IA841 – Modelagem de Sólidos

Curvas Espaciais

Farin: Capítulos 4, 5 e 8

Parábola

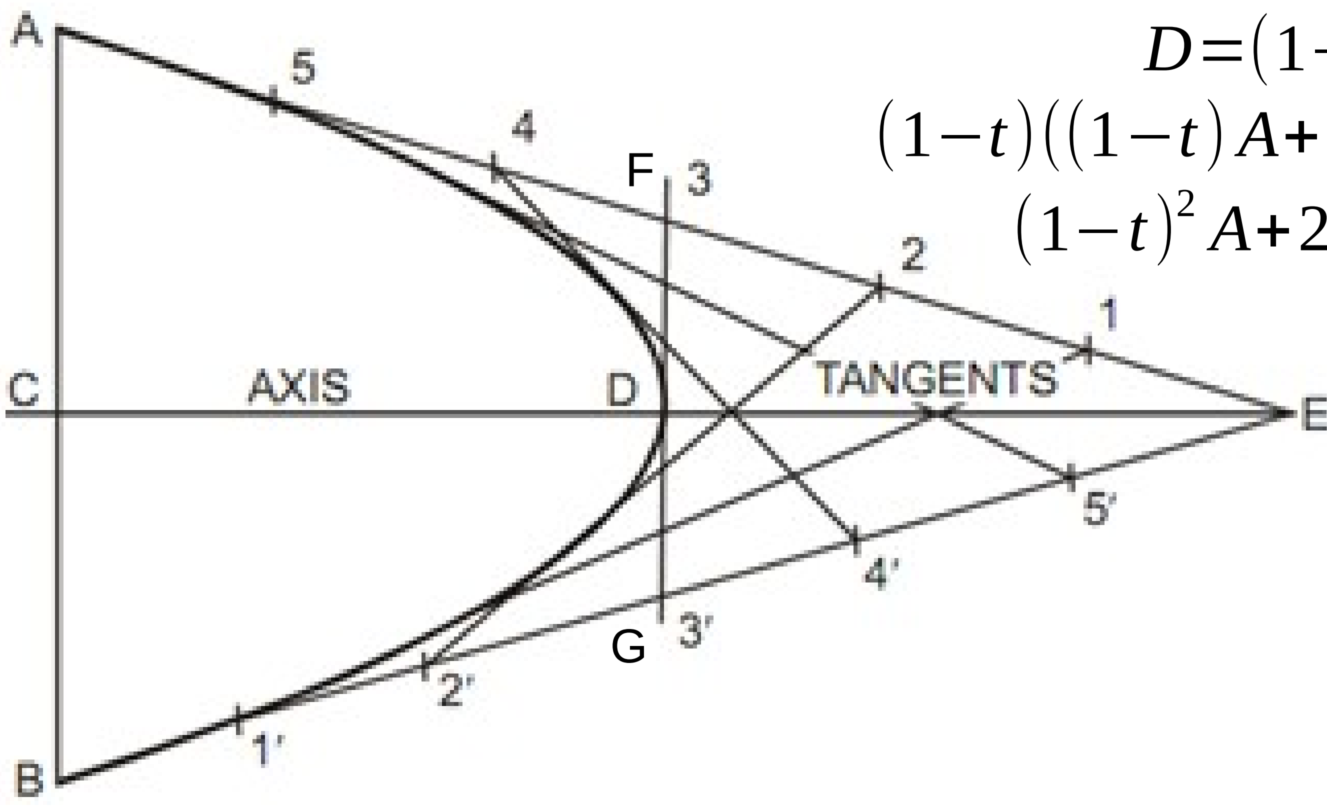
- Teorema das Três Tangentes

$$razão(A, F, E) = razão(F, D, G) = razão(E, G, B) = \frac{t}{1-t}$$

$$D = (1-t)F + tG$$

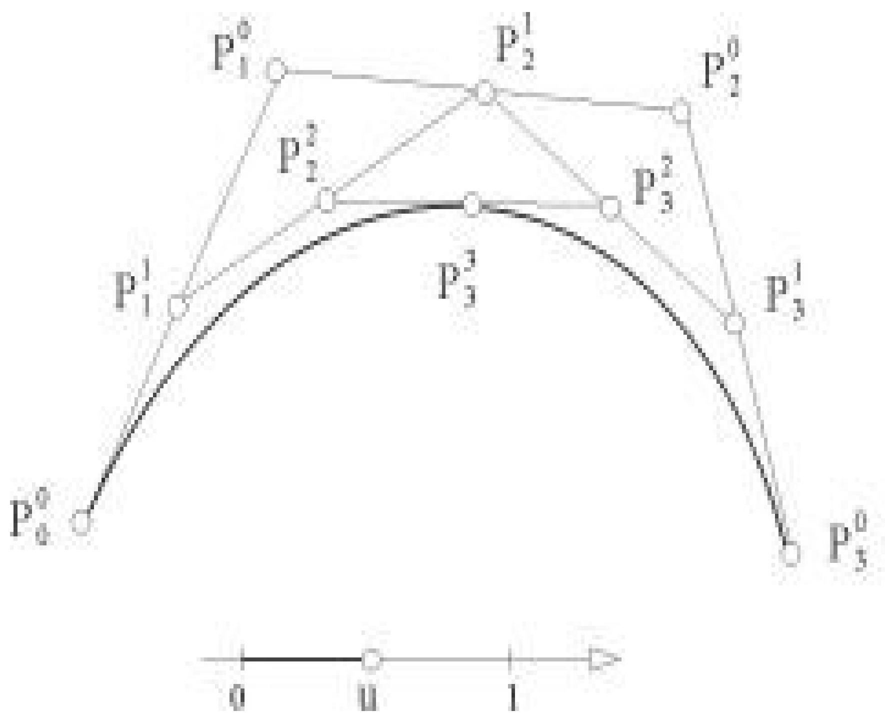
$$(1-t)((1-t)A + tE) + t((1-t)E + tB)$$

$$(1-t)^2 A + 2\mathrm{t}(1-t)E + t^2 B$$

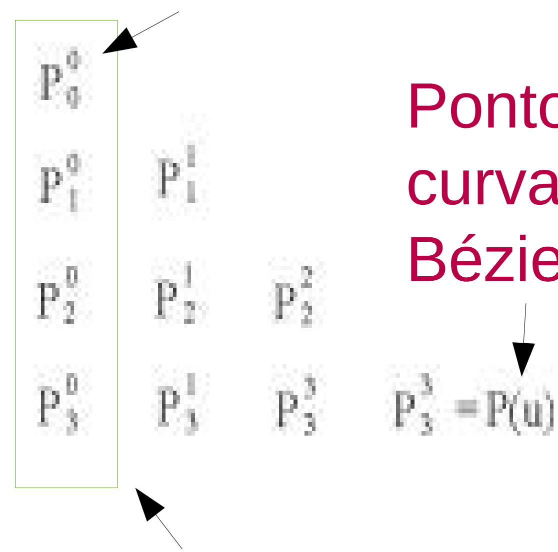


Esquema de Construção DeCasteljau

Interpolações lineares sucessivas



Pontos de Bézier ou
Pontos de controle

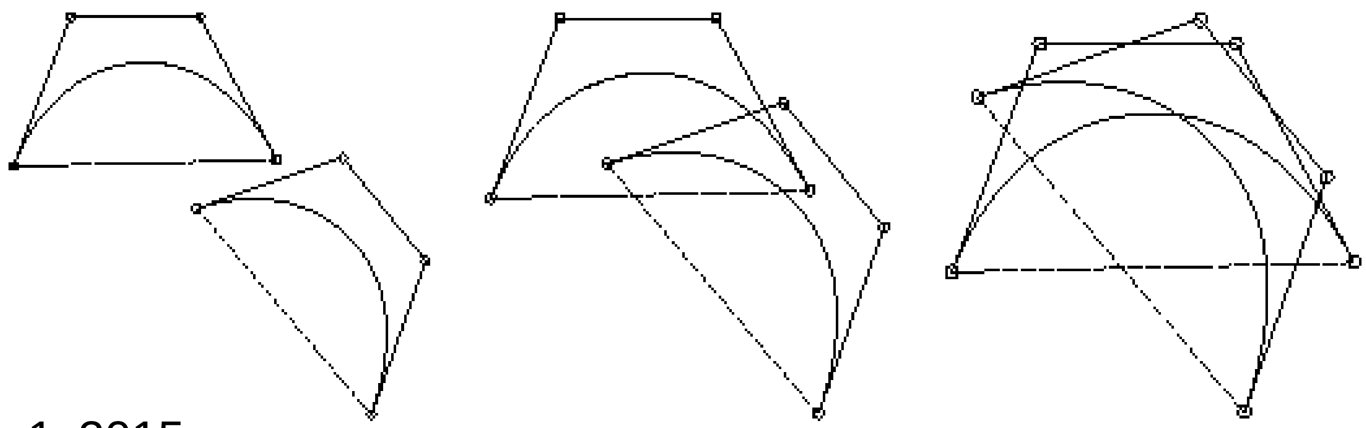


