Detection of Generic Conic Form Parameters Using Hough Transform

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Abstract

Hough Transform (HT) is a method for shape extraction that uses a parameter accumulator array. Based on the analysis of generic conic equation it is possible to establish a robust approach for conic shape identification in images if some aspects are respected. This paper introduces a unique methodology to detect any conic equation parameters using the HT idea. The basis of the formulation here presented is the use of polar coordinates to detect an open or closed form, and the parameters search sequence. In this way we can identify complex forms through the union of several conic detections.

1. Introduction

Detecting equations that describe objects in images is very useful in many applications [1]. Hough transform is a method for shape extraction that uses a parameter accumulator array to identify shapes in images.

Firstly, this method transforms an image edge from the spatial representation (x,y) to parametrical representation (ρ,β, x_o, y_o, ...). All points in the same curve are mapped as a specific cell in the parameter space. The biggest accumulator array value defines the detected curve. This method was proposed for line detection, and then improved to circles and ellipses detection [2] [3] [4]. It was also an issue to optimization [5]. The generalized HT considers an arbitrary set of points. However, in such generalization the main objective is just pattern recognition and the search of a curve equation is not the focus [6]. This paper extends the initial study of lines, circle or ellipses form detection to a generic detection valid for any conic equations, so this algorithm is able to detect both open and closed equation parameters.

2. Generic Conic Detection

The developed integrated approach is based on the use of polar coordinates to describe all curve equations and parameters; the search order of these parameters; and its respective level of approximation and range. It was observed that the search sequence is fundamental to the success of this unified formulation. It is better to start the search for the polar coordinates (polar radius, polar angle) of each image pixel, using given values to the angle intervals. Another simple but relevant consideration is that the definition of parameters range limits and number of bins is more important than the number of parameters itself. The accuracy in a conic detection depends on the chosen cell size. The smaller cell size is, the more accurate and slower the algorithm is.

Considering common aspects of all conics to establish an unified approach, the parameters of a curve are represented as a set P= (p_1, p_2, p_3, ..). Each element of P should have a value in an adequate range defined by L= (l_1, l_2, l_3, ...) and U= (u_1, u_2, u_3, ...), where L is the lower limit and U is the upper limit, according to the image domain. The accumulator matrix has a dimension given by:

\[ D = \begin{cases} 
\text{length}(P) + 1 & \text{if the conic has rotation} \\
\text{length}(P) & \text{otherwise} 
\end{cases} \]

The polar equation of each element in P is computed. If there is center or focus, these are calculated as the following:

\[
x_a = x - \rho \cos \beta \cos \omega + \rho \sin \beta \sin \omega \\
y_a = y - \rho \cos \beta \sin \omega - \rho \sin \beta \cos \omega
\]

where \( \rho \) is the polar form of the curve point, (x,y) is an image point, \( \beta \) is the slope between the curve center or focus and the curve point, and \( \omega \) is the curve rotation angle. Afterwards, the cell that corresponds to the parameters is incremented by 1, respecting the limits L and U. This algorithm allows the detection of
conics shapes in very noisy images. It is also possible to detect approximated conic forms from drawing. When all parameters of a curve are detected, it is easier to compute their geometric features like slopes, curvatures, focuses and centers, which can be useful in pattern recognition or shape analysis [1].

For closed curves it is not necessary to identify the initial or final points, the accumulator array it is enough, but considering open conic forms, it is necessary to create a limit point identification mechanism.

This work proposes a simple comparison between the pixels in the original image and in the detected image to elect the border points. However, if such pixels do not match, the Euclidian distance with a tolerance is used. In case of lacking points along the curve this distance is also considered. Moreover, if the matching does not occur, the final point is set as the last pixel matched successfully with the original image. When a conic form is closed, both initial and final points receive as default the value zero. Arcs of circles or ellipses can be determined by the same mechanism. Another advantage is the possibility of detecting more than one conic in the same image. The basic idea is to detect a conic and erase the corresponding pixels in the edge image in such a way that a next conic can be detected in the same image [2].

3. Experimental Results and Conclusion

In this section, the efficiency of this generic detection is shown. Figure 1 shows the original image, its edge detected by Canny method and the 4 main circles detected by unified approach commented on section 2. It shows the possibility of detecting conic forms in images with incomplete or lacking information. The detected radius are: 136, 35, 16, 16 pixels and center (142,131); (152, 107), (196, 66) and (223,148).

In this paper, an approach for the generic detection of conic forms based on the Hough transform was presented. Several tests illustrated the efficiency of the method with any conic form or any combination of conic shapes, as found in [2]. The conic form can be detected independently of the shapes quantity in the image; with any rotation; with close or open curves; with noisy information; for circle or ellipses arcs; and even for hand drawings, with the coarse manner that the form is drawn. Finally several different conics can be detected in the same image. The features that define the curve equation can also be computed. The union of all conic detection on a unique approach is important because permits the detection of any kind of conic in the same image.

4. References


Figure 1 – Original image and the 4 main circle detected (in red)