

IA725 – Computação Gráfica I

Modelos Geométricos

Watt: Capítulo 2

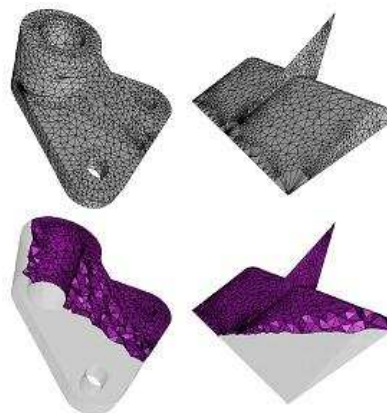
Redbook: Capítulo 2

Modelos Geométricos

Descrições da forma de um objeto
com uso de ferramentas da
Geometria

Sub-problemas

1. Lugar geométrico dos pontos: **Geometria**
2. Conectividade entre os pontos: **Topologia**

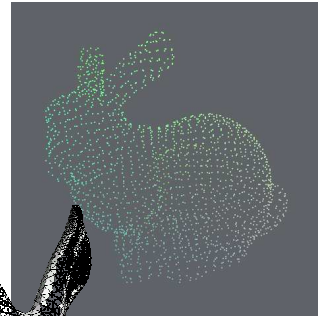
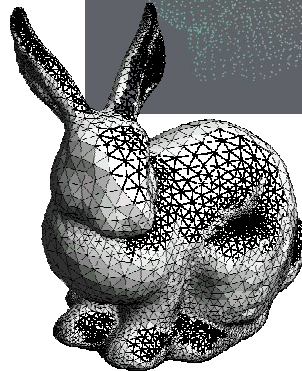


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Representação Processável

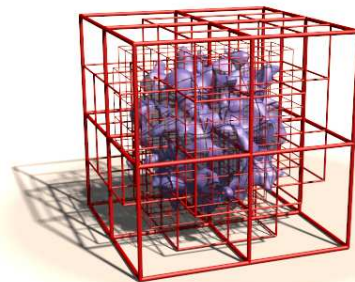
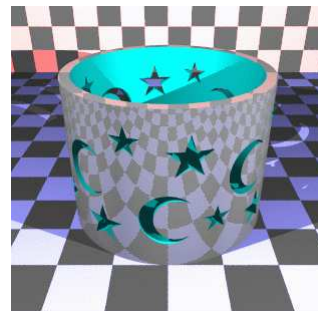
Principais Considerações

1. Dimensão
2. Representatividade/Precisão
3. Concisão
4. Univocidade
5. Interface
6. Complexidade
7. Estrutura de dados
8. Editabilidade

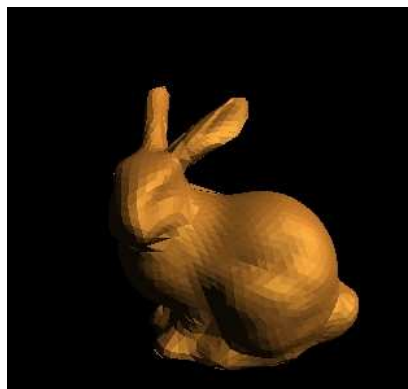
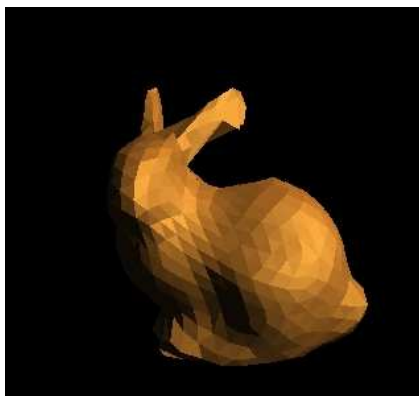


Modelos

- Malhas Poligonais
- Funções Paramétricas
- CSG
- Subdivisão Espacial
- Funções Implícitas



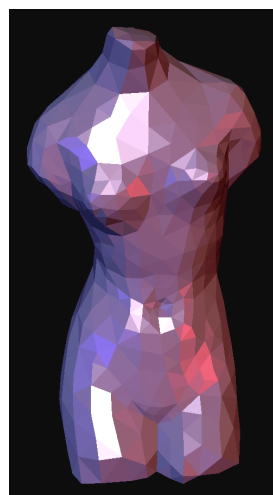
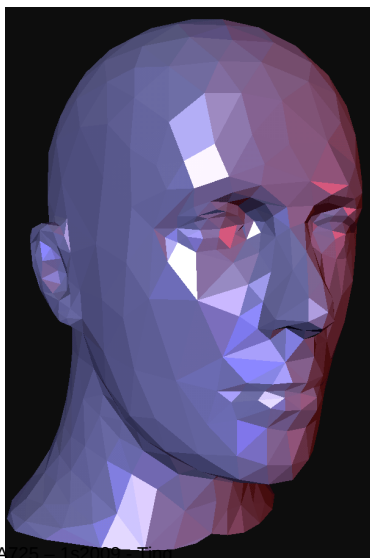
Malhas Poligonais



