



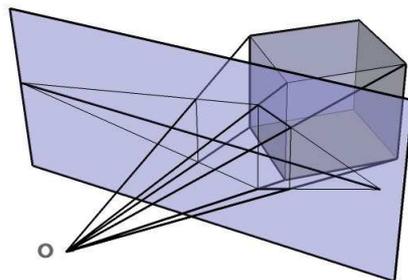
IA725 – Computação Gráfica I

Transformações Projetivas

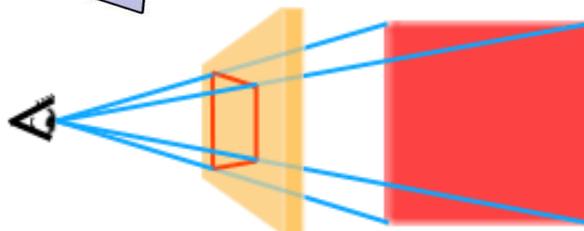
Shirley: Capítulo 7
Redbook: Capítulo 3



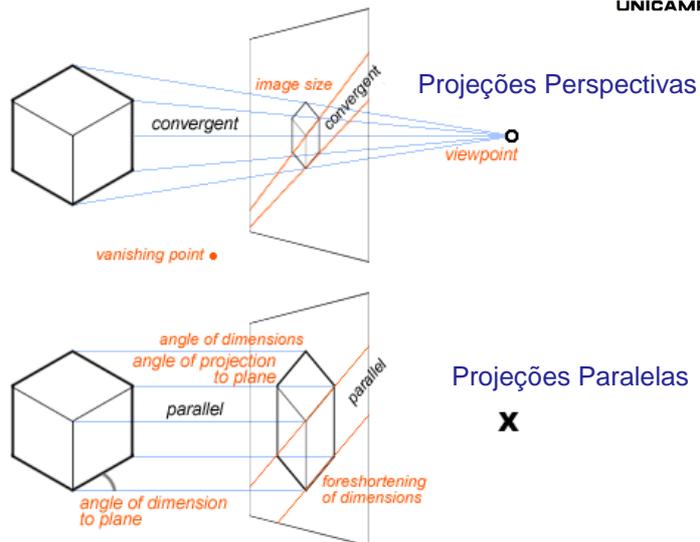
Transformações Projetivas



Projetar modelos
geométricos 3D numa
imagem 2D, exibível
em dispositivos de
saída 2D



- Aplicações
- Taxonomia de projeções
- Modelo Geométrico de Projeções
- Modelo de Câmera

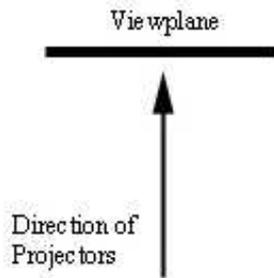




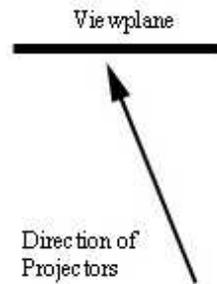
Projeções Paralelas



Retas e Oblíquas



Ortográficas/Axonométricas



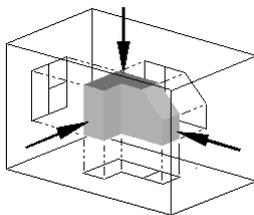
Oblíquas



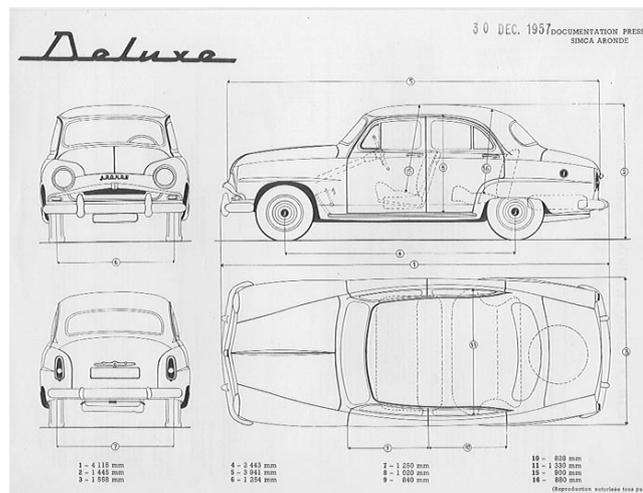
Projeções Paralelas



Desenhos técnicos: preserva a relação das medidas



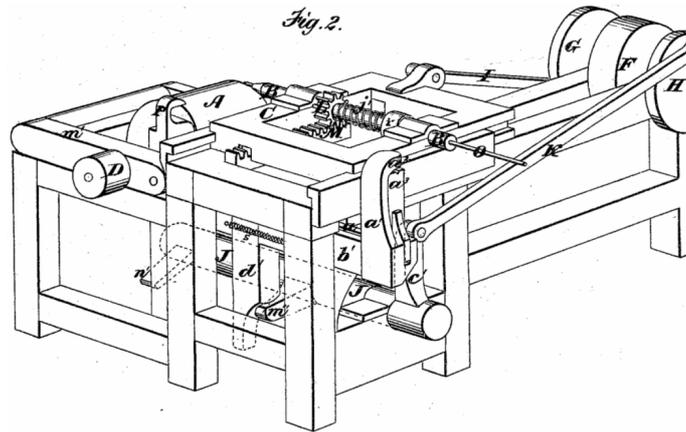
Projeções Paralelas Ortográficas



Projeções Paralelas

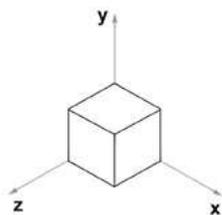
Projeções Paralelas Axonométricas percepção de profundidade

Fig. 2.

