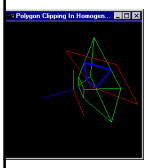
# Recorte em 4D
## Após a divisão por w

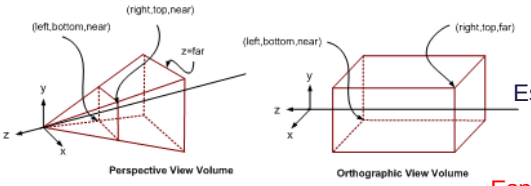



EA978 - 1s2009- Ting

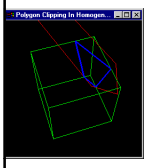
# Em qual espaço?
## w ≤ 0

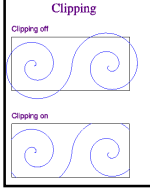


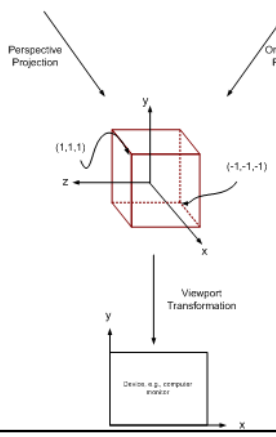



Espaço da Câmera

Espaço 4D: antes da divisão pela coordenada homogênea w





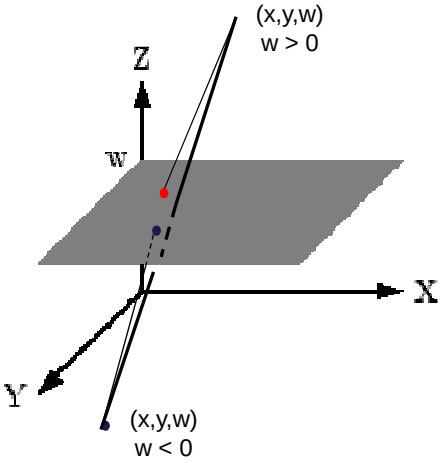




Espaço 3D Canônico: após a divisão pela coordenada homogênea w

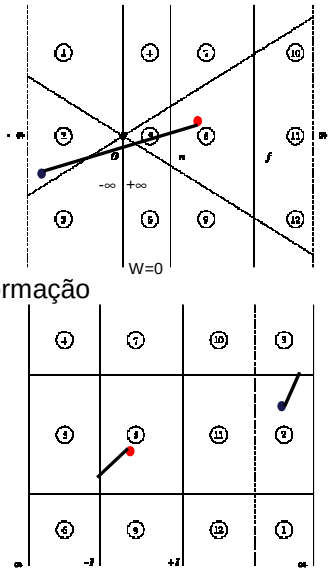
Espaço 2D de Exibição