Aproximação por facetas planas

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Malhas Poligonais



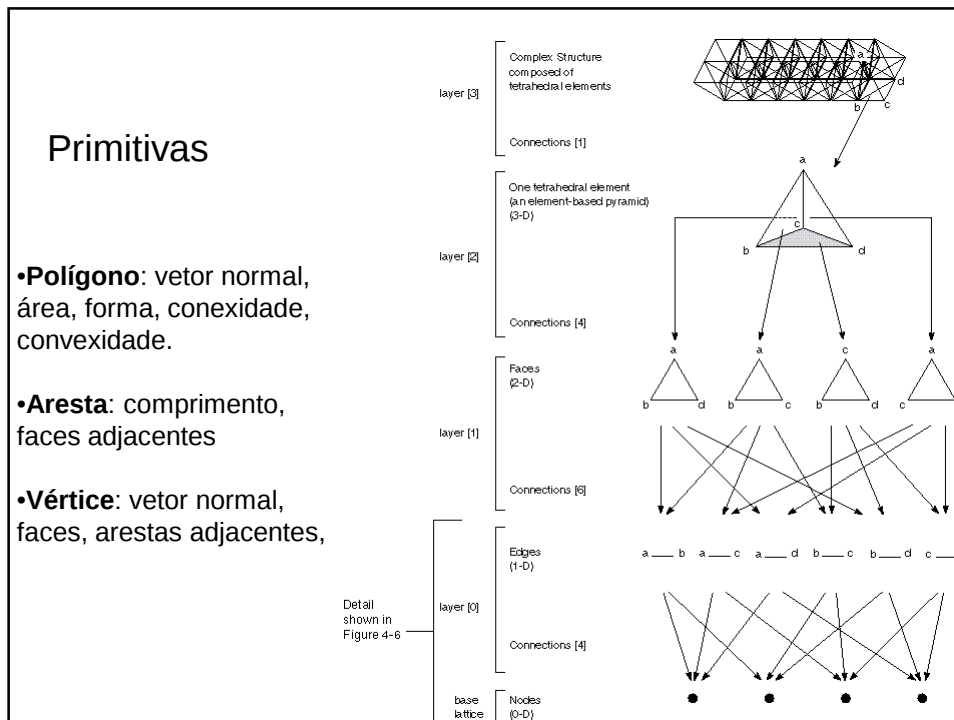
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Primitivas

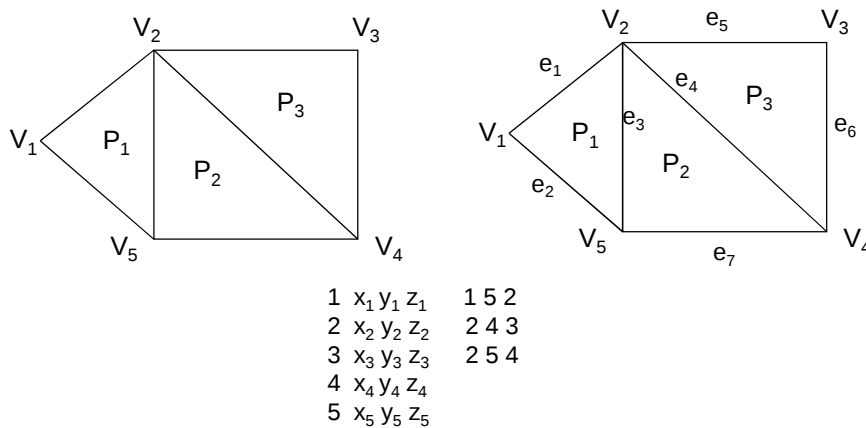
•**Polígono:** vetor normal, área, forma, conexidade, convexidade.

•**Aresta:** comprimento, faces adjacentes

•**Vértice:** vetor normal, faces, arestas adjacentes,



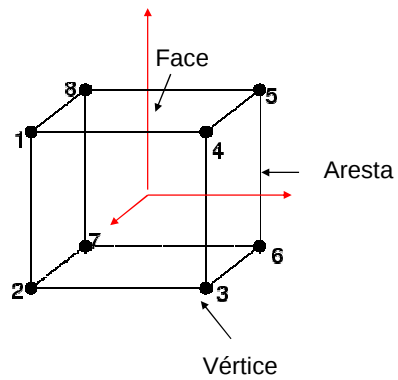
Malha Poligonal Topologia



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Malha Poligonal

Representação por Bordo

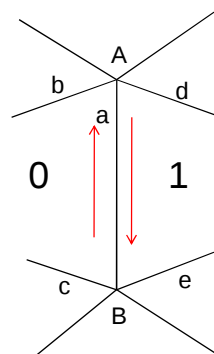


Boundary representation

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Malha Poligonal

Estrutura Alada

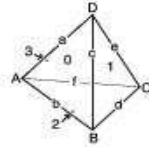


Winged-edge data structure

Aresta	Vértice 1	Vértice 2	Face direita	Face esquerda	Predecessor direito	Sucessor direito	Predecessor esquerdo	Predecessor direito
a	B	A	0	1	c	b	d	e

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Malha Poligonal Estrutura Alada

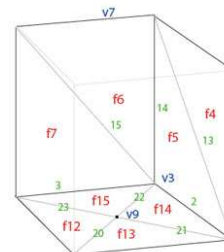
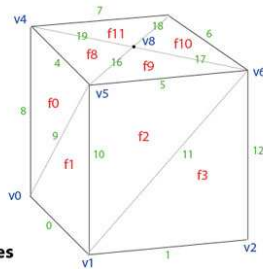


edge	vertex ₁	vertex ₂	face _{left}	face _{right}	pred _{left}	succ _{left}	pred _{right}	succ _{right}
a	A	D	3	0	f	e	c	b
b	A	B	0	2	a	c	d	f
c	B	D	0	1	b	a	e	d
d	B	C	1	2	c	e	f	b
e	C	D	1	3	d	c	a	f
f	C	A	3	2	e	a	b	d

Face	Aresta
0	a
1	c
2	d
3	a

Vértice	Aresta
A	a
B	d
C	e
D	c

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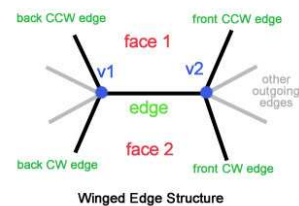