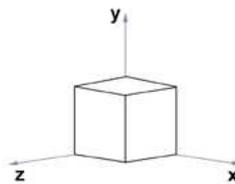


Projeções Paralelas

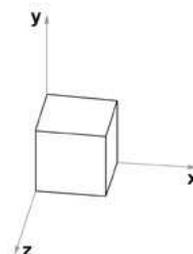
Projeções Paralelas Axonométricas



Isométricas
(mesmo fator
de redução
em 3
direções)



Dimétricas
(mesmo fator
de redução
em 2
direções)



Trimétricas



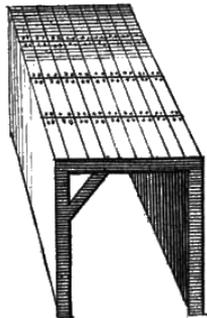
Projeções Paralelas



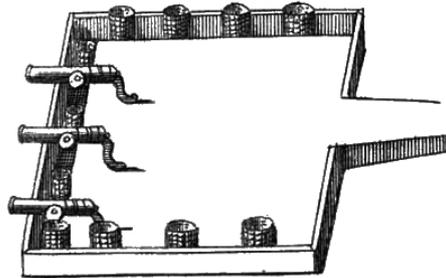
UNICAMP

Projeção Paralela Oblíqua Cavalier : “Vista Aérea”

Gallery



Battery

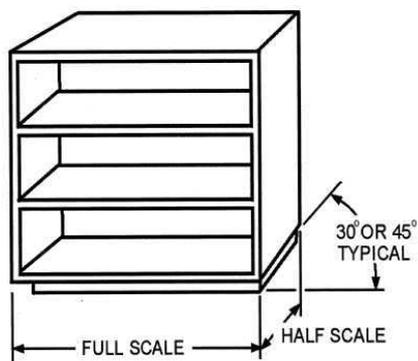


Projeções Paralelas



UNICAMP

“Vista Perspectiva” dos Móveis:
Projeção Paralela Oblíqua Cabinet



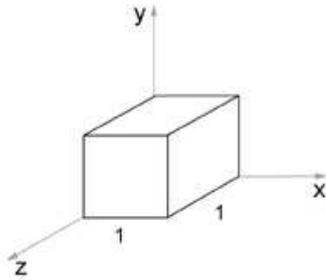
DMV2Ch06f06



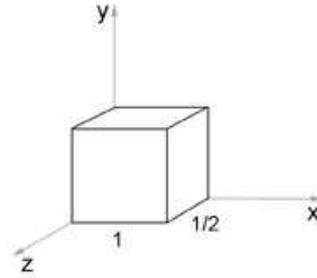
Projeções Paralelas



Cavalier e Cabinet



Cavalier



Cabinet

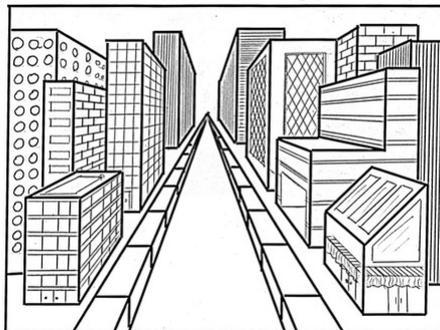


Projeções Perspectivas

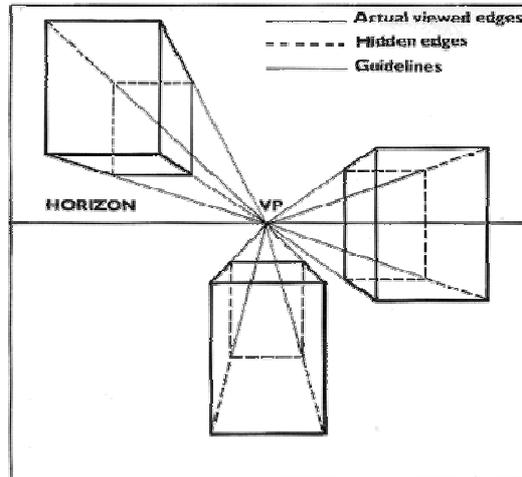


Geometria Perspectiva (Desargues, Pascal, Monge)

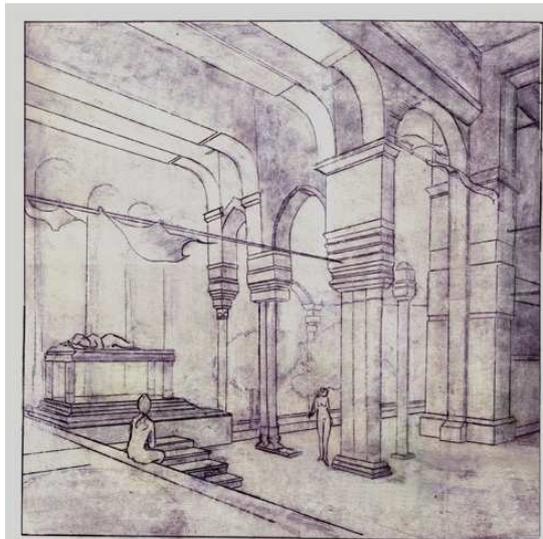
Linhas paralelas podem se cruzar em um ponto!