Ponto da curva de Bézier

Polígono de Bézier ou
Polígono de controle

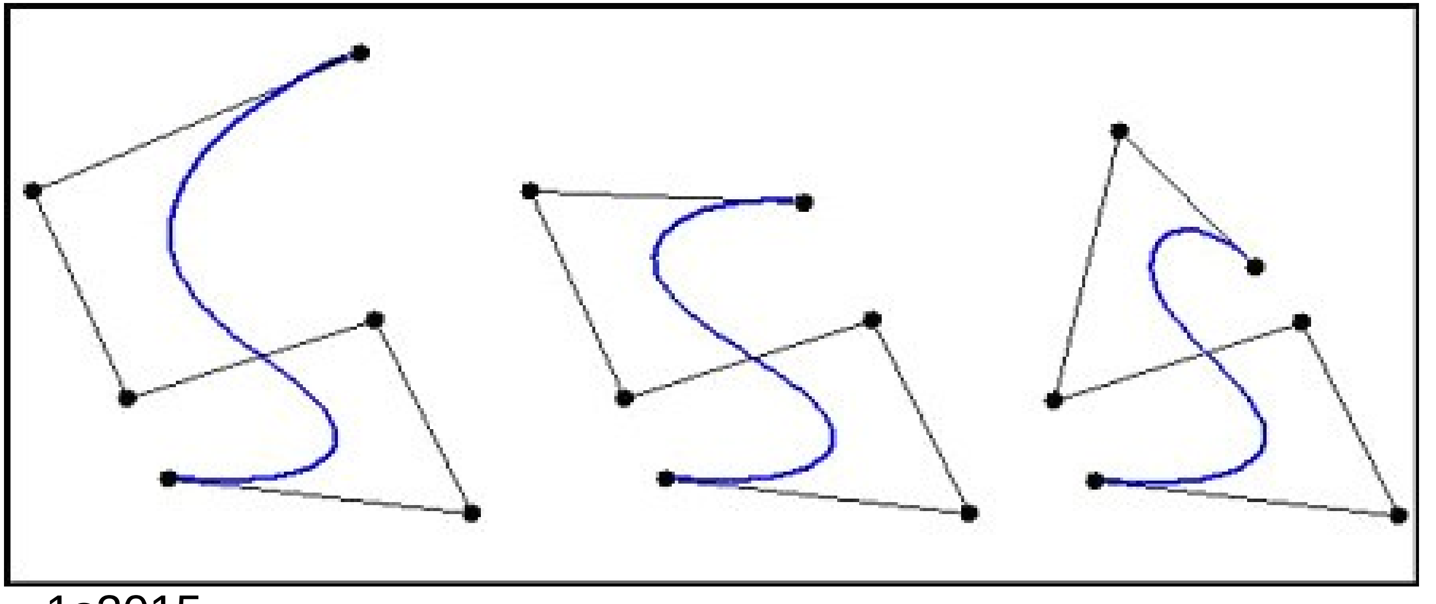
Propriedades do Algoritmo de DeCasteljau

- Curvas invariantes sob transformações afins, mas não são invariantes sob transformações projetivas!
- Curvas invariantes sob transformações afins no domínio.
- Curvas contidas no fecho convexo do seu polígono de controle.

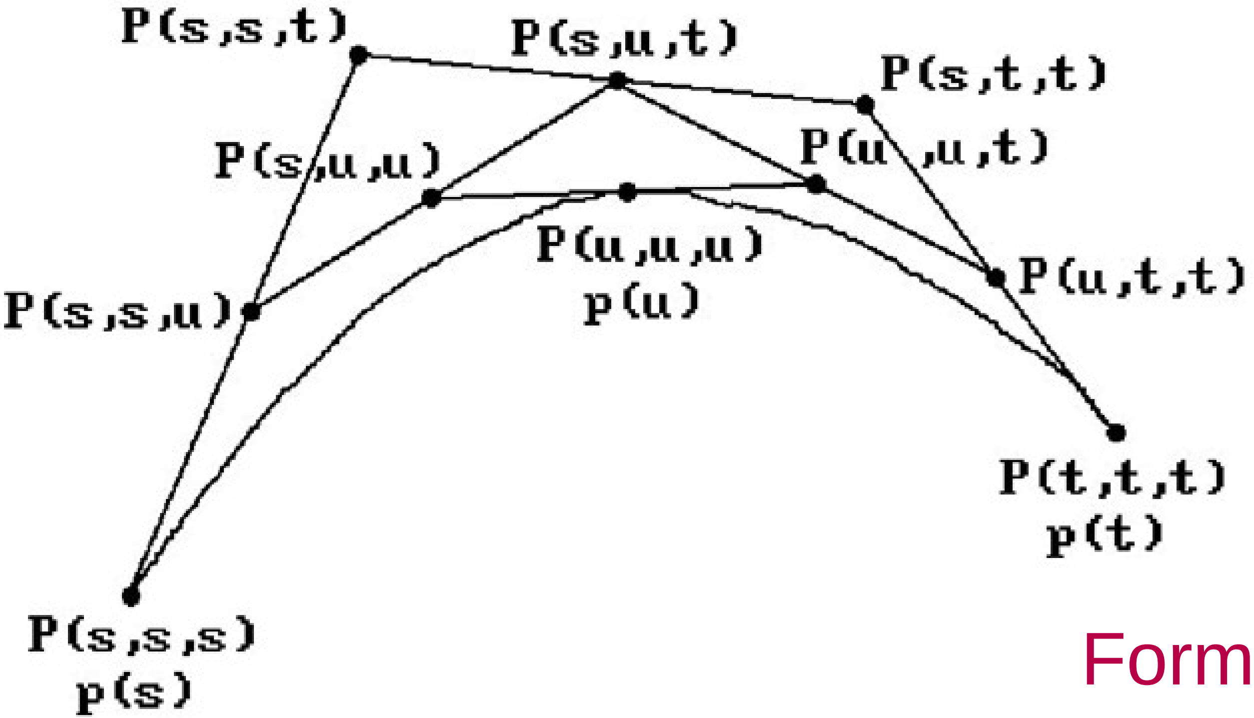


Propriedades do Algoritmo de DeCasteljau

- Número de combinaçãoe afins = (número de pontos de controle –1).
- Curvas acompanham a forma do seu polígono de controle.