Winged-Edge Meshes

Face List
f0: 4 8 9
f1: 0 10 9
f2: 5 10 11
f3: 1 12 11
f4: 6 12 13
f5: 2 14 13
f6: 7 14 15
f7: 3 8 15
f8: 4 16 19
f9: 5 17 16
f10: 6 18 17
f11: 7 19 18
f12: 0 23 20
f13: 1 20 21
f14: 2 21 22
f15: 3 22 23

Edge List
e0: v0 v1 f1 f12 9 23 10 20
e1: v1 v2 f3 f13 11 20 12 21
e2: v2 v3 f5 f14 13 21 14 22
e3: v3 v0 f7 f15 15 22 8 23
e4: v4 v5 f0 f8 19 8 16 9
e5: v5 v6 f2 f9 16 10 17 11
e6: v6 v7 f4 f10 17 12 18 13
e7: v7 v4 f6 f11 18 14 19 15
e8: v0 v4 f7 f0 3 9 7 4
e9: v0 v5 f0 f1 8 0 4 10
e10: v1 v5 f1 f2 0 11 9 5
e11: v1 v6 f2 f3 10 1 5 12
e12: v2 v6 f3 f4 1 13 11 6
e13: v2 v7 f4 f5 12 2 6 14
e14: v3 v7 f5 f6 2 15 13 7
e15: v3 v4 f6 f7 14 3 7 15
e16: v5 v8 f8 f9 4 5 19 17
e17: v6 v8 f9 f10 5 6 16 18
e18: v7 v8 f10 f11 6 7 17 19
e19: v4 v8 f11 f8 7 4 18 16
e20: v1 v9 f12 f13 0 1 23 21
e21: v2 v9 f13 f14 1 2 20 22
e22: v3 v9 f14 f15 2 3 21 23
e23: v0 v9 f15 f12 3 0 22 20

Vertex List
v0: 0,0,0 8 9 0 23 3
v1: 1,0,0 10 11 1 20 0
v2: 1,1,0 12 13 2 21 1
v3: 0,1,0 14 15 3 22 2
v4: 0,0,1 8 15 7 19 4
v5: 1,0,1 10 9 4 16 5
v6: 1,1,1 12 11 5 17 6
v7: 0,1,1 14 13 6 18 7
v8: 5,5,0 16 17 18 19
v9: 5,5,1 20 21 22 23



Malha Poligonal Geração/Criação

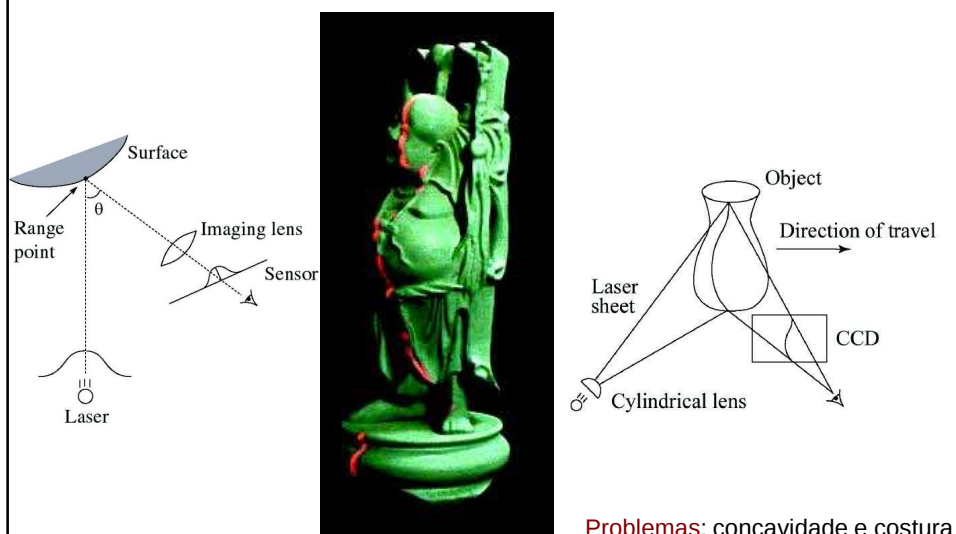
- Manualmente ou digitalizador
- Automáticamente
- Funções matemáticas



A Volkswagen Beetle becomes the subject of a 1970 simulation project. Ivan Sutherland (left) and assistants plot coordinates for digitizing the car.

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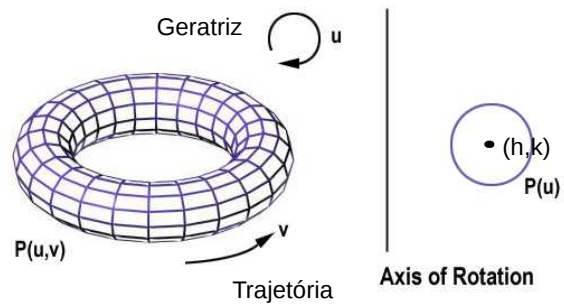
Malha Poligonal Dispositivo de Aquisição



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Problemas: concavidade e costura

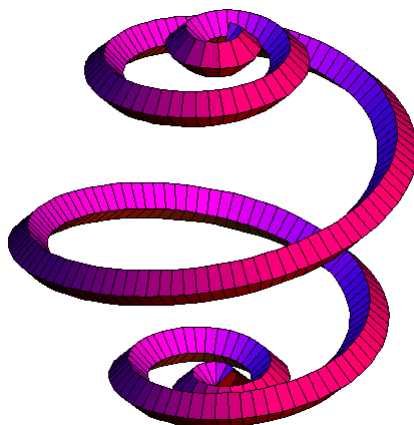
Malha Poligonal Varredura



$$\begin{aligned}x &= h+r \cos u \\y &= (k+r \sin u)\cos v \\z &= (k+r \sin u)\sin v\end{aligned}$$