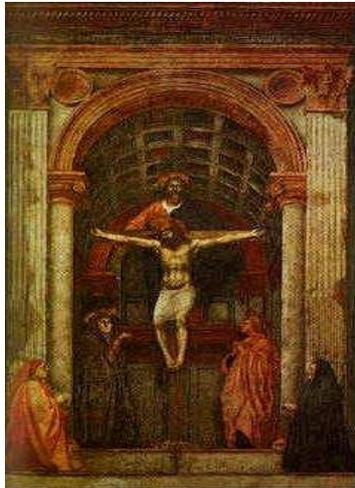


Um ponto de fuga



Exemplos

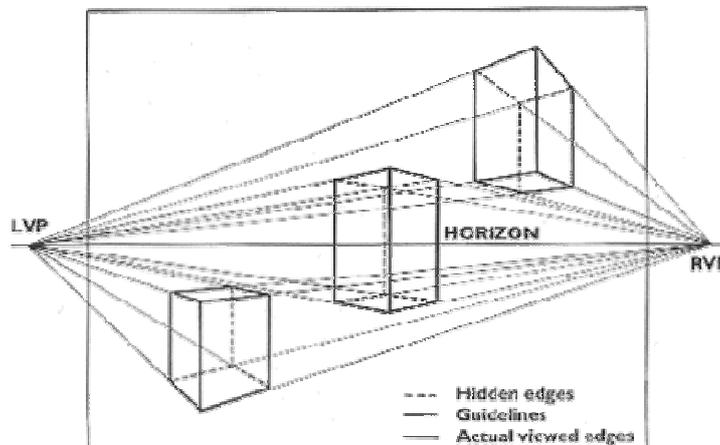




Trinity with the Virgin, St. John and Donors de Masaccio em 1427



The Piazza of St. Mark, Venice de Canaletto em 1735-45





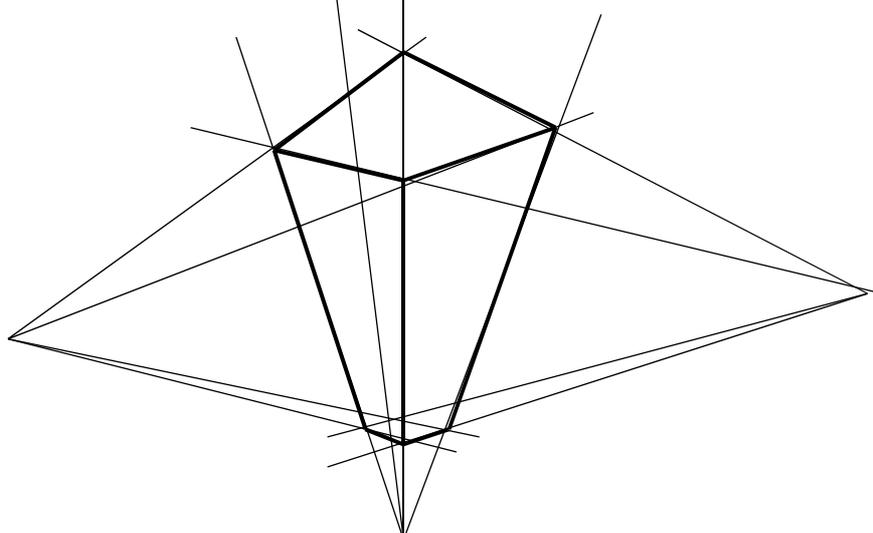
Projeções Perspectivas



Projeções Perspectivas

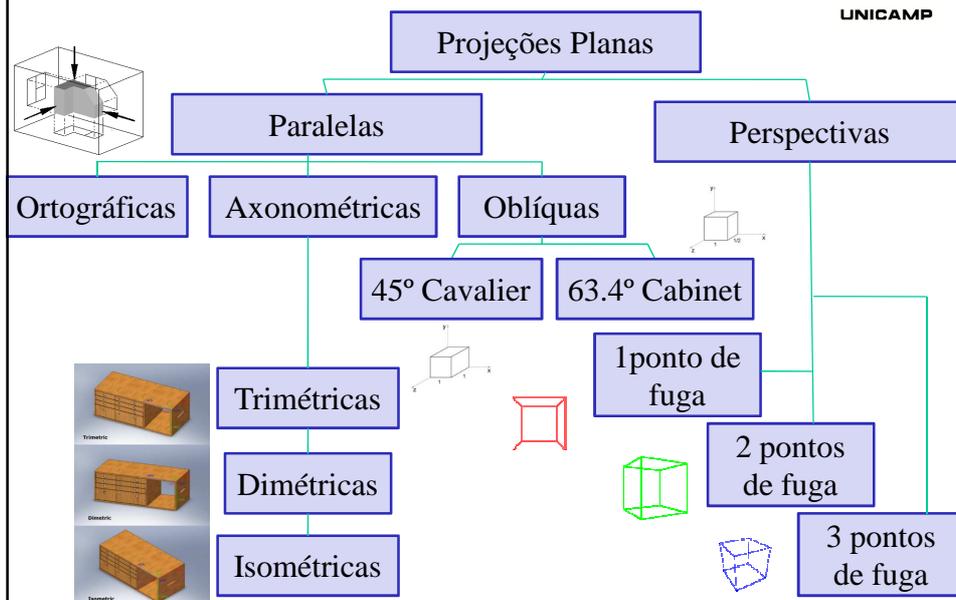


Três pontos de fuga





City Night de Georgia
O'Keefe em 1926





With a Camera	With a Computer
 tripod	 viewing positioning the viewing volume in the world
 model	 modeling positioning the models in the world
 lens	 projection determining shape of viewing volume
 photograph	 viewport



Geração de Imagens

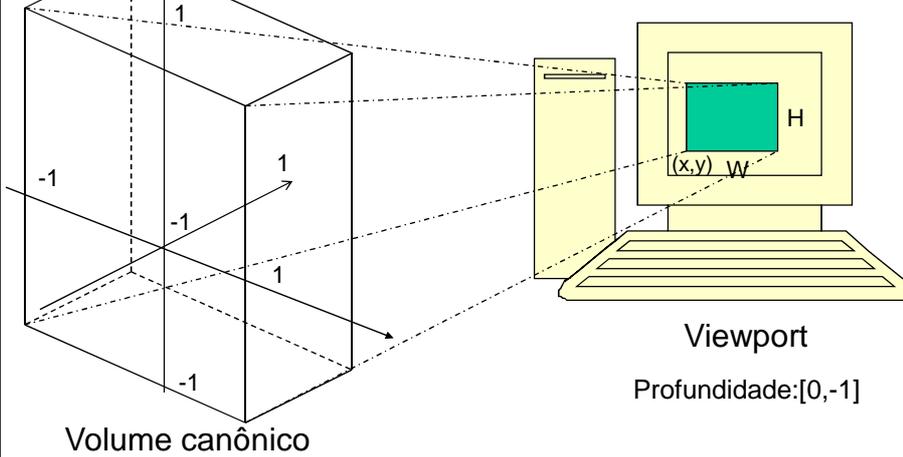


With a Camera	With a Computer
 tripod	 viewing positioning the viewing volume in the world
 model	 modeling positioning the models in the world
 lens	 projection determining shape of viewing volume
 photograph	 viewport



Modelo da Câmera

Espaço Normalizado → Espaço do Dispositivo
 Aspect ratio (relação de aspecto) = W/H



$$\begin{bmatrix} 1 & 0 & 0 & x \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
 \begin{bmatrix} W/2 & 0 & 0 & 0 \\ 0 & H/2 & 0 & 0 \\ 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
 =
 \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$(-1, -1, -1, 1, -1, 1) \rightarrow (x, x+W, y, y+H, 0, -1)$$