Blossom



Formulação multiafim

$$P(s,s,s)=P(0,0,0)$$

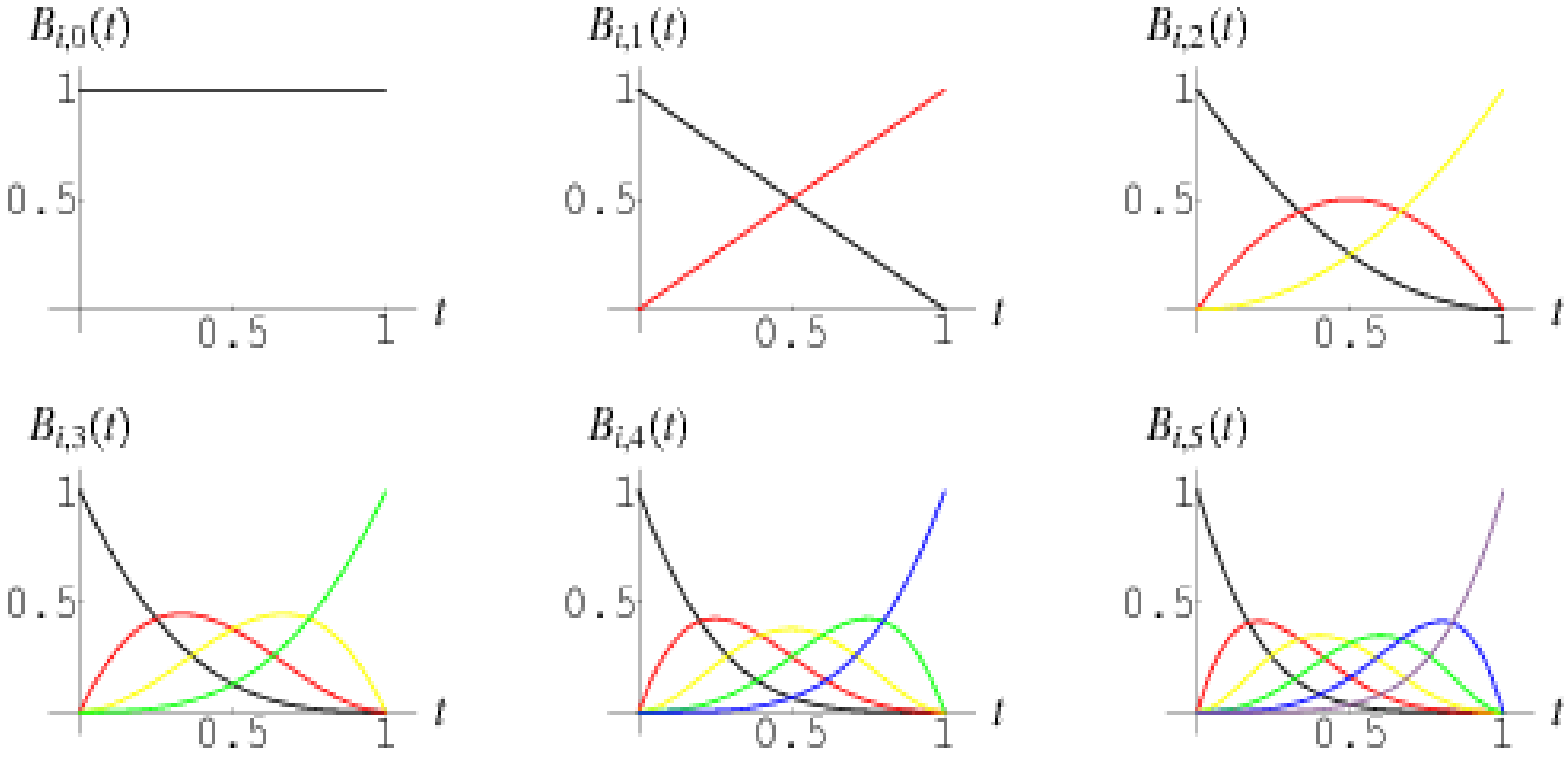
$$P(s,s,t)=P(0,0,1) \quad P(s,s,u)$$

$$P(s,t,t)=P(0,1,1) \quad P(s,u,t) \quad P(s,u,u)$$

$$P(t,t,t)=P(1,1,1) \quad P(u,t,t) \quad P(u,u,t) \quad P(u,u,u)$$

Funções de Bernstein

$$B_i^n(t) = \binom{n}{i} t^i (1-t)^{n-i} = \frac{n!}{i!(n-i)!} t^i (1-t)^{n-i}$$



Recursividade

$$B_i^n(t)=\binom{n}{i}t^i(1-t)^{n-i}=\frac{n!}{i!(n-i)!}t^i(1-t)^{n-i}$$

$$\frac{(n-i+i)(n-1)!}{i(i-1)!(n-i)(n-1-i)!}t^i(1-t)^{n-i}$$

$$\left(\frac{(n-i)(n-1)!}{i(n-i)(i-1)!(n-1-i)!}+\frac{i(n-1)!}{i(i-1)!(n-i)(n-1-i)!}\right)t^i(1-t)^{n-i}$$

$$\frac{(n-1)!}{i!(n-1-i)!}t^i(1-t)^{n-i}+\frac{(n-1)!}{(i-1)!(n-i)!}t^i(1-t)^{n-i}$$

$$\frac{(n-1)!}{i!(n-1-i)!}t^i(1-t)^{n-1-i}(1-t)+\frac{(n-1)!}{(i-1)!(n-i)!}t^{(i-1)}(1-t)^{(n-1)-(i-1)}t$$

$$B_i^n(t)=(1-t)B_i^{n-1}(t)+tB_{i-1}^{n-1}(t)$$

Algoritmo de DeCasteljau ↔ Funções de Bernstein

$$b(t)=\sum_{i=0}^{n} b_i B_i^n(t)=\sum_{i=0}^{n} b_i\left[(1-t)B_i^{n-1}(t)+tB_{i-1}^{n-1}(t)\right]$$

t=0

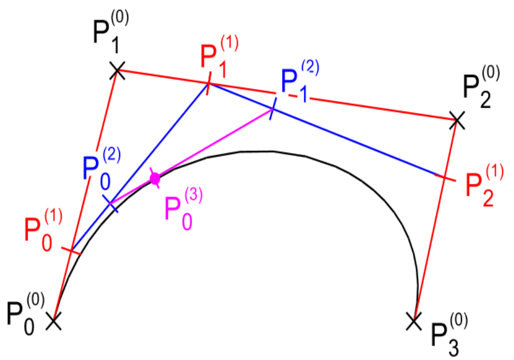
$$b_0 tB_{-1}^{n-1}(t)+\underbrace{b_0(1-t)B_0^{n-1}(t)+b_1 tB_0^{n-1}(t)}_{b_0^1}+\underbrace{b_1(1-t)B_1^{n-1}(t)+b_2 tB_1^{n-1}(t)}_{b_1^1}+\dots$$

$$\dots+\underbrace{b_{i-1}(1-t)B_{i-1}^{n-1}(t)+b_i tB_{i-1}^{n-1}(t)}_{b_{i-1}^1}+\underbrace{b_i(1-t)B_i^{n-1}(t)+b_{i+1}tB_i^{n-1}(t)}_{b_i^1}+\dots$$

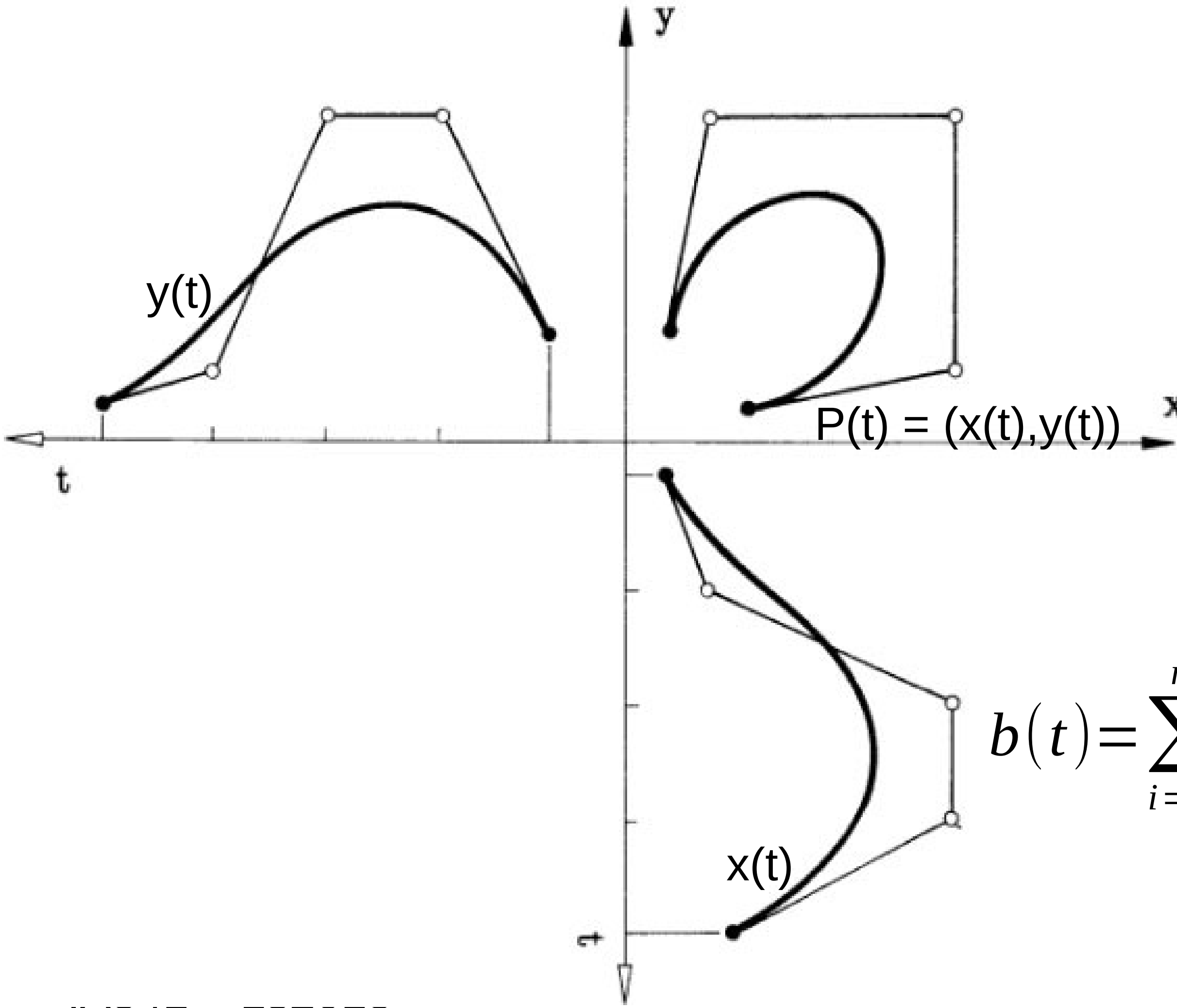
$$\dots+\underbrace{b_{n-1}(1-t)B_{n-1}^{n-1}(t)+b_n tB_{n-1}^{n-1}(t)}_{b_{n-1}^1}+b_n(1-t)B_n^{n-1}(t)$$