# Recorte em 4D

## Antes da divisão por w



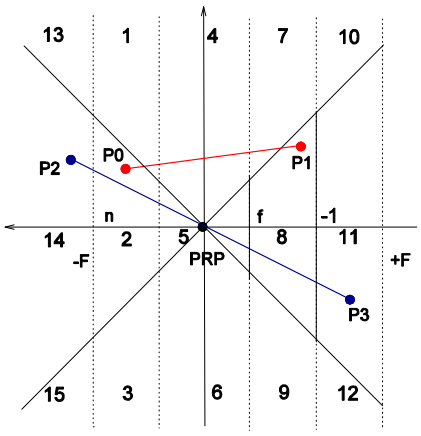

Transformação

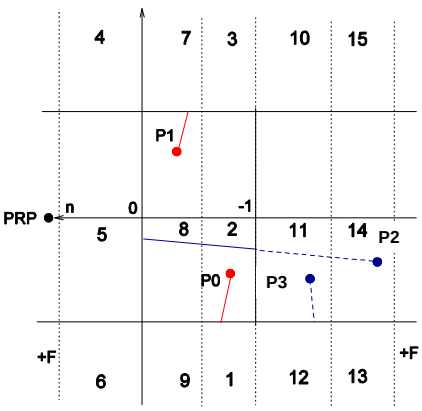



EA978 - 1s2009- Ting

# Recorte em 4D

## Antes da divisão por w




EA978 - 1s2009- Ting

# Recorte em 4D

$$-1 \leq \frac{\bar{x}, \bar{y}, \bar{z}}{\bar{w}} \leq +1$$

$-w \leq x,y,z \leq w$

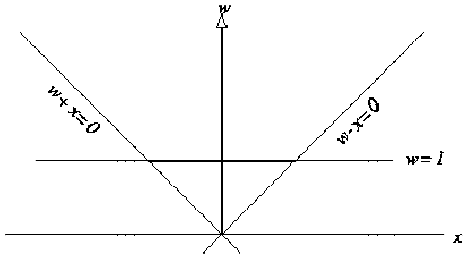
Dado: P(t) = a + t (b-a)

Interseção com x=-w:
$w_a + t\,(w_b - w_a) = -(x_a + t\,(x_b - x_a))$

Possibilidades:
(entra,sai)
(sai,entra)

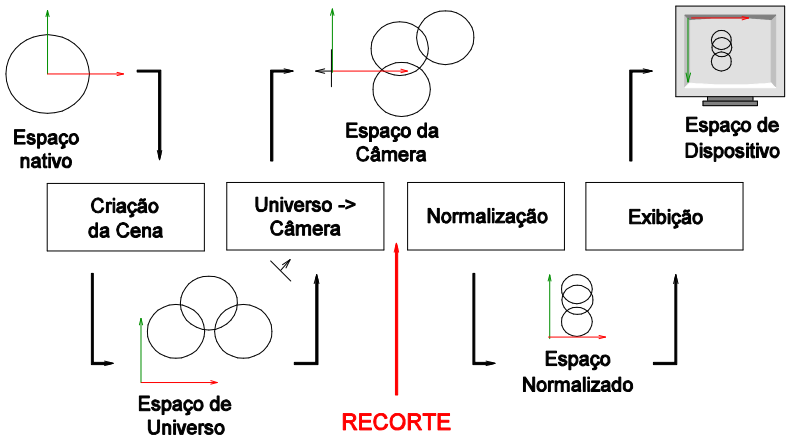
$$t = \frac{w_a + x_a}{(w_a + x_a) - (w_b + x_b)}$$