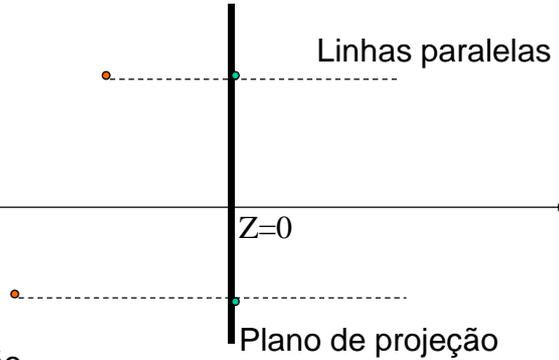
$$\begin{bmatrix} W/2 & 0 & 0 & W/2+x \\ 0 & H/2 & 0 & H/2+y \\ 0 & 0 & -1/2 & -1/2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Modelo Geométrico de Projeções Paralelas

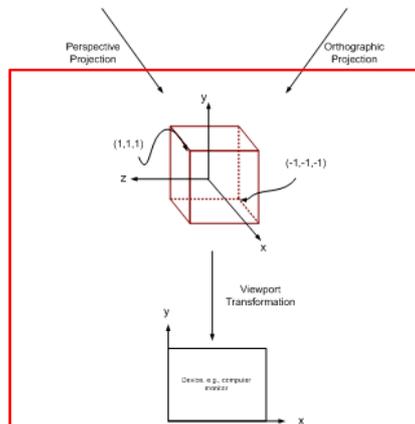
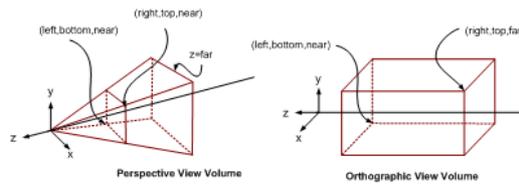
$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array} \right]$$

zerar a coordenada z!

Centro de projeção
no infinito



Volume Canônico



$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2/(r-l) & 0 & 0 & 0 \\ 0 & 2/(t-b) & 0 & 0 \\ 0 & 0 & 2/(n-f) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & -(r+l)/2 \\ 0 & 1 & 0 & -(t+b)/2 \\ 0 & 0 & 1 & -(n+f)/2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

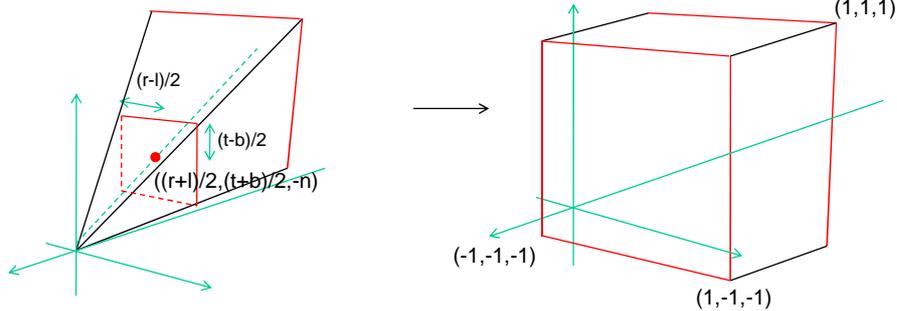
$$(l,r,b,t,-n,-f) \longrightarrow (-1,1,-1,1,-1,1) \begin{bmatrix} 2/(r-l) & 0 & 0 & -(r+l)/(r-l) \\ 0 & 2/(t-b) & 0 & -(t+b)/(t-b) \\ 0 & 0 & -2/(f-n) & -(f+n)/(f-n) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$n, f \geq 0$

$$(l,r,b,t,-n,-f) \longrightarrow (-1,1,-1,1,-1,1)$$

Volume Perspectivo Canônico

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{Per} \cdot S_2 \cdot S_1 \cdot \text{Sh} = \begin{bmatrix} 2n/(r-l) & 0 & (r+l)/(r-l) & 0 \\ 0 & 2n/(t-b) & (t+b)/(t-b) & 0 \\ 0 & 0 & -(f+n)/(f-n) & -2fn/(f-n) \\ 0 & 0 & -1 & 0 \end{bmatrix}$$



1º Passo: eixo OC = eixo z

Transformação de 3 vetores LI:

$$\begin{pmatrix} 1,0,0 \\ 0,1,0 \\ ((r+l)/2, (t+b)/2, -n) \end{pmatrix}$$

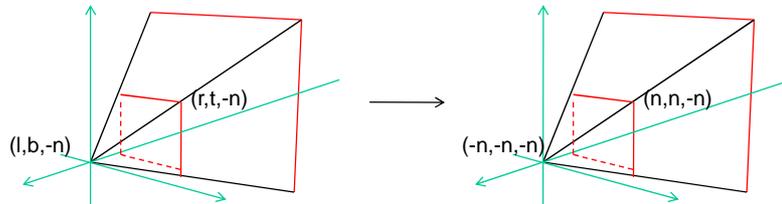
Em notação matricial:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -n \end{pmatrix} = \text{Sh}' \begin{pmatrix} 1 & 0 & (r+l)/2 \\ 0 & 1 & (t+b)/2 \\ 0 & 0 & -n \end{pmatrix}$$

Diagram showing the transformation of a perspective volume into a canonical volume. The left diagram shows a perspective volume with dimensions (r-l)/2 and (t-b)/2, and a point C = ((r+l)/2, (t+b)/2, -n). The right diagram shows a canonical volume with dimensions (r-l)/2 and (t-b)/2, and a point C' = (0, 0, -n).

