Fuzzy Granular Evolving Modeling for Trading Strategies with Exchange Rates

Leandro Maciel, Fernando Gomide (Orientador)

Department of Computer Engineering and Industrial Automation (DCA)
School of Electrical and Computer Engineering (FEEC)
University of Campinas (Unicamp)
Av. Albert Einstein, 400 Campinas, SP, Brazil CEP: 13083-852

Abstract – This work addresses a fuzzy set based evolving modeling (FBeM) approach and the task of trading strategies performance. FBeM is a granular computing technique that uses fuzzy information granules to model nonstationary functions providing functional and linguistic approximations. As an application, we consider the BRL/USD exchange rate for the period from January 2000 to October 2012. Comparisons based on trading performance indicators includes the granular model against a Multi-Layer Perceptron (MLP), an autoregressive moving average (ARMA), a naïve strategy and some state-of-the-art evolving fuzzy systems. Computational results suggest that the FBeM model outperforms the alternative approaches.

Keywords – Granular Computing, Evolving Systems, Exchange Rates, Trading, Forecasting.

1. Introduction

Forecasting exchange rate trend is a challenging task due to its high nonlinear, time-varying and noisy environment. Several factors such as economic activity, trader’s expectations, political events, and inflation influence the dynamic of currency markets. Therefore, new tools and techniques are needed in dealing with exchange rate prediction, since the traditional econometric approach based on autoregressive moving average (ARMA) models has been criticized by their limitations to reply the dynamics inherent to exchange rate time series [15, 14].

On the other hand, the main drawback with ANNs, as alternative models, is their black-box nature, revealing difficulty in interpreting the results by not providing an insight into the dynamic of the interactions between the technical indicators and the currency market fluctuations.

To overcome these limitations, the use of fuzzy-based models provides both predictive accuracy and interpretation to deal with complex real-world problems. Unlike traditional sets, fuzzy sets allow for the concept of partial membership. This enables discrimination between elements that are relevant to the phenomenon of interest and those of borderline importance that involve imprecision and uncertainty. Moreover, information granules can be processed using fuzzy logic whereby each linguistic term describes a fuzzy set, providing an interpretable system [6]. Recent studies have been revealing the high potential of fuzzy models to deal with financial time series, including exchange rate forecasting [7, 10, 13]. In general, they considered evolving fuzzy systems, constructed by fuzzy rules in a form of “If-Then” statements, comprising an adaptive structure based on streaming data [3, 4, 12].

In this paper, we investigate the trading performance of fuzzy granular evolving modeling by benchmarking their trading results over the Brazilian BRL/USD daily exchange rate with a MLP, an ARMA model, a naïve trading strategy and some state-of-the-art evolving fuzzy modeling approaches.

2. Fuzzy Set Based Evolving Modeling

The fuzzy set based evolving modeling (FBeM) produces higher level information granules in an evolving modeling structure based on streaming data and recursive learning algorithm. A FBeM model is comprised by a set of If-Then fuzzy rules extracted from the data, managing information granules and gradually evolved over time. Each granule correspond to a rule. Rules $R^t$ governing information granules $\gamma^t$ are of the type:

$$\text{IF} \ (x_1 \text{ is } A^1_1) \ \text{AND} \ \ldots \ \text{AND} \ (x_j \text{ is } A^1_j) \ \text{AND} \ \ldots \ \text{AND} \ (x_n \text{ is } A^1_n) \ \text{THEN} \ (y \text{ is } B^1) \ \text{AND} \ y = p^t(x_j \forall j)$$

where $x_j$ and $y$ are variables of the data stream $(x, y)^{[t]}$, $j = 1, 2, \ldots, n$, $t = 1, \ldots$; $A^1_j$ and $B^1_k$ are membership functions; $p^t$ are approximation poly-
nominals. One must note that the consequent part
\( (y = B^i) \) comprises the output linguistic part, pro-
viding interpretability of the results, and the term
\( y = p^i(x, \forall j) \) is the functional output, offering pre-
cision\(^1\). The collection of rules \( R^i, i = 1, 2, \ldots, c \),
forms the rule base. FBeM takes advantage of both,
linguistic and functional systems, within a single
modeling framework \[8\].

In this paper, scattering-type mechanism
was used for granulation of data into fuzzy objects\(^2\).
Specifically, FBeM uses Gaussian fuzzy subsets
\( A_j^i = G(\mu_j^i, \sigma_j^i) \), where \( \mu_j^i \) is the modal value and \( \sigma_j^i \)
the spread. The Gaussian representation are useful
since their necessary parameters are set straightforward
from a data stream, the infinite support does not ignore the data and comprises a smoothness and
continuously differentiable surface.