Analogamente, com x=w, y=-w, y=w, z=-w, z=w





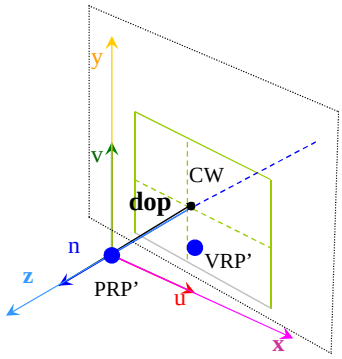

# Fluxo de Projeção
## Espaço de Recorte

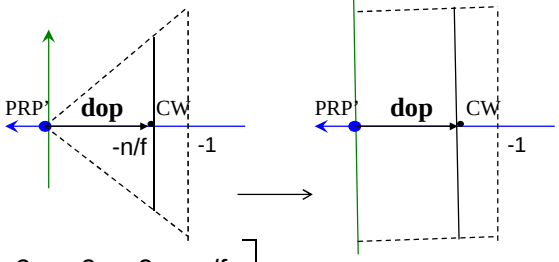


# VRC → NDC

## Espaço Normalizado

$(u_{min}, u_{max}, v_{min}, v_{max}, F, B)$



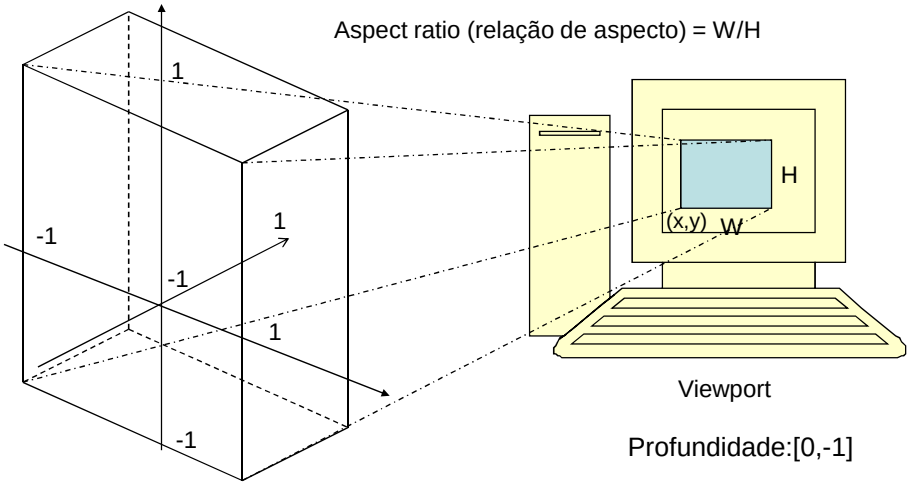

$$\begin{bmatrix} 0 & 0 & 0 & n/f \\ 0 & 0 & n/f & 0 \\ -n/f & -1 & -n/f & -n/f \\ 1 & 1 & 1 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$



# NDC → DC

**Normalizado** → **Espaço do Dispositivo (DC)**

Aspect ratio (relação de aspecto) = W/H

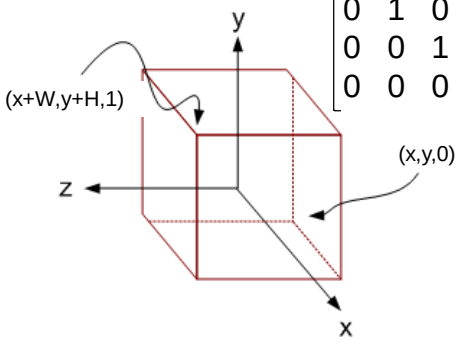



Viewport

Profundidade:[0,-1]

# NDC → DC

$$\begin{bmatrix} 1 & 0 & 0 & x \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} W/2 & 0 & 0 & 0 \\ 0 & H/2 & 0 & 0 \\ 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$




$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

(-1,1,-1,1,-1,1) ⟶ (x,x+W,y,y+H,0,1)

$$\begin{bmatrix} W/2 & 0 & 0 & W/2+x \\ 0 & H/2 & 0 & H/2+y \\ 0 & 0 & -1/2 & 1/2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



# Matriz de Transformação

Transformações lineares

Translação

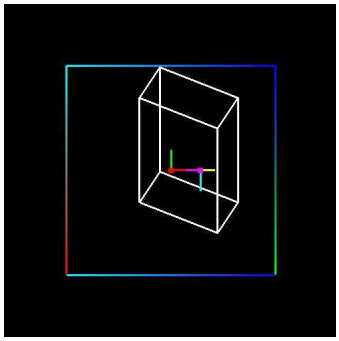
$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline p_x & p_y & p_z & 1 \end{array}\right)$$

Perspectivo

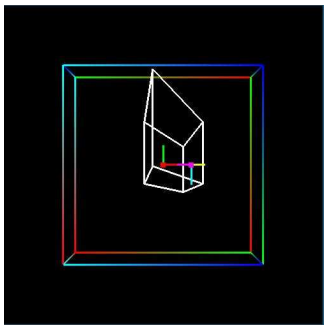
Número de pontos de fuga: número de elementos p da última linha diferentes de zero

# Projeção Passo a Passo

Nas próximas transparências será ilustrado cada passo com as imagens do programa projecao.c. O programa gera a projeção paralela e perspectiva de um cubo (em branco). São mostrados os referenciais WC (magenta,ciano,amarelo) e VRC (vermelho, verde,azul). O volume de visão é desenhado por linhas coloridas.