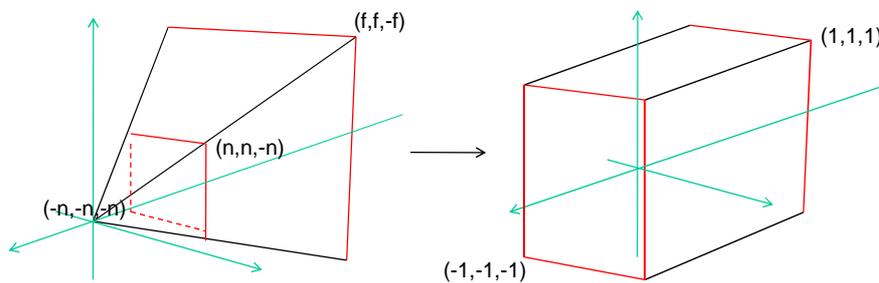
$$\text{Sh} = \begin{bmatrix} 1 & 0 & (r+l)/2n & 0 \\ 0 & 1 & (t+b)/2n & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2º Passo: base quadrada e altura igual ao lado



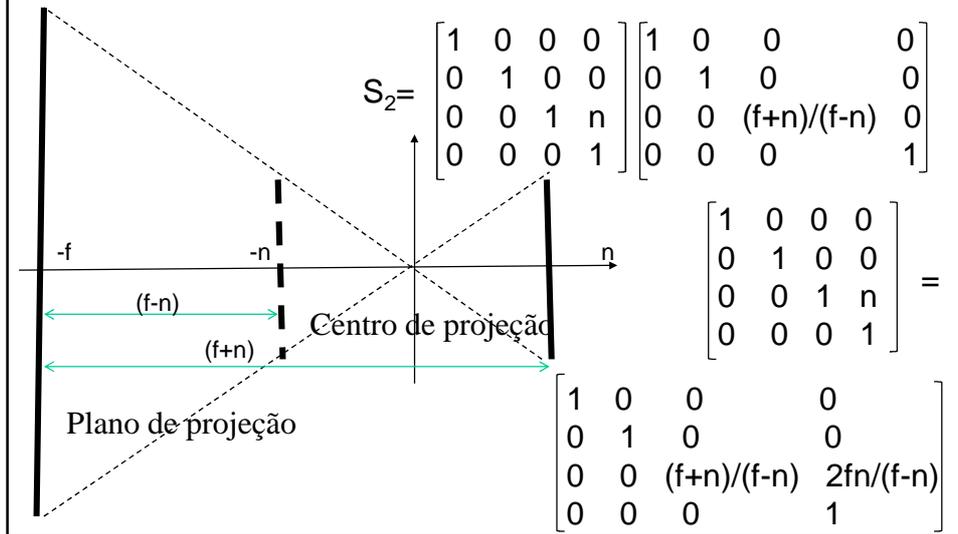
$$S_1 = \begin{bmatrix} 2n/(r-l) & 0 & 0 & 0 \\ 0 & 2n/(t-b) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3º Passo: trapezoedro em paralelepípedo

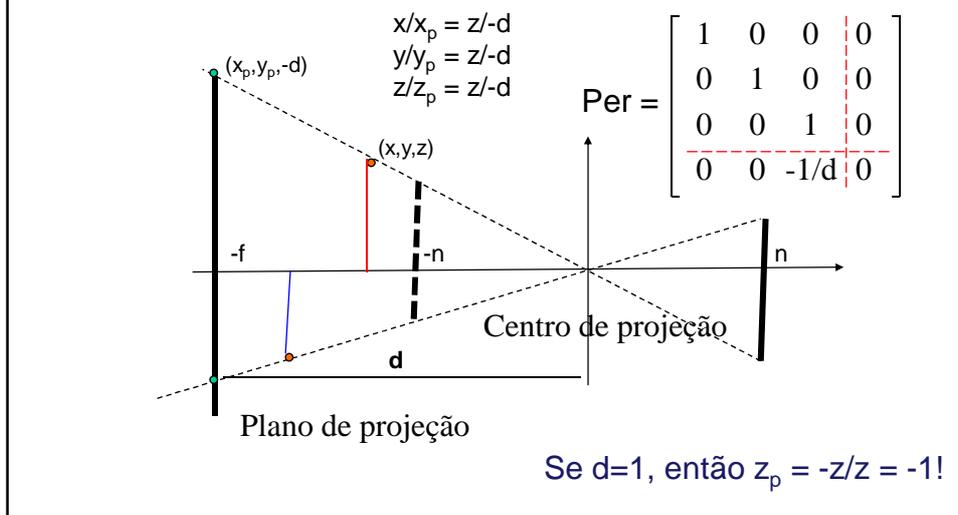


A ser feito em 2 sub-passos ...

3.a. Alargar o volume em eixo z (de (f-n) para (f+n))

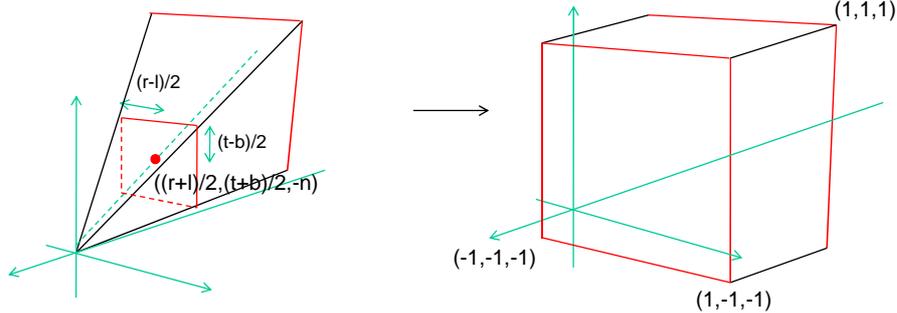


3.b. Projeções Perspectivas = Semelhança de Triângulos



Volume Perspectivo Canônico

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{Per} \cdot S_2 \cdot S_1 \cdot S_h = \begin{bmatrix} 2n/(r-l) & 0 & (r+l)/(r-l) & 0 \\ 0 & 2n/(t-b) & (t+b)/(t-b) & 0 \\ 0 & 0 & -(f+n)/(f-n) & -2fn/(f-n) \\ 0 & 0 & -1 & 0 \end{bmatrix}$$



Propriedades

Projeções Paralelas → Linhas paralelas em linhas paralelas
 Projeções Perspectivas → Linhas paralelas em linhas convergentes

Colinearidade é preservada?