t=1

$$b(t)=\sum_{i=0}^{n-r} b_i^r B_i^{n-r}(t)$$



Coordenadas → Curvas

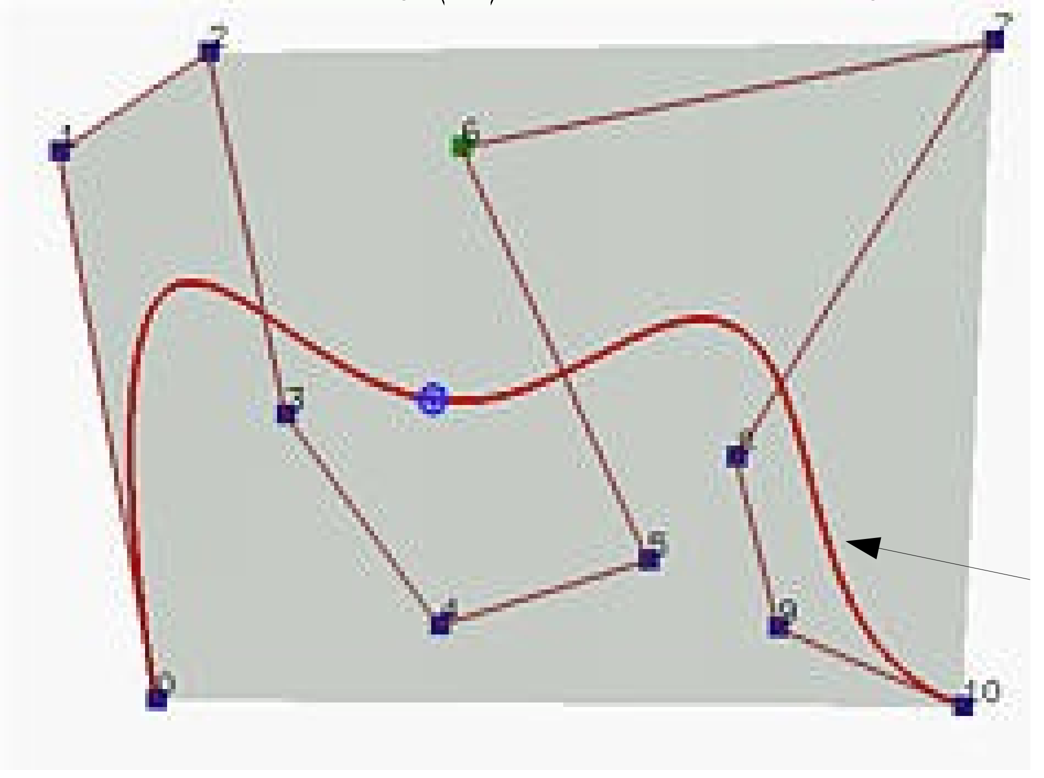


$$b(t)=\sum_{i=0}^{n} b_i B_i^n(t)=\begin{pmatrix}\sum_{i=0}^{n} x_i B_i^n(t)\\ \sum_{i=0}^{n} y_i B_i^n(t)\end{pmatrix}$$

Propriedades das Funções de Bernstein

- Curvas contidas no fecho convexo do polígono de controle.

$$1=(t+(1-t))^n=\sum_{i=0}^{n}\binom{n}{i}t^i(1-t)^{n-i}=\sum_{i=0}^{n}B_i^n(t)$$



Propriedades das Funções de Bernstein

- Curvas invariantes sob transformações afins
  - ➔ Combinações baricêntricas
- Curvas invariantes sob transformações afins dos parâmetros no domínio

$$\sum_{i=0}^{n} b_i B_i^n(t) = \sum_{i=0}^{n} b_i B_i^n\left(\frac{u-a}{b-a}\right)$$

$$B_i^n(t) = \frac{n!}{i!(n-i)!} t^i (1-t)^{n-i}$$

$$B_i^n\left(\frac{u-a}{b-a}\right) = \frac{n!}{i!(n-i)!} \left(\frac{u-a}{b-a}\right)^i \left(1-\frac{u-a}{b-a}\right)^{n-i}$$

Propriedades das Funções de Bernstein

- Curvas interpolam os pontos extremos do seu polígono de controle.

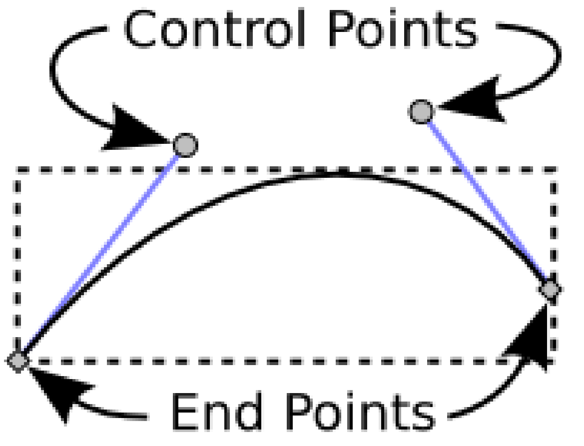
$$b(t)=\sum_{i=0}^{n} b_i B_i^n(t) \qquad \sum_{i=0}^{n} B_i^n(t)=1\,;\,B_0^n(0)=1\,;\,B_n^n(1)=1$$

- Curvas tangenciam os segmentos extremos do seu polígono de controle.

$$b'(t)=\sum_{i=0}^{n} b_i \frac{d\,B_i^n(t)}{dt}$$

$$b'(0)=b_1-b_0$$

$$b'(1)=b_n-b_{n-1}$$



Propriedades das Funções de Bernstein

- Curvas simétricas em relação ao domínio.

$$B_i^n(t) = \frac{n!}{i!(n-i)!} t^i (1-t)^{n-i}$$

$$\frac{n!}{(n-i)!(n-(n-i))!}(1-t)^{n-i}(1-(1-t))^{n-(n-i)} = B_{n-i}^n(1-t)$$

- Curvas invariantes sob combinações baricêntricas.

$$\alpha + \beta = 1$$

$$\alpha b(t) + \beta c(t) = \alpha \sum_{i=0}^{n} b_i B_i^n(t) + \beta \sum_{i=0}^{n} c_i B_i^n(t) = \sum_{i=0}^{n} (\alpha b_i + \beta c_i) B_i^n(t)$$