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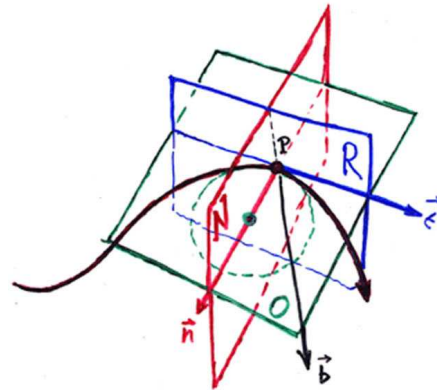
Malha Poligonal Varredura



Passo?
Orientação?

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Malha Poligonal Triedro de Frenet

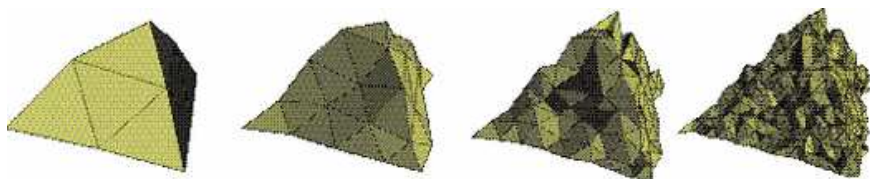


Frenet - Frame

$$\vec{T} = \frac{\vec{P}'}{|\vec{P}'|} \quad \vec{B} = \frac{\vec{P}' \times \vec{P}''}{|\vec{P}' \times \vec{P}''|} \quad \vec{N} = \vec{B} \times \vec{T}$$

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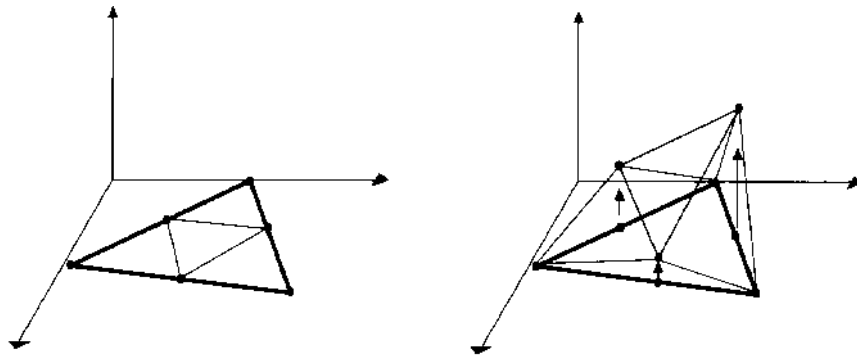
Malha Poligonal Terreno Fractal



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Malha Poligonal

Terreno Fractal (Fournier, Fussel e Carpenter)



Para cada segmento

$$x_{\text{novo}} = 1/2(x_i + x_{i+1})$$

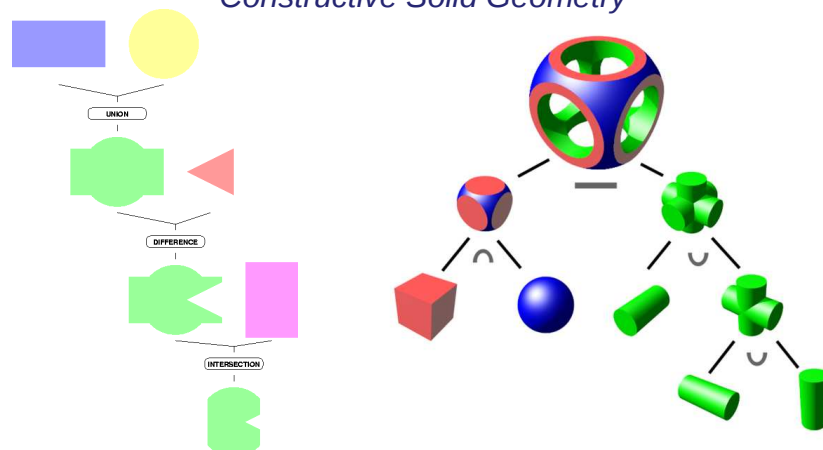
$$y_{\text{novo}} = 1/2(y_i + y_{i+1}) + P(x_{i+1} - x_i) \text{ Random}(x_{\text{novo}})$$