Rule’s consequent combines functional and
linguistic fuzzy information. The functional part
of the consequent, \( p^i \), concerns singular local func-
tions whereas the linguistic part describes information
granules \( B^i \) along the domain of output vari-
able. This paper assumes affine local functions for
functional part of the consequent:

\[
p^i = a^i_0 + \sum_{j=1}^{n} a_j^i x_i \quad (1)
\]

where \( a^i_0 \) and \( a_j^i \) are the corresponding coefficients.

Since Gaussian representation allows all
granules to overlap, each rule in FBeM contributes
to the system output. The model singular output is
obtained as the weighted mean value:

\[
p = \frac{\sum_{i=1}^{c} \min(A_1^i, \ldots, A_n^i) p^i}{\sum_{i=1}^{c} \min(A_1^i, \ldots, A_n^i)} \quad (2)
\]

Consequents of rules, \( B^i \), also assume
Gaussian fuzzy subsets \( B^i = G(\mu_j^i, \sigma_j^i) \) to assem-
ble granular objects in the output space.

There are two main sub-tasks related to
FBeM identification: learning rules antecedents and

\(^1\)This paper considers the FBeM model on its single output
form, the multivariate structure extension is straightforward.
See \[9\].

\(^2\)Granulation of data into fuzzy objects can be based on
grid, tree or scatter partitioning.

consequents. These sub-tasks are described as fol-
lows.

2.1. Learning Antecedents
FBeM learns online from a stream of instances
\((x, y)^{[t]}\), where \( y^{[t]} \) is known given \( x^{[t]} \) or will be
known at some latter step. The recursive algorithm
associated decide when and how to proceed struc-
tural and parametric adaptation of models. The
learning procedure to evolve fuzzy granular systems
FBeM decides on how to accommodate new infor-
mation given an instance \((x, y)^{[t]}\). When a new
instance does not fit current knowledge, the model
creates a new information granule and a rule gov-
erning the granule. Otherwise, if a new instance fits
current knowledge, the procedure adapts existing
granules and rules. Furthermore, the quotient
structure of fuzzy rules may be optimized, coarsed
or refined, according to inter-granules relationships.

FBeM model can start from scratch. Rules
are constructed and evolve when data are in-
put. When a new granule \( \gamma^{c+1} \) is created, a rule
\( R^{c+1} \) is added to the current rule base \( R = \{ R^1, \ldots, R^c \} \). If a instance \( x^{[t]} \) does not
activate the current collection of rules, a new gran-
ule is created. It is assumed that \( x^{[t]} \) brings new
information about the system.

As a new instance \( x^{[t]} \) arrives, a new granule
is created if the following condition holds:

\[
\min(A_1^i, \ldots, A_n^i) \leq \rho \forall i, \quad (3)
\]

where \( \rho \in [0, 1] \) is a threshold value determining
the granularity of FBeM models. If \( \rho \) is set to zero,
the system is structurally stable and unable to cap-
ture eventual concept shift. On the other hand, if \( \rho \)
equals 1, FBeM creates a rule for each new instance,
which is not practical \[8\].

Since a new granule \( \gamma^{c+1} \) is created, mem-
bership functions \( A_j^{c+1} \) and \( B^{c+1} \) are initiated as
follows:

\[
\mu_j^{c+1} = x_j^{[t]} \\
\mu_y^{c+1} = y^{[t]} \\
\sigma_j^{c+1} = \sigma_y^{c+1} = \frac{1}{2\pi} \quad (4)
\]
The coefficients of local-valued polynomials \( p^{c+1} \) are set to:

\[
a^{c+1}_0 = y^{[t]}, \quad a^{c+1}_j = 0, \quad j \neq 0
\]

(5)

As stated by [9], this initial parametrization gives preference to design of granules balanced along all dimensions rather than granules with unbalanced geometry, following the principle of balanced information granularity [5].

If a new instance \( x^{[t]} \) does not satisfy the condition in (3), the rule base need adaptation, i.e.: i) expand or contract \( A^i_j \) and \( B^i \) to accommodate new data; ii) move granules \( \gamma^i \) toward denser regions of data over the input and output domains; iii) adjust coefficients of local approximation function \( p^i \).

A rule \( R^i \) is adapted if a data instance \( x^{[t]} \) holds:

\[
\min(A^i_1, \ldots, A^i_n) > \rho
\]

(6)

The most active rule for \( x^{[t]} \) is then chosen for adaptation by updating the model value and the spread of membership functions \( A^i_j \) recursively:

\[
\mu^i_j(\text{new}) = \frac{(\omega^i - 1)\mu^i_j(\text{old}) + x_j}{\omega^i}
\]

(7)

\[
\sigma^i_j(\text{new}) = \frac{(\omega^i - 1)}{\omega^i} \sigma^i_j(\text{old}) + \frac{1}{(\omega^i - 1)}(x_j - \mu^i_j(\text{new}))^2
\]

(8)

where \( \omega^i \) is the number of times that the granule \( \gamma^i \) has been activated by the data stream.