$$P(t) = (1-t)P_1 + tP_2$$

$$M[(1-t)P_1 + tP_2] = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \left((1-t) \begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{bmatrix} + t \begin{bmatrix} x_2 \\ y_2 \\ z_2 \\ 1 \end{bmatrix} \right)$$

$$\Delta(t) = \begin{bmatrix} (1-t)(a_{11}x_1 + a_{12}y_1 + a_{13}z_1 + a_{14}) + t(a_{11}x_2 + a_{12}y_2 + a_{13}z_2 + a_{14}) \\ (1-t)(a_{21}x_1 + a_{22}y_1 + a_{23}z_1 + a_{24}) + t(a_{21}x_2 + a_{22}y_2 + a_{23}z_2 + a_{24}) \\ (1-t)(a_{31}x_1 + a_{32}y_1 + a_{33}z_1 + a_{34}) + t(a_{31}x_2 + a_{32}y_2 + a_{33}z_2 + a_{34}) \\ (1-t)(a_{41}x_1 + a_{42}y_1 + a_{43}z_1 + a_{44}) + t(a_{41}x_2 + a_{42}y_2 + a_{43}z_2 + a_{44}) \end{bmatrix}$$

$$\sim 1/\Delta(t) \left((1-t) \begin{bmatrix} (a_{11}x_1 + a_{12}y_1 + a_{13}z_1 + a_{14}) \\ (a_{21}x_1 + a_{22}y_1 + a_{23}z_1 + a_{24}) \\ (a_{31}x_1 + a_{32}y_1 + a_{33}z_1 + a_{34}) \\ 1 \end{bmatrix} + t \begin{bmatrix} (a_{11}x_2 + a_{12}y_2 + a_{13}z_2 + a_{14}) \\ (a_{21}x_2 + a_{22}y_2 + a_{23}z_2 + a_{24}) \\ (a_{31}x_2 + a_{32}y_2 + a_{33}z_2 + a_{34}) \\ 1 \end{bmatrix} \right)$$

Coplanaridade é preservada?

$$P(t) = \alpha P_1 + \beta P_2 + \gamma P_3$$

$$M[\alpha P_1 + \beta P_2 + \gamma P_3] = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \left(\alpha \begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{bmatrix} + \beta \begin{bmatrix} x_2 \\ y_2 \\ z_2 \\ 1 \end{bmatrix} + \gamma \begin{bmatrix} x_3 \\ y_3 \\ z_3 \\ 1 \end{bmatrix} \right)$$

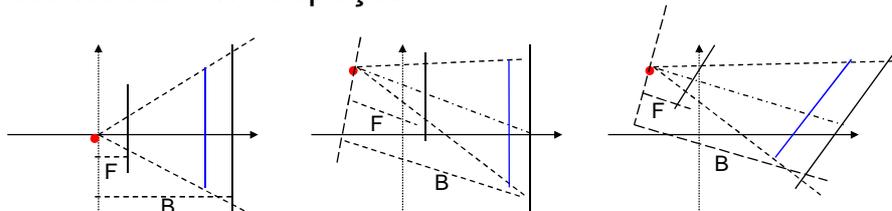
$$= \begin{bmatrix} \alpha(a_{11}x_1 + a_{12}y_1 + a_{13}z_1 + a_{14}) + \beta(a_{11}x_2 + a_{12}y_2 + a_{13}z_2 + a_{14}) + \gamma(a_{11}x_3 + a_{12}y_3 + a_{13}z_3 + a_{14}) \\ \alpha(a_{21}x_1 + a_{22}y_1 + a_{23}z_1 + a_{24}) + \beta(a_{21}x_2 + a_{22}y_2 + a_{23}z_2 + a_{24}) + \gamma(a_{21}x_3 + a_{22}y_3 + a_{23}z_3 + a_{24}) \\ \alpha(a_{31}x_1 + a_{32}y_1 + a_{33}z_1 + a_{34}) + \beta(a_{31}x_2 + a_{32}y_2 + a_{33}z_2 + a_{34}) + \gamma(a_{31}x_3 + a_{32}y_3 + a_{33}z_3 + a_{34}) \\ \alpha(a_{41}x_1 + a_{42}y_1 + a_{43}z_1 + a_{44}) + \beta(a_{41}x_2 + a_{42}y_2 + a_{43}z_2 + a_{44}) + \gamma(a_{41}x_3 + a_{42}y_3 + a_{43}z_3 + a_{44}) \end{bmatrix}$$

$$\Delta =$$

$$= 1/\Delta \left(\alpha \begin{bmatrix} a_{11}x_1 + a_{12}y_1 + a_{13}z_1 + a_{14} \\ a_{21}x_1 + a_{22}y_1 + a_{23}z_1 + a_{24} \\ a_{31}x_1 + a_{32}y_1 + a_{33}z_1 + a_{34} \\ 1 \end{bmatrix} + \beta \begin{bmatrix} a_{11}x_2 + a_{12}y_2 + a_{13}z_2 + a_{14} \\ a_{21}x_2 + a_{22}y_2 + a_{23}z_2 + a_{24} \\ a_{31}x_2 + a_{32}y_2 + a_{33}z_2 + a_{34} \\ 1 \end{bmatrix} + \gamma \begin{bmatrix} a_{11}x_3 + a_{12}y_3 + a_{13}z_3 + a_{14} \\ a_{21}x_3 + a_{22}y_3 + a_{23}z_3 + a_{24} \\ a_{31}x_3 + a_{32}y_3 + a_{33}z_3 + a_{34} \\ 1 \end{bmatrix} \right)$$

Formulação Algébrica: Três Casos

- Plano de projeção tem o vetor normal na direção do eixo z e o centro de projeção sobre o eixo z.
- Plano de projeção tem o vetor normal na direção do eixo z e o centro de projeção arbitrariamente posicionado.
- Tanto o plano quanto o centro são arbitrariamente posicionados no espaço.