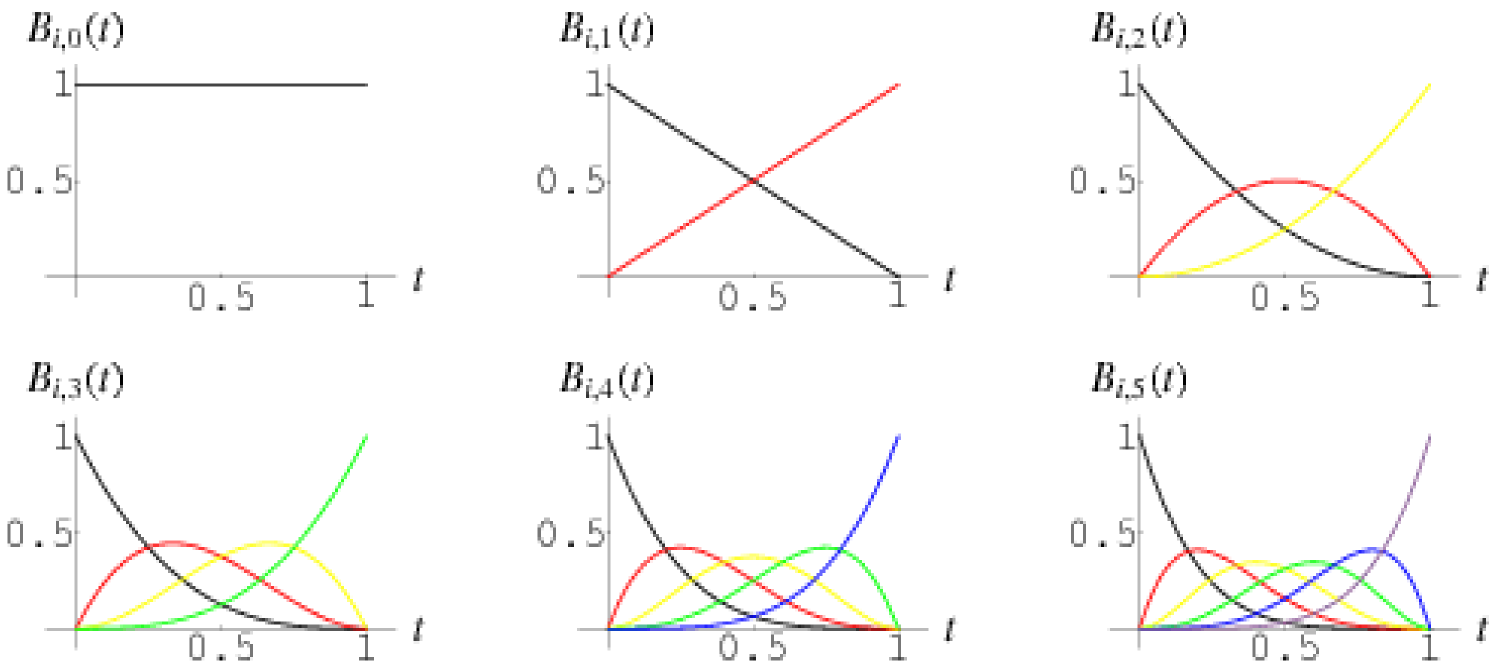
- Grau das funções = número de pontos de controle - 1

Propriedades das Funções de Bernstein

- Precisão linear: pontos de controle colineares → segmento de reta

$$b(t)=\sum_{i=0}^{n}\frac{i}{n}B_i^n(t)=\sum_{i=0}^{n}\frac{i}{n}\frac{n!}{i!(n-i)!}t^i(1-t)^{n-i}=t$$

- Previsibilidade.



Notação Matricial

- Curvas Cúbicas

$$B_0^3(t)=(1-t)^3=1-3\mathrm{t}+3\mathrm{t}^2-1$$

$$B_1^3(t)=3\mathrm{t}(1-t)^2=3\mathrm{t}(1-2\mathrm{t}+t^2)=3\mathrm{t}-6\mathrm{t}^2+3\mathrm{t}^3$$

$$B_2^3(t)=3\mathrm{t}^2(1-t)=3\mathrm{t}^2-3\mathrm{t}^3$$

$$B_3^3(t)=t^3$$

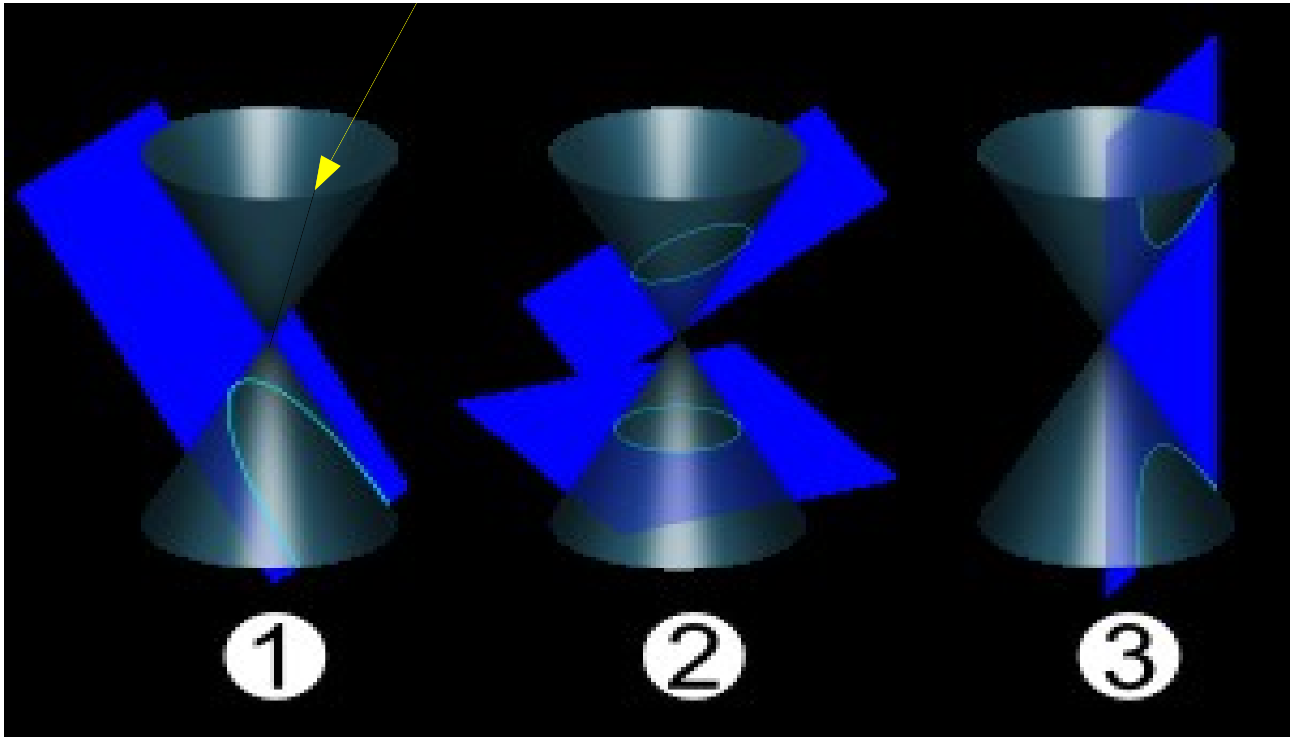
$$\begin{pmatrix} B_0^3(t) \\ B_1^3(t) \\ B_2^3(t) \\ B_3^3(t) \end{pmatrix} = \begin{pmatrix} 1 & -3 & 3 & -1 \\ 0 & 3 & -6 & 3 \\ 0 & 0 & 3 & -3 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ t \\ t^2 \\ t^3 \end{pmatrix}$$

$$b(t)=\sum_{i=0}^{n} b_i B_i^n(t)=\begin{pmatrix} b_0 & b_1 & b_2 & b_3 \end{pmatrix} \begin{pmatrix} B_0^3(t) \\ B_1^3(t) \\ B_2^3(t) \\ B_3^3(t) \end{pmatrix}$$