Paralela



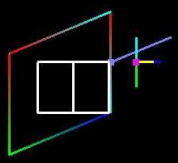
Perspectiva



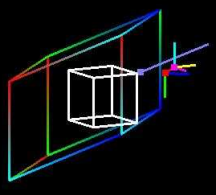
# WC (Universo)

Projeção Paralela
(Várias Vistas)

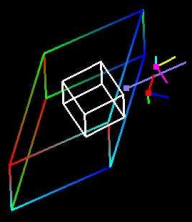
Observe que o referencial WC está centrado no centro da janela



Observe que o volume de visão é um paralelepípedo



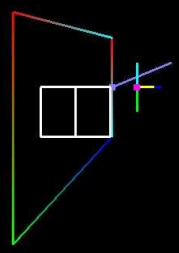
Observe que os referenciais WC e VRC são distintos



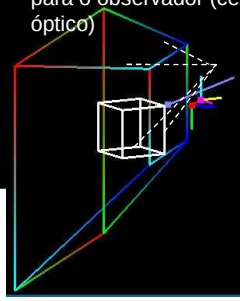
# WC (Universo)

Projeção Perspectiva
(Várias Vistas)

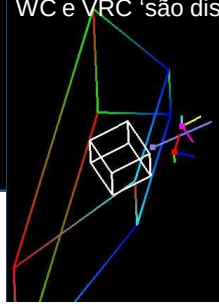
Observe que o referencial WC está no centro da janela



Observe que o volume de visão é agora um trapezóide com as arestas convergindo para o observador (centro óptico)



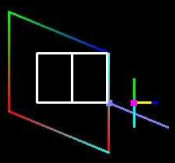
Observe que os referenciais WC e VRC 'são distintos





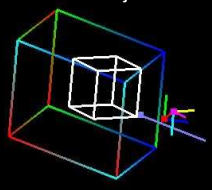
# VRC

Projeção Paralela
(Várias Vistas)



Observe que o eixo y do VRC mudou de sentido e as coordenadas da cena são dadas em termos do VRC

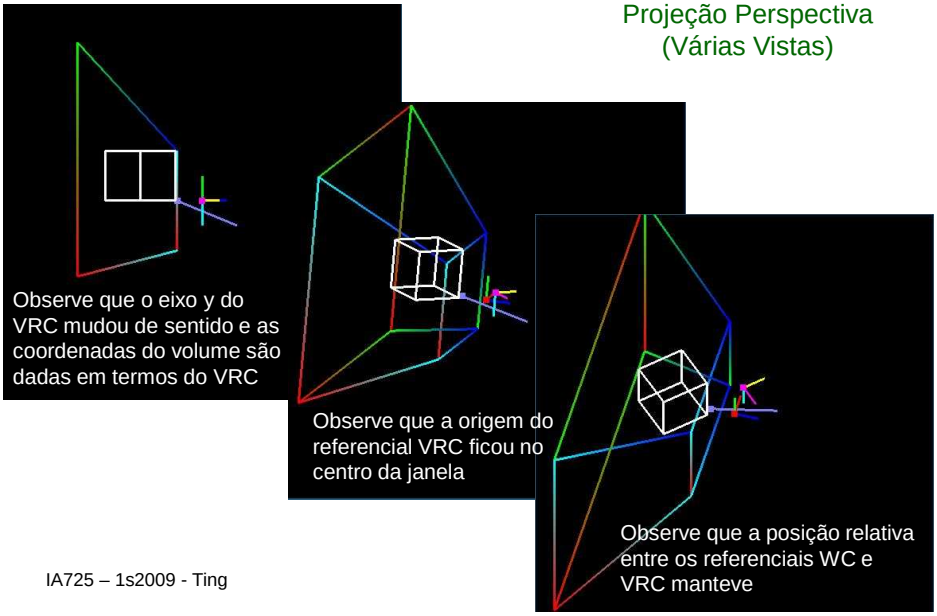
Observe que a origem do referencial VRC ficou no centro da janela



