Optimal selection of free parameters in expansions of Volterra models using Kautz functions

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Abstract – A new solution for the problem of selecting poles of the two-parameter Kautz functions in Volterra models of any order is proposed. The usual large number of parameters required to represent the Volterra kernels can be reduced by describing each kernel using a basis of orthonormal functions, such as the Kautz basis. The resulting model can be truncated into fewer terms if the Kautz functions are properly designed. The underlying problem is how to select the arbitrary complex poles that fully parameterize these functions. This problem is approached by minimizing an upper bound for the error resulting from the truncation of the kernel expansion when each multidimensional kernel is decomposed into a set of independent Kautz bases, each of which is parameterized by an individual pair of conjugate Kautz poles intended to represent the dominant dynamic of the kernel along a particular dimension. An analytical solution for one of the Kautz parameters is derived.

Keywords – Nonlinear systems, Volterra series, Kautz functions, Optimization, System identification.

1. Introduction

In the past few decades, there has been a growing interest in the use of orthonormal basis functions (OBF) in studies involving the identification and control of dynamic processes [6, 1, 5]. One important issue regarding the use of an orthonormal basis model structure is the incorporation of approximate knowledge about the dynamics of the system into the identification process. This allows a significant reduction in the number of model parameters to be estimated.

Laguerre and Kautz bases [1, 5] are the orthonormal bases functions most commonly used in the modeling and approximation of systems and signals. The Laguerre functions are preferable for representing well-damped systems, whereas the two-parameter Kautz functions provide better approximations for systems with oscillatory behavior. If the basis poles are set close to the dominant modes of the system, a good approximation can be obtained with fewer coefficients. Optimizations that lead to analytical solutions have been investigated for the Laguerre basis [4, 2] and for the Kautz basis [7, 3].

In this work, an optimal selection for one of the parameters related to the Kautz pole is obtained when the Volterra kernels are decomposed into a set of independent orthonormal bases, each of which parameterized by an individual pair of conjugate Kautz poles associated with the dominant dynamic of the kernel along a particular dimension. Using independent bases for each kernel dimension is expected to reduce the truncation error when the dominant dynamics along the multiple dimensions are different from one another.

2. Preliminaries

A Volterra model is an input-output functional expansion of a nonlinear system whose structure is a straightforward generalization of the unit-impulse response model. In the discrete-time domain, the mathematical description of a Volterra model relates the output $y(k)$ of a physical process to its input $u(k)$ as [6]:

$$y(k) = \sum_{\tau_1=0}^{k} h_1(\tau_1)u(k - \tau_1) + \sum_{\tau_1=0, \tau_2=0}^{k} h_2(\tau_1, \tau_2)u(k - \tau_1)u(k - \tau_2) + \cdots \tag{1}$$

where $h_1(\tau_1), h_2(\tau_2, \tau_2), \ldots$ are the Volterra kernels. The drawback of requiring a large number of parameters in the representation of these models can be circumvented by describing every kernel using an orthonormal basis of functions $\{\psi_{l,n}\}$, as follows:

$$h_{\eta}(k_1, \ldots, k_\eta) = \sum_{i_1=1}^{\infty} \cdots \sum_{i_\eta=1}^{\infty} \alpha_{i_1, \ldots, i_\eta} \prod_{l=1}^{\eta} \psi_{l,i_l}(k_l) \tag{2}$$

Discrete-time generalized orthonormal basis functions (GOBF) are defined in the $z$-domain as [5]:

$$\Psi_{l,n}(z) = \frac{z\sqrt{1 - |\beta_{l,n}|^2}}{z - \beta_{l,n}} \prod_{j=1}^{n-1} \frac{1 - \beta_{k,j}z}{z - \beta_{k,j}} \tag{3}$$
where $\beta_{l,n}, \bar{\beta}_{l,n} \in \mathbb{C}$ are the poles of the GOBF. The particular case of GOBF in which the set of poles \{\beta_{l,n}\} in (3) is \{\beta_l, \bar{\beta}_l, \beta_1, \bar{\beta}_1, \ldots\} results in the so-called two-parameter Kautz functions [7, 5]. These functions are usually defined in terms of the real-valued parameters $b_l$ and $c_l$, which are related to the pair of conjugate Kautz poles ($\beta_l, \bar{\beta}_l$) as $b_l = (\beta_l + \bar{\beta}_l)/(1 + \beta_l \bar{\beta}_l)$ and $c_l = -\beta_l \bar{\beta}_l$.