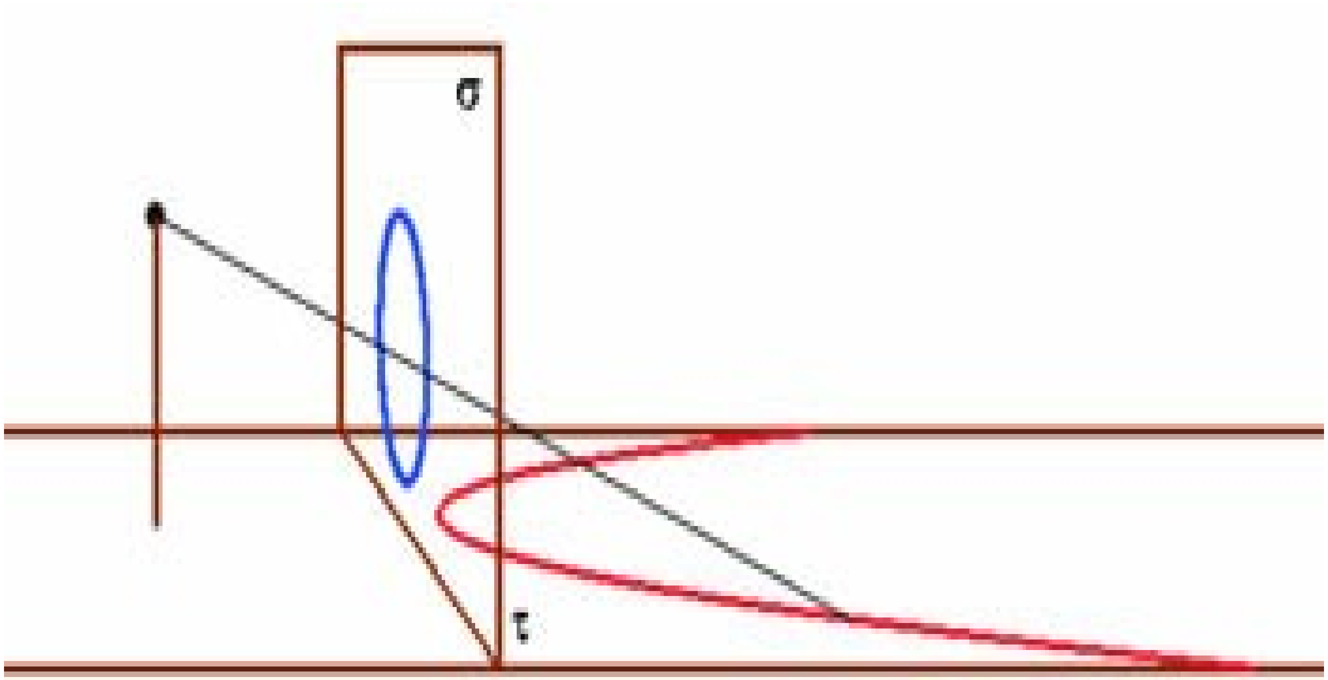
Seções Cônicas

- Interseções entre um cone e um plano
  - (1) Parábola
  - (2) Elipse
  - (3) Hipérbole

No espaço projetivo a reta corresponde a um ponto!



Projeções de uma Parábola



http://archiviomacmat.unimore.it/PAWeb/Sito/Inglese/247i.htm

Curvas de Bézier Racionais

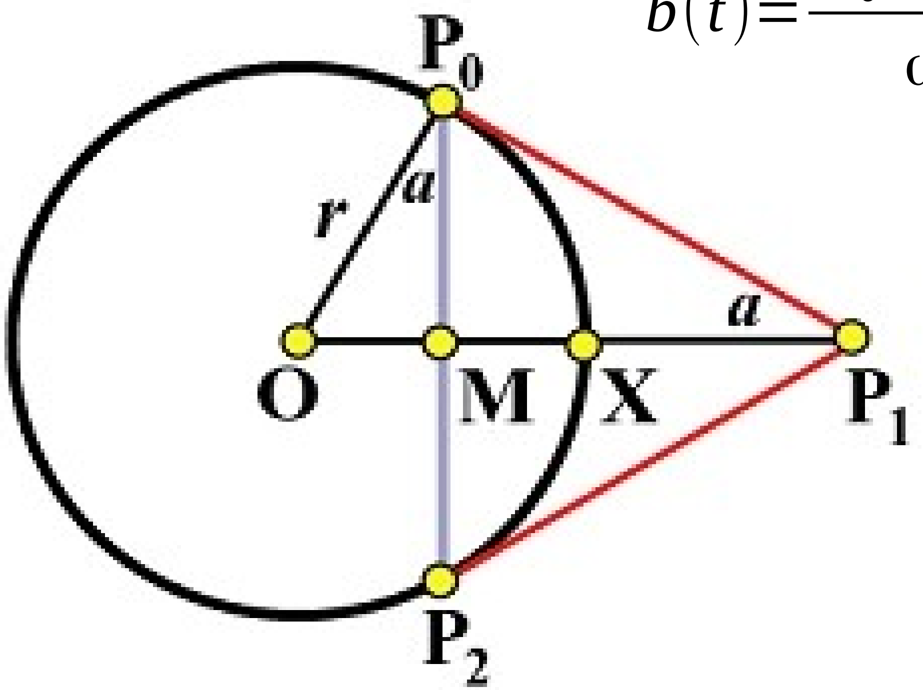
- Pontos em coordenadas homogêneas

$$\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} \Rightarrow \begin{pmatrix} x(t) \\ y(t) \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} \omega\, x(t) \\ \omega\, y(t) \\ \omega \end{pmatrix}$$

- Cônicas: projeções de curvas de Bézier quádricas em $\omega = 1$:

$$b(t) = \frac{\sum_{i=0}^{2} \omega_i b_i B_i^2(t)}{\sum_{i=0}^{2} \omega_i B_i^2(t)} = \frac{\omega_0 b_0 B_0^2(t) + \omega_1 b_1 B_1^2(t) + \omega_2 b_2 B_2^2(t)}{\omega_0 B_0^2(t) + \omega_1 B_1^2(t) + \omega_2 B_2^2(t)}$$

Arcos Circulares



$$b(t)=\frac{\omega_0 b_0 B_0^2(t)+\omega_1 b_1 B_1^2(t)+\omega_2 b_2 B_2^2(t)}{\omega_0 B_0^2(t)+\omega_1 B_1^2(t)+\omega_2 B_2^2(t)}$$

$$\omega_0=\omega_2=1\,;\frac{MX}{MP_1}=\frac{\omega_1}{\omega_1+1}$$

$$\omega_1=sen(\alpha)$$

http://www.cs.mtu.edu/~shene/COURSES/cs3621/NOTES/spline/NURBS/RB-circles.html

OpenGL

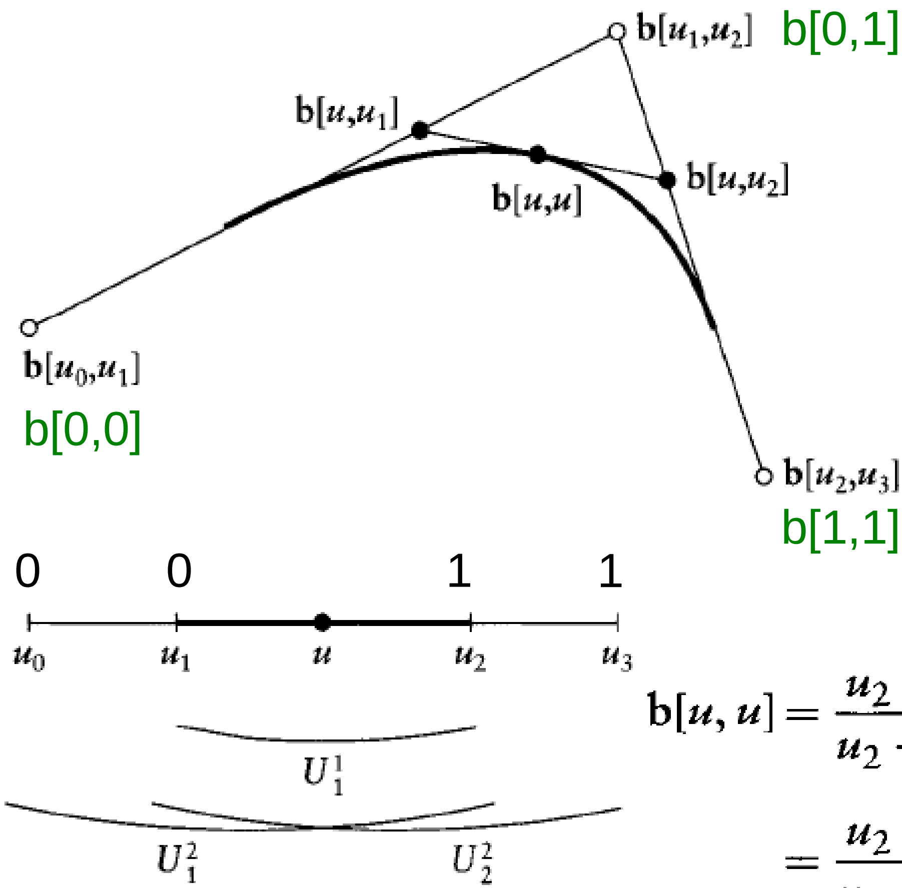
http://en.wikibooks.org/wiki/OpenGL_Programming/Modern_OpenGL_Tutorial_07

http://www.me.berkeley.edu/~mcmains/pubs/SPM07KrishnamurthyKhardMcMains.pdf

http://www.informatik.uni-marburg.de/~guthe/Publications/guthe-2005-gpu-based.pdf

http://codeflow.org/entries/2010/nov/07/opengl-4-tessellation/

Generalização do Algoritmo de DeCasteljau



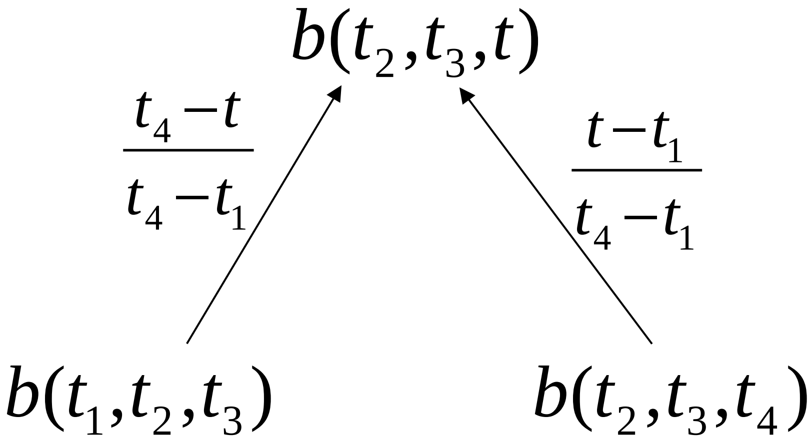
Generalização do algoritmo de DeCasteljau