Observe que a posição relativa entre os referenciais WC e VRC manteve

# VRC

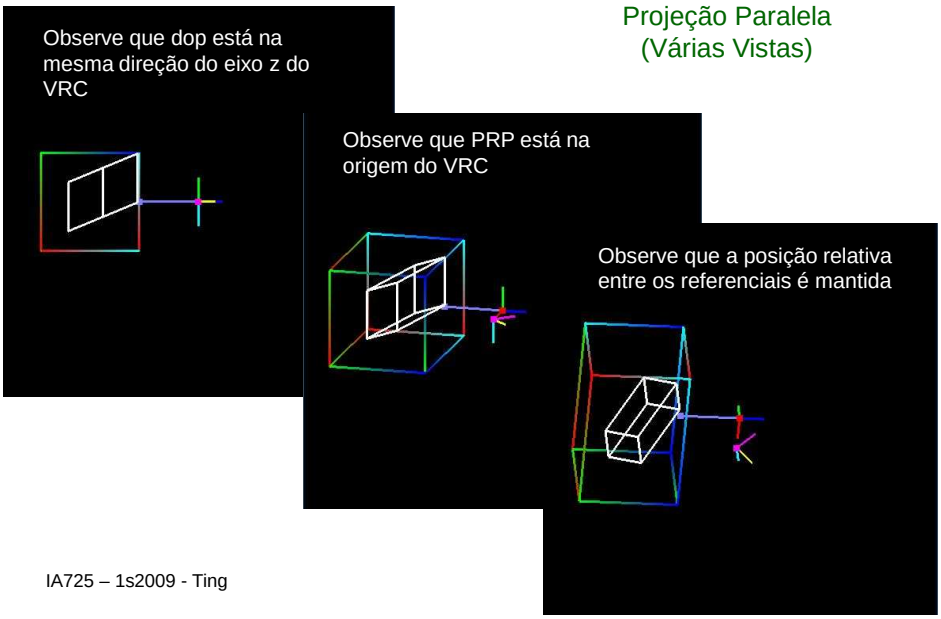
Projeção Perspectiva
(Várias Vistas)




IA725 – 1s2009 - Ting

# VRC Cisalhado

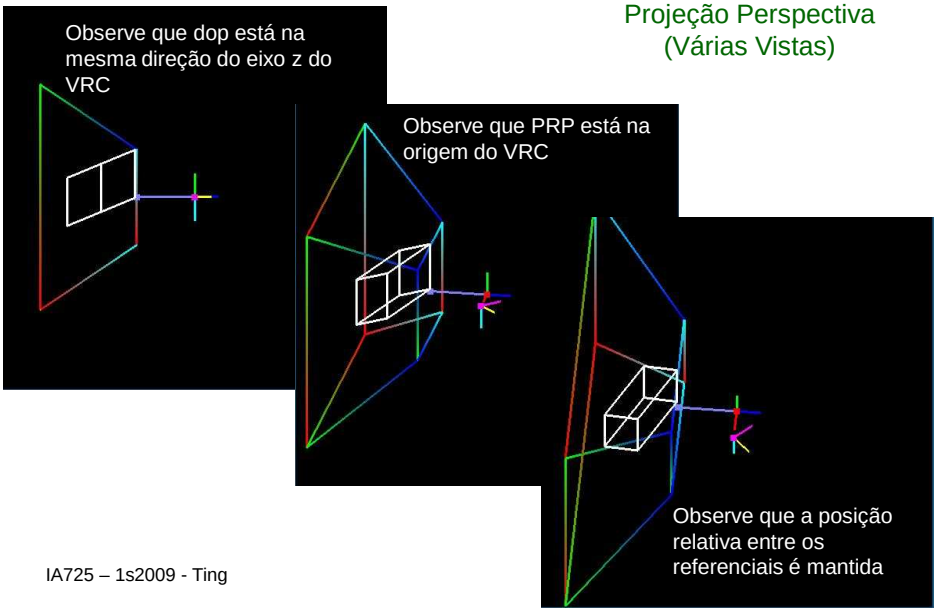
Projeção Paralela
(Várias Vistas)