3. Optimization of the Kautz parameters

The central problem considered here is how to select the Kautz parameters $b_l$ and $c_l$ so as to minimize the upper bound resulting from truncating the series expansion (2). The normalized quadratic error (NQE) of the approximation of kernel $h_\eta$ decomposed into $\eta$ $M$-term Kautz bases (one independent basis along each kernel dimension), is bounded by [3]:

$$\text{NQE} \leq L(c_1, \ldots, c_\eta)$$

where $L(c_1, \ldots, c_\eta)$ is a multidimensional function which depends on the $\eta$-order Volterra kernel $h_\eta(k_1, \ldots, k_\eta)$. By minimizing $L(c_1, \ldots, c_\eta)$ with respect to the parameters $c_l, \forall l = \{1, \ldots, \eta\}$, one obtains an analytical solution for an optimal selection of the parameter $c_l$ of the Kautz functions. After the setting of the value of $b_l$, this solution can be used to minimize the upper bound $L$ for the quadratic error resulting from the truncated expansion of the Volterra kernels.

4. Simulation results

Suppose that a specific system has the second-order Volterra kernel given by $h_2(k_1, k_2) = (k_1 - 2k_2) \exp(-0.45k_1 - 0.7k_2) \cos(100k_1 + k_2)$ for $k_1, k_2 \geq 0$. For instance, the values of $b_1 = 0.6$ for the first axial direction and $b_2 = 0.5$ for the second have been chosen. Computational simulations provided the optimal values $c_{\text{opt},1} = -0.2321$ and $c_{\text{opt},2} = -0.3058$. With these parameters, the error associated with the approximation of $h_2$ is shown in Table 1 for different numbers of Kautz functions. The values of the upper bound in (4) are presented as well.

<table>
<thead>
<tr>
<th>No. functions ($M$)</th>
<th>NQE</th>
<th>$L(c_1, c_2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.8110</td>
<td>0.8774</td>
</tr>
<tr>
<td>4</td>
<td>0.1036</td>
<td>0.2106</td>
</tr>
<tr>
<td>6</td>
<td>0.0077</td>
<td>0.0112</td>
</tr>
<tr>
<td>8</td>
<td>$8.1136 \times 10^{-4}$</td>
<td>0.0045</td>
</tr>
</tbody>
</table>

Table 1. Approximation errors and upper bounds for the orthonormal expansion of $h_2$.

The best approximation of $h_2$ is obtained by choosing $b_1 = 0.770$ and $b_2 = 0.345$, which results in $c_{\text{opt},1} = -0.4583$ and $c_{\text{opt},2} = -0.2473$, respectively. The corresponding Kautz poles are $\beta_1 = 0.5615 \pm 0.3783$ and $\beta_2 = 0.2152 \pm 0.4483$. The error associated with this approximation when using $M = 8$ functions is NQE = $3.6082 \times 10^{-7}$.

5. Conclusions

An optimal choice for one of the Kautz parameters in OBF Volterra models of any order has been addressed. It involves the decomposition of each multidimensional Volterra kernel using a set of independent orthonormal bases, each of which parameterized by an individual pair of conjugate Kautz. The resulting solution is based on the minimization of an upper bound for the error resulting from the truncated approximation of Volterra kernels using the two-parameter Kautz functions. Simulation results have shown that the use of an independent basis for each kernel dimension reduces the truncation error when the kernel has different dominant dynamics along its multiple dimensions.

References