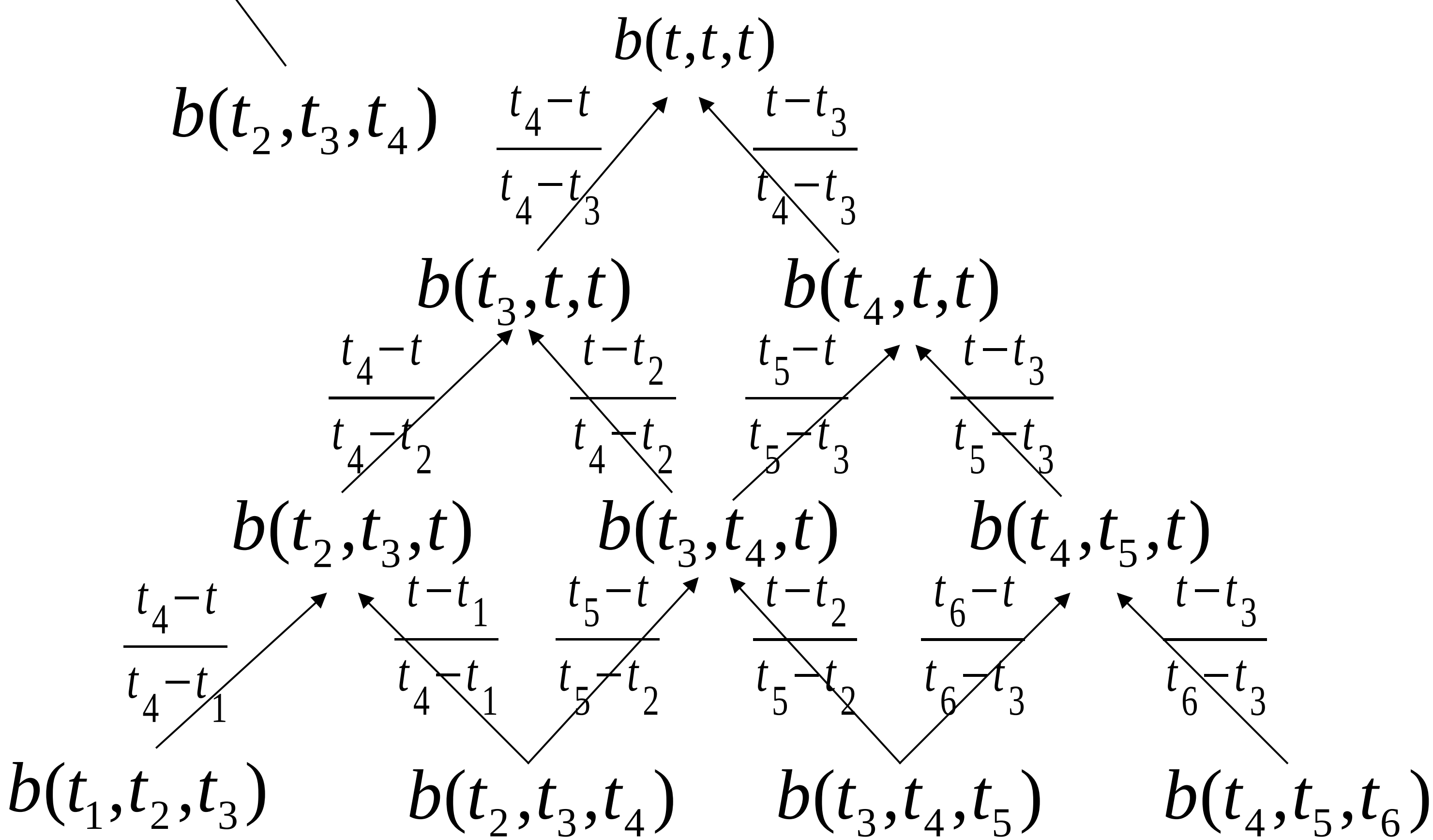
$$\mathbf{b}[u,u] = \frac{u_2 - u}{u_2 - u_1}\mathbf{b}[u_1,u] + \frac{u - u_1}{u_2 - u_1}\mathbf{b}[u,u_2]$$

$$= \frac{u_2 - u}{u_2 - u_1}\left(\frac{u_2 - u}{u_2 - u_0}\mathbf{b}[u_0,u_1] + \frac{u - u_0}{u_2 - u_0}\mathbf{b}[u_1,u_2]\right)$$

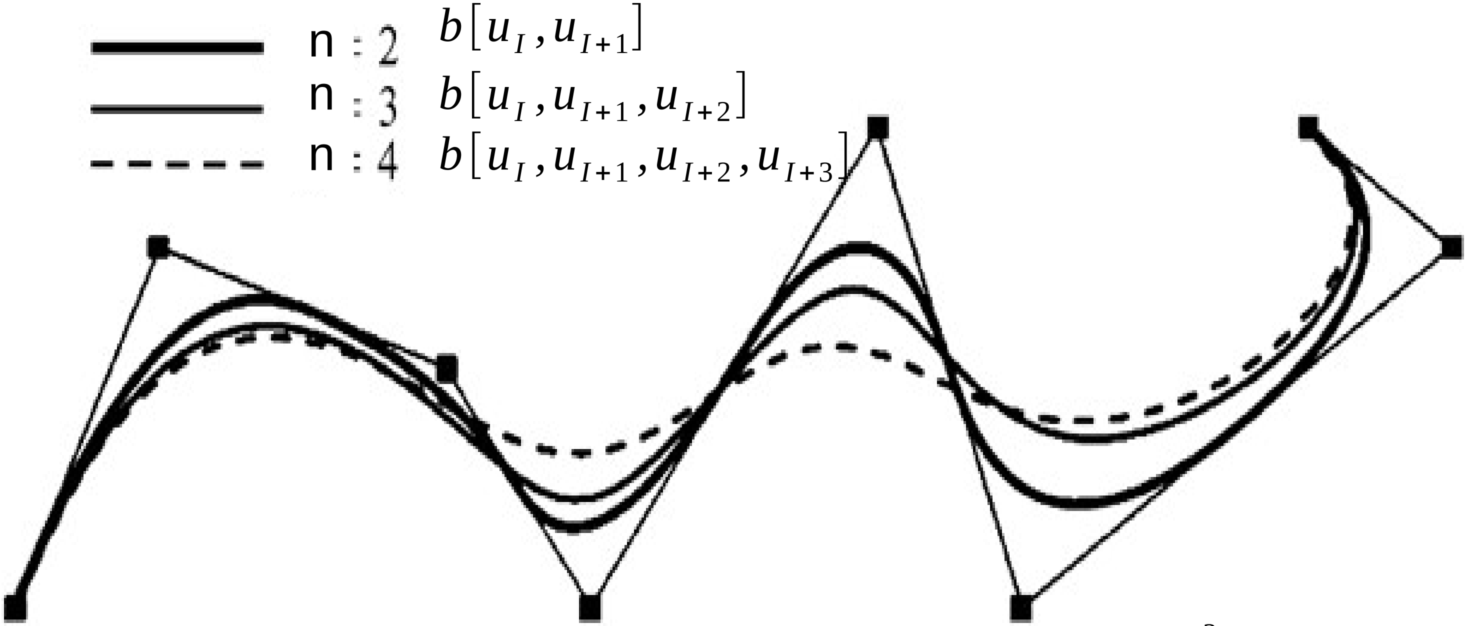
$$+ \frac{u - u_1}{u_2 - u_1}\left(\frac{u_3 - u}{u_3 - u_1}\mathbf{b}[u_1,u_2] + \frac{u - u_1}{u_3 - u_1}\mathbf{b}[u_2,u_3]\right)$$