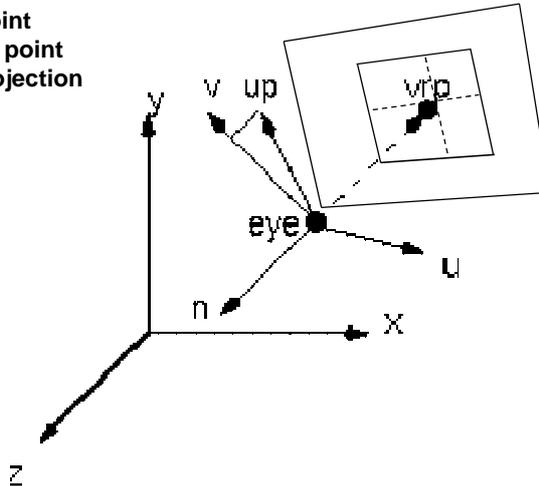


Modelo de Câmera

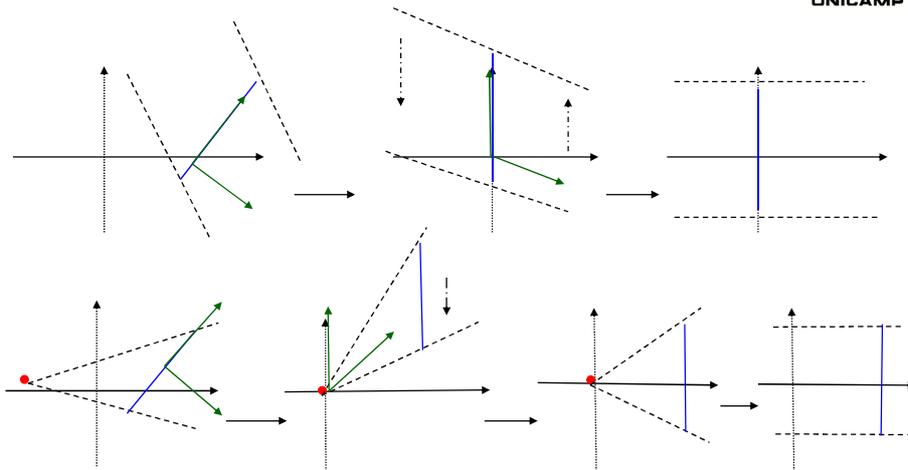
Plano e Centro de Projeção



VUP (up): view up vector
 VRP (center): view reference point
 PRP (eye): projection reference point
 dop (PRP-VRP): direction of projection



Transformação de Espaços

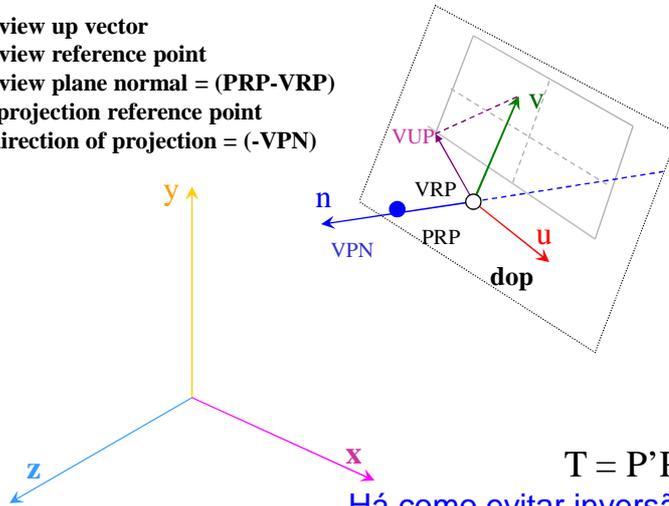


$$BP = (TB)P = B(TP)$$

\uparrow matriz-identidade

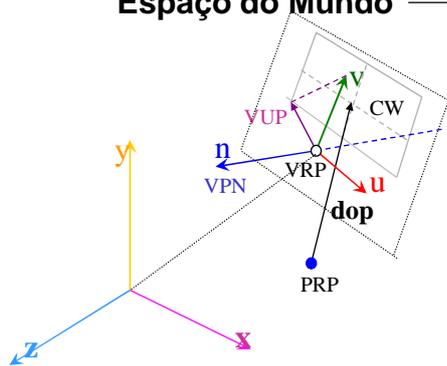
Espaço do Mundo → Espaço da Câmera

VUP: view up vector
 VRP: view reference point
 VPN: view plane normal = (PRP-VRP)
 PRP: projection reference point
 dop: direction of projection = (-VPN)



Há como evitar inversão de matriz?

Espaço do Mundo → Espaço da Câmera



(1) VRP na origem

$$D = \begin{bmatrix} 1 & 0 & 0 & -VRP_x \\ 0 & 1 & 0 & -VRP_y \\ 0 & 0 & 1 & -VRP_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(2) $(\vec{u}, \vec{v}, \vec{n})$ em vetores base ortonormais

$$\vec{n} = \frac{\vec{VPN}}{|\vec{VPN}|}$$

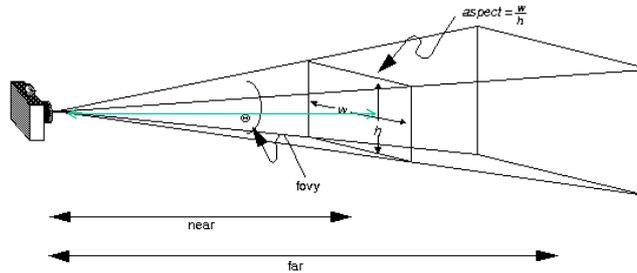
$$\vec{u} = \frac{\vec{VUP} \times \vec{n}}{|\vec{VUP} \times \vec{n}|}$$

$$\vec{v} = \vec{n} \times \vec{u}$$

$$R = \begin{bmatrix} u_x & u_y & u_z & 0 \\ v_x & v_y & v_z & 0 \\ n_x & n_y & n_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Campo de Abertura

Outro Modelo de Câmera



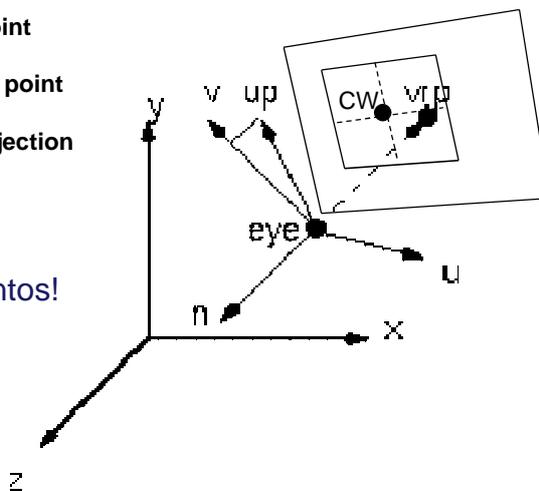
fovy (*field of view*) = campo de abertura em ângulo
 $\text{tg}(\Theta/2) = h/2 \cdot \text{near}$
 $w = \text{aspect} \cdot h$