IA725 – 1s2009 - Ting

# VRC Cisalhado

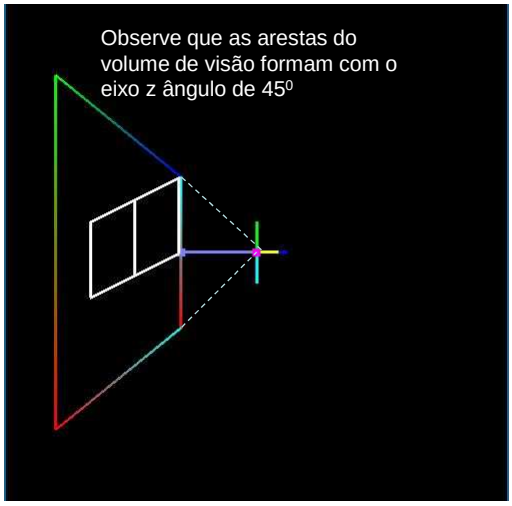
Projeção Perspectiva
(Várias Vistas)





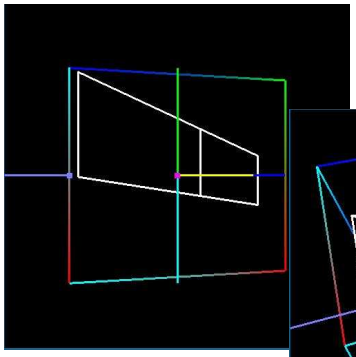

# VRC com abertura $45^0$

Projeção Perspectiva

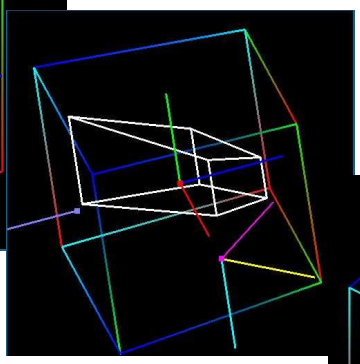


# Normalizado

Projeção Perspectivas
(Várias Vistas)

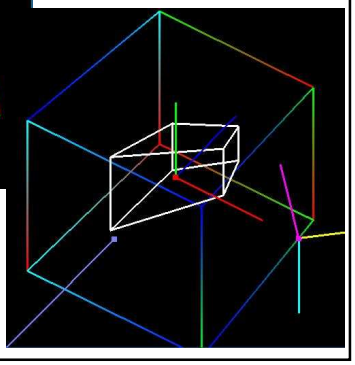


Observe que a relação do volume com o referencial



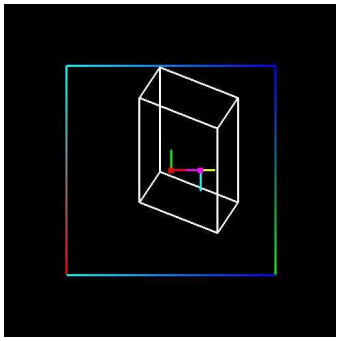
Observe que o volume de visão ficou um cubo centrado na origem do referencial VRC e o cubo dentro do volume ficou distorcido

Observe que PRP foi para "infinito"

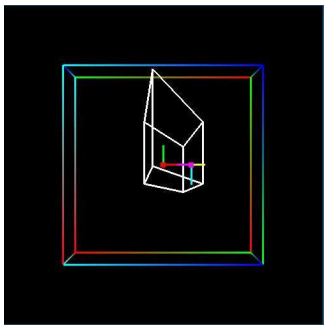




# Projeções Paralelas em DC



Paralela



Perspectiva