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CSG

Operações

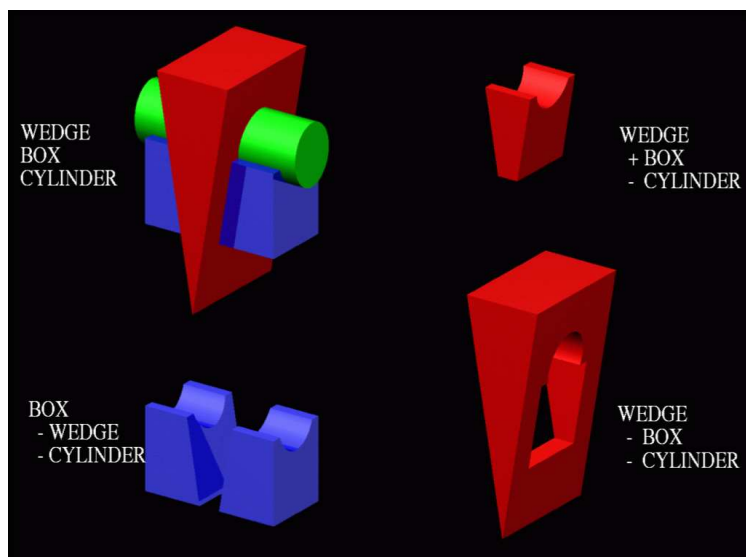
Constructive Solid Geometry



Operações Booleanas entre as primitivas

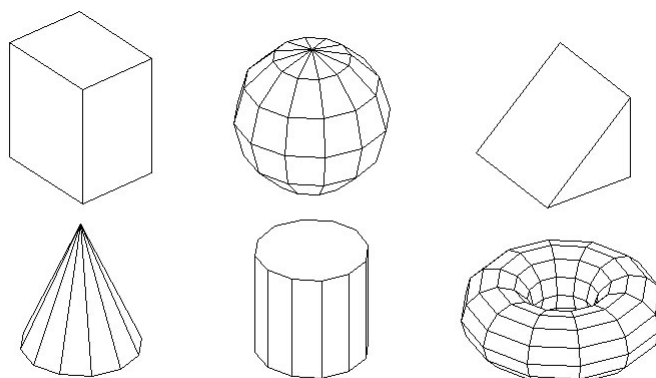
CSG

Operações

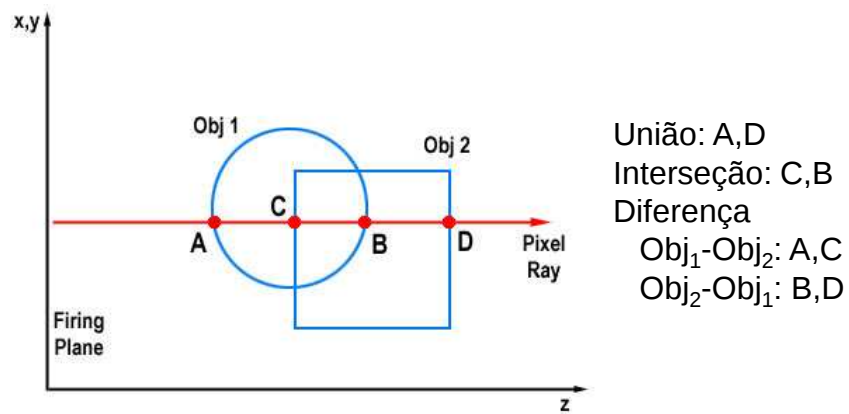


CSG

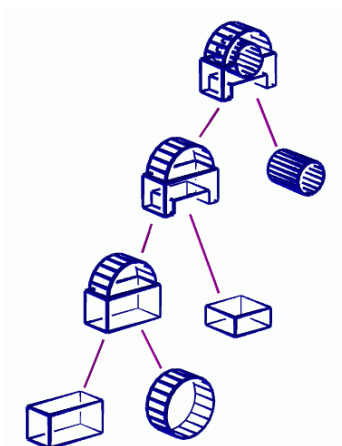
Primitivas



CSG Desenvolvimento



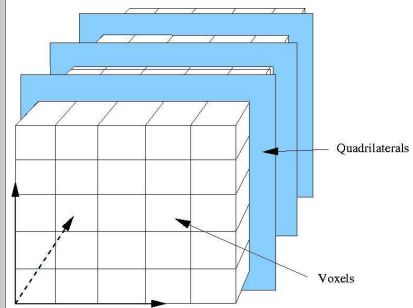
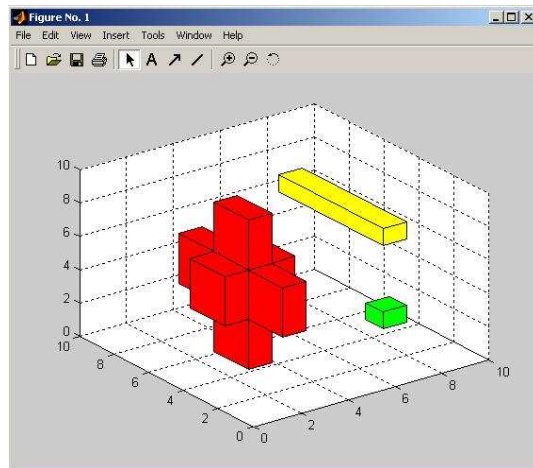
CSG



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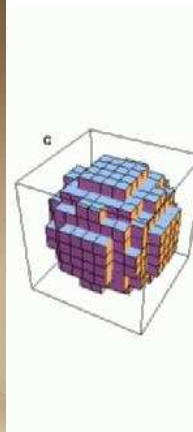
Operações?

Subdivisão Espacial



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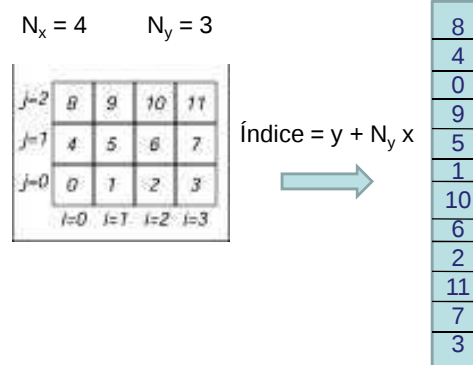
Subdivisão Espacial