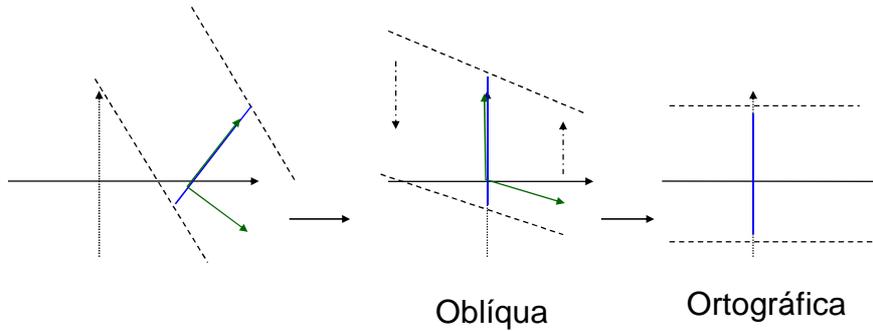
Modelo de Câmera

Plano e Centro de Projeção

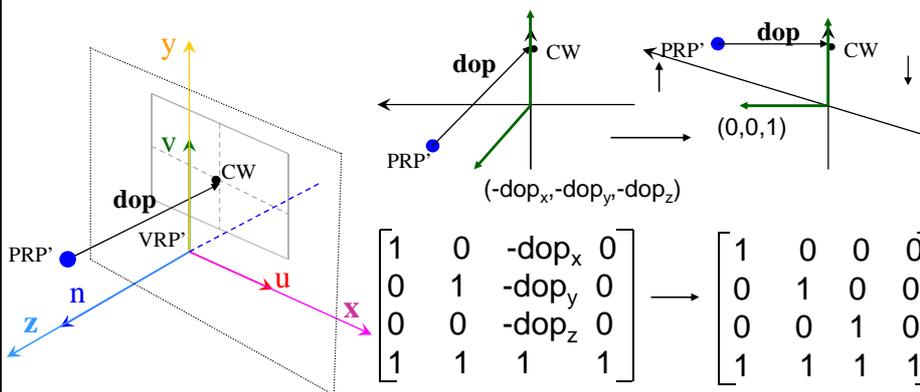
- VUP (up): view up vector
- VRP (center): view reference point
- VPN (n): view plane normal
- PRP (eye): projection reference point
- CW: centro de janela
- dop (PRP-CW): direction of projection

dop e VPN são distintos!

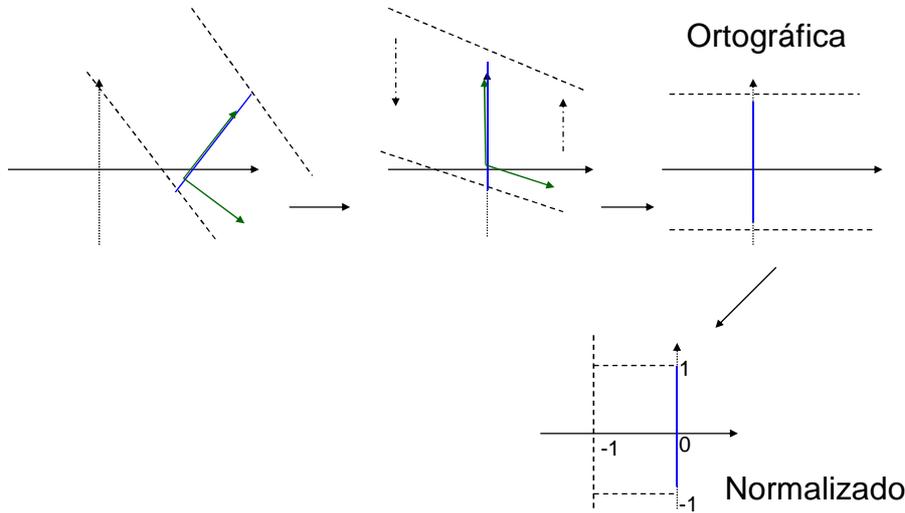




Raios Obliquos Raios Paralelos



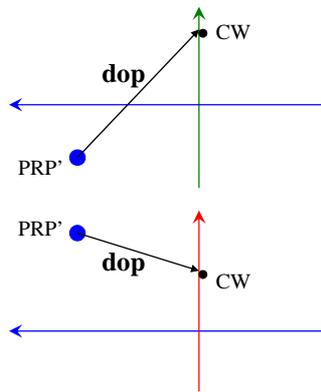
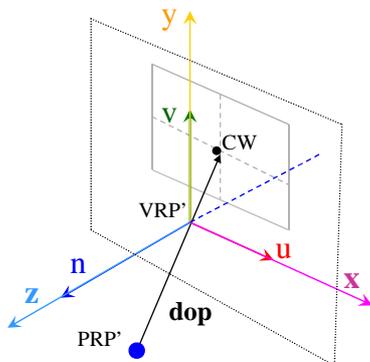
$$Sh = \begin{bmatrix} 1 & 0 & -(dop_x/dop_z) & 0 \\ 0 & 1 & -(dop_y/dop_z) & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Raios Obliquos

Raios Paralelos

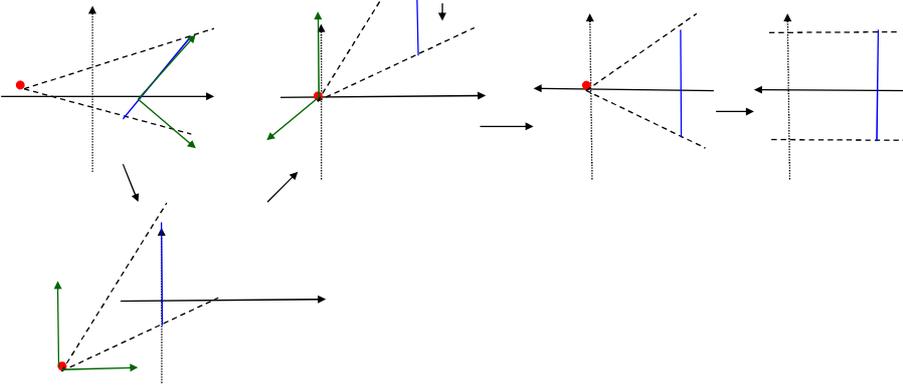
VRP': view reference point in VRC
 PRP': projection reference point in VRC
 dop: direction of projection
 CW: center of window



$$\vec{dop} = CW - PRP'$$

Mundo

Câmera



Raios Obliquos

Raios Paralelos