Adaptation of fuzzy sets of rule consequents \( B^i \) uses output data \( y^{[t]} \). Moreover, polynomial coefficients \( a^0_i \) and \( a^1_j \) are updated according to recursive least squares (RLS).

Let \( E \) be the squared error between predictions \( p(x^{[t]}) \) and actual value \( y^{[t]} \), then

\[
E = (y^{[t]} - p(x^{[t]}))^2
\]

(9)

Hence, \( \rho \) learning values for itself from

\[
\rho(\text{new}) = \rho(\text{old}) + \eta(E_D - E)
\]

(10)

where \( \eta \) is a learning rate and \( E_D \) the desired prediction error. We also assume \( \rho^{[0]} = 0.5 \) as default initial value.

Once the structure of the FBeM is defined and established, the problem of parameter identification relies on the use of RLS algorithm.

### 3. Results

In this paper, we considered the daily closing BRL/USD (Real/Dollar) exchange rate from 3 January 2000 through 26 October 2012. The data period is partitioned into sub-periods namely training and validation data sets. The training data set covers the period from 3 January 2000 to 31 December 2009. The remaining data comprises the validation set (out-of-sample). The FBeM model was compared with MLP, ARMA models, a naïve strategy and some state-of-the-art evolving fuzzy models such as eTS [2], xTS [3], ePL [11] and eTS+ [1].

Model’s trading performance were based on annualized returns \( (R^A) \), cumulative returns \( (R^C) \), annualized volatility \( (\sigma^A) \) and maximum drawdown \( (MD) \), described as follows:

\[
R^A = 252 \cdot \frac{1}{T} \sum_{t=1}^{T} y_t
\]

(11)

\[
R^C = \sum_{t=1}^{T} y_t
\]

(12)

\[
\sigma^A = \sqrt{252 \cdot \sqrt{T - 1 \sum_{t=1}^{T} (y_t - \bar{y})^2}}
\]

(13)

\[
\text{MD} = \min_{i=1,\ldots,T; j=1,\ldots,T} \left( \sum_{j=i}^{T} y_j \right)
\]

(14)

where \( \bar{y}_t \) is the mean value of returns.

In order to evaluate the results in a real-world application, trading strategies were applied
for all forecasting methods. The trading strategy is to go or stay “long” when the forecasting return is above zero and go or stay “short” when the forecast return is below zero. The “long” and “short” BRL/USD positions are defined as buying and selling Brazilian Reals at the current price respectively. Table 1 reports the out-of-sample trading performance of all models. The FBeM model performs better than all other models in terms of trading performance. It presents a slightly higher annualized ($R^A$) and cumulative ($R^C$) returns than the other models. On the other hand, naïve and ARMA strategies provide lower results. The annualized volatility ($\sigma^A$) and maximum drawdown (MD) were similar for all models.

<table>
<thead>
<tr>
<th>Models</th>
<th>$R^A$</th>
<th>$R^C$</th>
<th>$\sigma^A$</th>
<th>MD</th>
</tr>
</thead>
<tbody>
<tr>
<td>naïve</td>
<td>12.86%</td>
<td>32.14%</td>
<td>12.54%</td>
<td>-5.32%</td>
</tr>
<tr>
<td>ARMA</td>
<td>25.34%</td>
<td>55.87%</td>
<td>11.13%</td>
<td>-4.57%</td>
</tr>
<tr>
<td>MLP</td>
<td>44.50%</td>
<td>112.84%</td>
<td>10.94%</td>
<td>-4.13%</td>
</tr>
<tr>
<td>eTS</td>
<td>60.85%</td>
<td>130.69%</td>
<td>10.50%</td>
<td>-4.23%</td>
</tr>
<tr>
<td>xTS</td>
<td>66.47%</td>
<td>144.95%</td>
<td>9.25%</td>
<td>-4.12%</td>
</tr>
<tr>
<td>ePL</td>
<td>63.12%</td>
<td>127.84%</td>
<td>9.66%</td>
<td>-4.96%</td>
</tr>
<tr>
<td>eTS+</td>
<td>86.09%</td>
<td>163.04%</td>
<td>10.01%</td>
<td>-3.72%</td>
</tr>
<tr>
<td>FBeM</td>
<td>94.74%</td>
<td>171.41%</td>
<td>9.43%</td>
<td>-3.57%</td>
</tr>
</tbody>
</table>

Table 1. Summary of trading performance results.

4. Conclusion

This paper applies a fuzzy set based evolving model (FBeM) to perform trading strategies of the Brazilian Real/Dollar (BRL/USD) exchange rate. Results have shown the effectiveness of the FBeM approach by its best trading measures in terms of annualized and cumulative returns than traditional approaches.

References