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Subdivisão Espacial

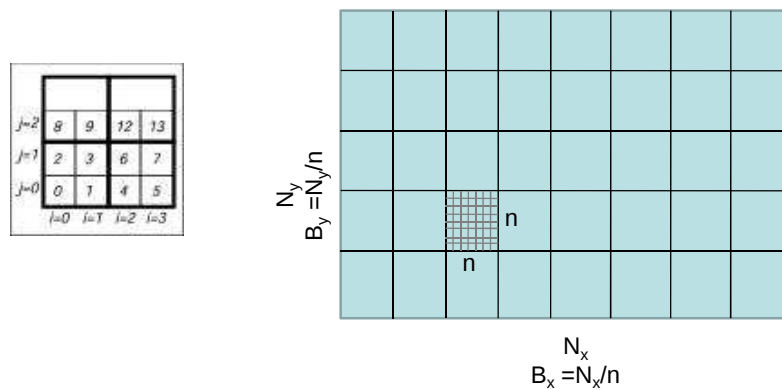
Arranjos Multidimensionais



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Subdivisão Espacial

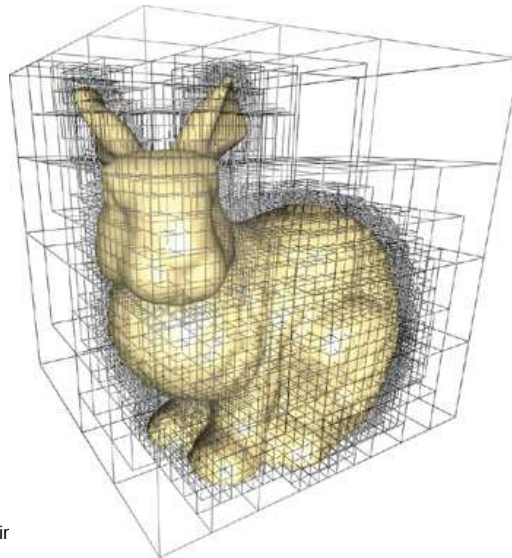
Arranjos Multidimensionais

Ladrilhamento (*Tiling*)

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Subdivisão Espacial

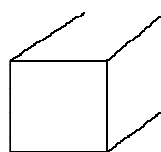
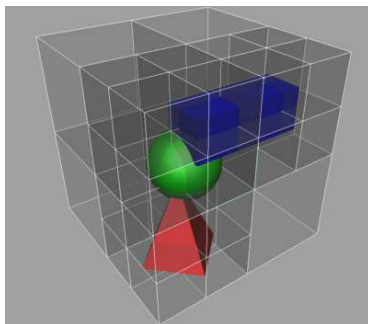
Octree



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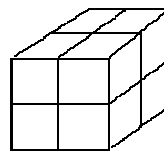
Subdivisão Espacial

Octree

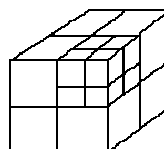
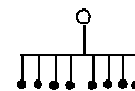


(root)

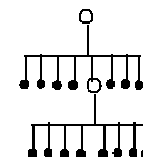
•



(1 level)



(2 levels)



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Representação Analítica

➤ Funções Explícitas:

Uma das coordenadas é explicitamente dada em função das outras.

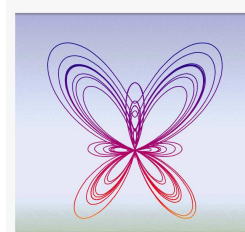
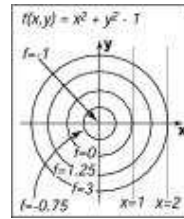
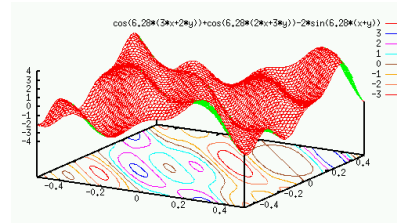
➤ Funções Implícitas:

As coordenadas são relacionadas por uma função.

➤ Funções Paramétricas:

As coordenadas são dadas em termos de um conjunto de parâmetros.

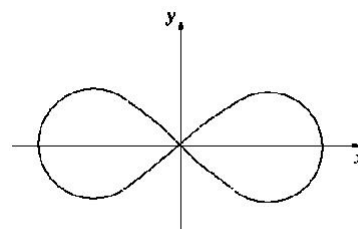
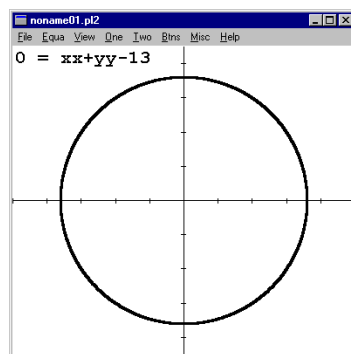
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Função Explícita

$$y = f(x) \text{ ou } z = f(x,y)$$

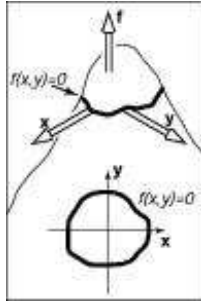
Não permite representar correspondências sobrejetivas!



$$\text{Lemniscate: } (x^2 + y^2)^2 - (x^2 - y^2) = 0$$

Função Implícita

$$f(x,y)=0 \text{ ou } f(x,y,z)=0$$



Exemplo: $x^2+y^2=R^2$

$$f(x,y) = x^2+y^2-R^2 = 0$$

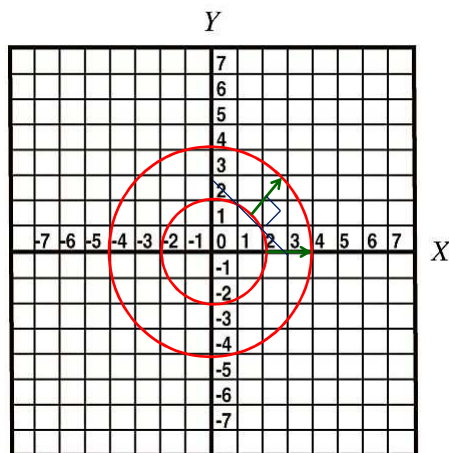
Para um dado R:

- (x,y) sobre a curva
 $x^2+y^2 - R^2 = 0$
- (x,y) dentro da curva
 $x^2+y^2 - R^2 < 0$
- (x,y) fora da curva
 $x^2+y^2 - R^2 > 0$

Função de classificação de pertinência

Função Implícita

Gradiente = Vetor Normal



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Função Implícita

$$f(x,y,z) = 0$$

➤ Vetor Normal

$$\text{grad}(f)$$

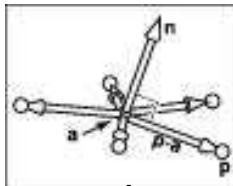
➤ Planos Implícitos

$$(p-a) \cdot \mathbf{n} = 0$$

➤ Cônicas: elipsóides, hiperbolóides e parabolóides

$$ax^2+by^2+cz^2+dxy+eyz+fxz+gx+hy+jz+k=0$$

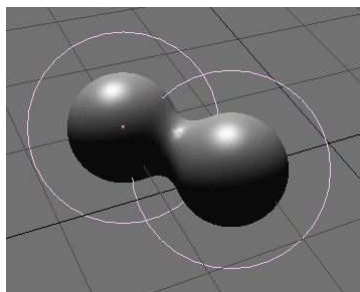
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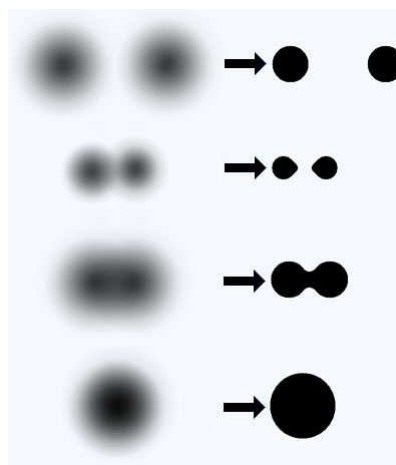
Função Implícita

Metaball

$$f(x,y,z) = 0$$



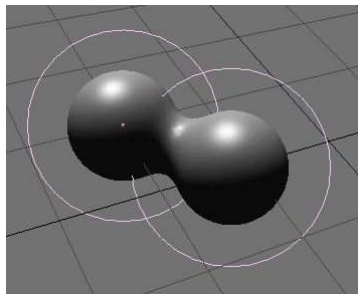
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(Two on top of each other)

Função Implícita *Metaball*

$$f(x,y,z) = 0$$



Campo de Influência de cada primitiva num raio R

$$f(P) = \begin{cases} (1-(P-P_i)^2/R^2)^2, & \text{se } d \leq R \\ 0, & \text{se } d > R \end{cases}$$

$$\sum f_i(P)$$

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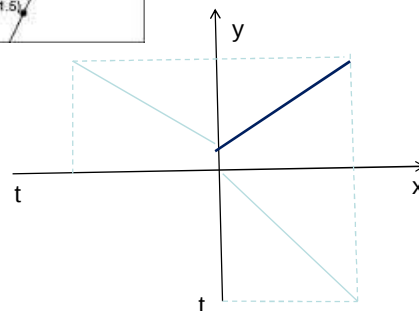
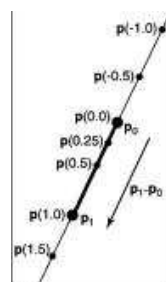
Função Paramétrica

➤ Segmentos

$$p(t) = p_0 + t(p_1 - p_0)$$

➤ Curvas

$$p(t) = (x(t), y(t))$$

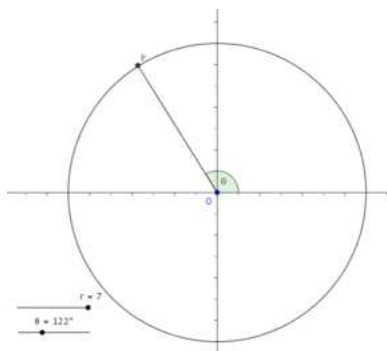


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Função Paramétrica

Circuferências

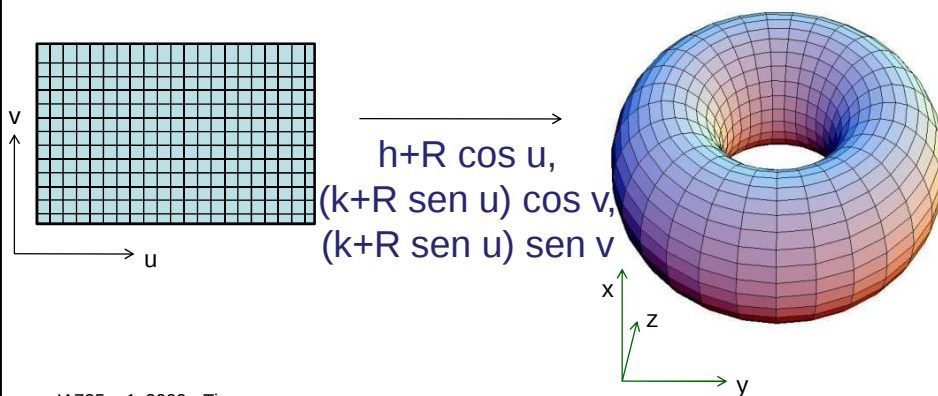
$$p(t) = (r \cos t, r \sin t)$$



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Função Paramétrica

$$(x(u,v), y(u,v), z(u,v))$$



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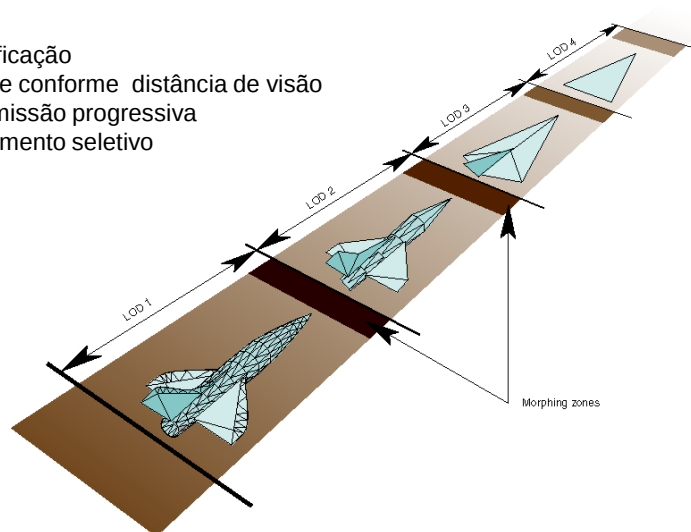
Quadro Comparativo

	Malha	F.Paramétrica	CSG	S.Espacial	F. Implícita
Dimensão	2D	2D	3D	3D (voxel)	2D/3D
Representatividade	Abrangente	restrito/preciso	restrito/preciso	abrangente	restrito/preciso
Concisão	Baixa	boa	Boa	baixa	boa
Univocidade	não	não	não	sim	não
Interface	tediosa	Conhecimento matemático	intuitiva	tediosa	Conhecimento matemático
Complexidade	simples	"complexa"	+ -	simples	"complexa"
Estrutura de dados	lista		árvore	arranjo	
Editabilidade	baixa	alta	alta	baixa	baixa

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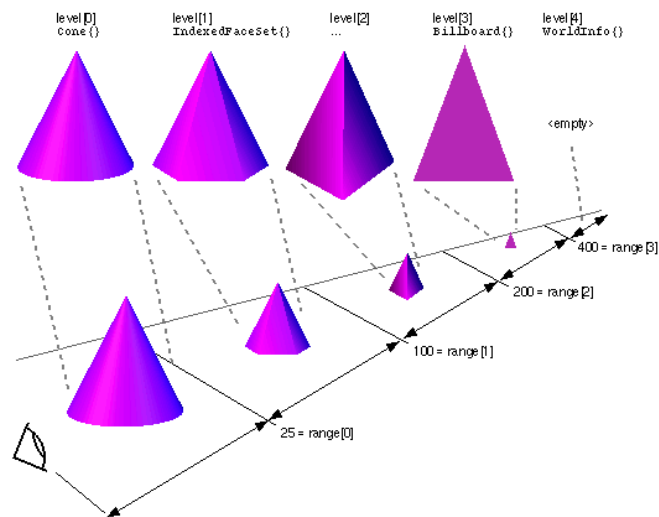
Níveis de Detalhe

Simplificação
 Detalhe conforme distância de visão
 Transmissão progressiva
 Refinamento seletivo



IA725 – 1s2009 - Ting

Níveis de Detalhe



IA725 – 1s2009 - Ting

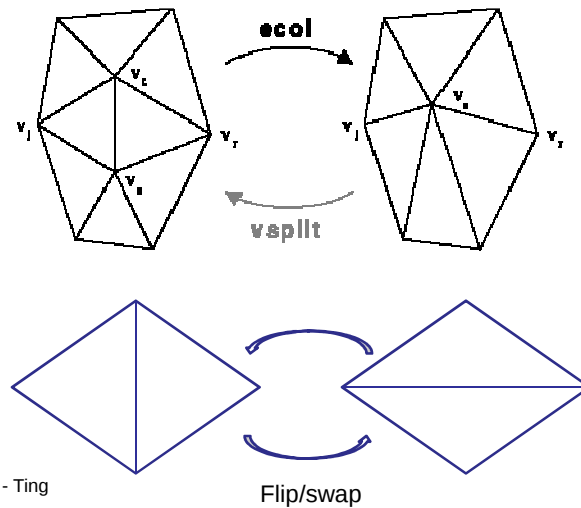
Níveis de Detalhe

Imagem					
Vértices	~5500	~2880	~1580	~670	140
Nota	Máximo de detalhes (próximo)				Mínimo de detalhes (distante)

IA725 – 1s2009 - Ting

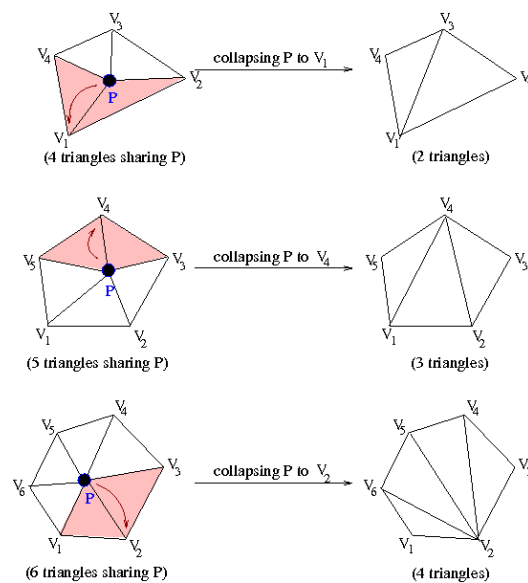
Níveis de Detalhe

Operações Básicas (Lawson)



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Níveis de Detalhe



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