Blossom por Partes





Grau das curvas



$b[u_I, u_{I+1}]$

$b[u_I, u_{I+1}, u_{I+2}]$

$b[u_I, u_{I+1}, u_{I+2}, u_{I+3}]$

$$b[u_I, u_{I+1}] \rightarrow D_0^2[u,u] = b[u,u], u \in [u_I, u_{I+1}]$$

$$b[u_I, u_{I+1}, u_{I+2}] \rightarrow D_0^3[u,u,u] = b[u,u,u], u \in [u_I, u_{I+1}]$$

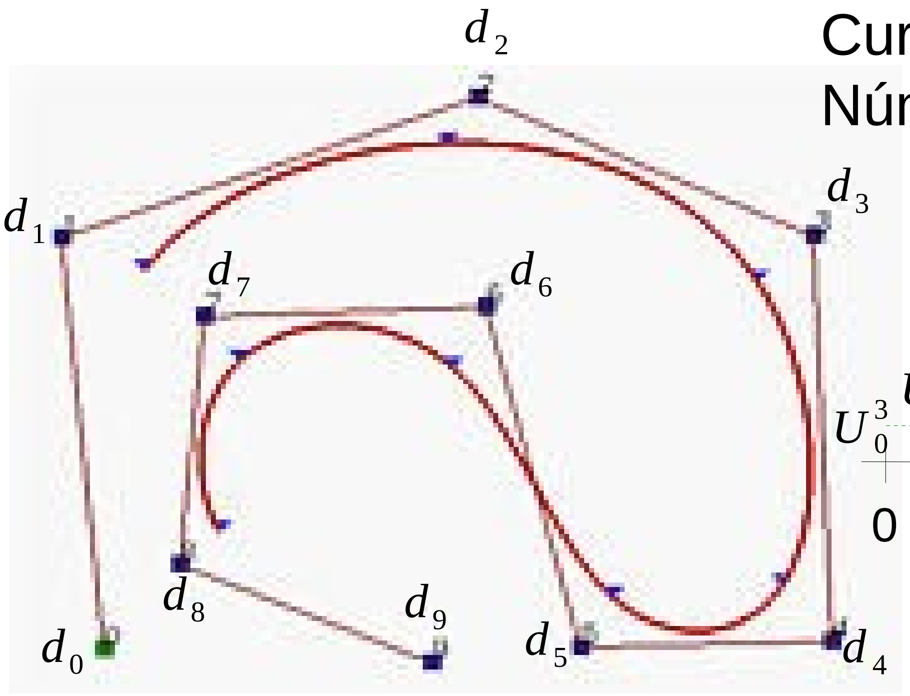
$$b[u_I, u_{I+1}, u_{I+2}, u_{I+3}] \rightarrow D_0^4[u,u,u,u] = b[u,u,u,u], u \in [u_I, u_{I+1}]$$

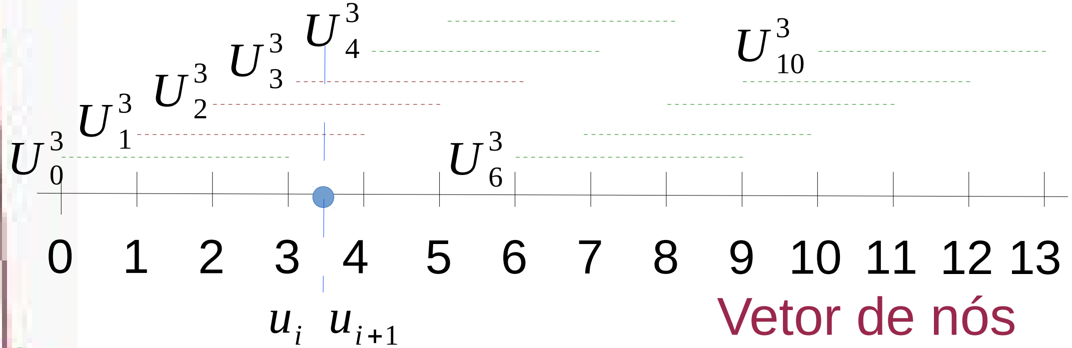
$$D_i^k(u) = \frac{u_{I+i} - u}{u_{I+i} - u_{I+i-k}} D_i^{k-1}(u) + \frac{u - u_{I+i-k}}{u_{I+i} - u_{I+i-k}} D_{i+1}^{k-1}(u)$$

Algoritmo de De Boor

Curva Cúbica: n = 3

Número de pontos de controle: L+1 = 10





d_j, onde j = i-n

$$d_0 = D_0^0 = b[1,2,3]$$

$$d_1 = D_1^0 = b[2,3,4] \quad D_0^1 = b[u,2,3]$$

$$d_2 = D_2^0 = b[3,4,5] \quad D_1^1 = b[u,3,4] \quad D_0^2 = b[u,u,3]$$

$$d_3 = D_3^0 = b[4,5,6] \quad D_2^1 = b[u,4,5] \quad D_1^2 = b[u,u,4] \quad D_0^3 = b[u,u,u]$$

Exemplo

$$n=2\,;\, b[1,2]=\begin{bmatrix}0\\0\end{bmatrix}, b[2,3]=\begin{bmatrix}8\\8\end{bmatrix}, b[3,4]=\begin{bmatrix}8\\0\end{bmatrix}$$

$$b[2.5,2.5]=?$$

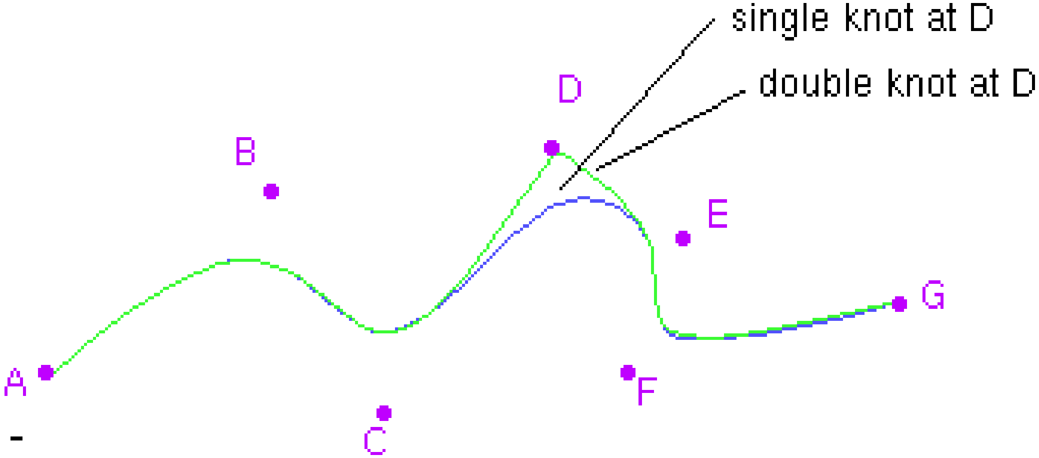
$$b[2,2.5]=\frac{(3-2.5)}{(3-1)}\begin{bmatrix}0\\0\end{bmatrix}+\frac{(2.5-1)}{(3-1)}\begin{bmatrix}8\\8\end{bmatrix}=\begin{bmatrix}6\\6\end{bmatrix}$$

$$b[2.5,3]=\frac{(4-2.5)}{(4-2)}\begin{bmatrix}8\\8\end{bmatrix}+\frac{(2.5-2)}{(4-2)}\begin{bmatrix}8\\0\end{bmatrix}=\begin{bmatrix}8\\6\end{bmatrix}$$

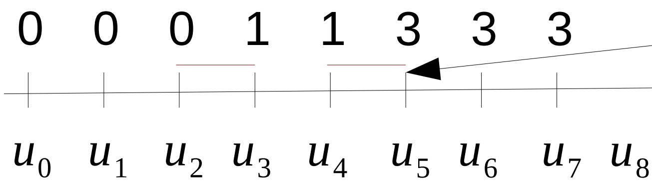
$$b[2.5,2.5]=\frac{(3-2.5)}{(3-2)}\begin{bmatrix}6\\6\end{bmatrix}+\frac{(2.5-2)}{(3-2)}\begin{bmatrix}8\\6\end{bmatrix}=\begin{bmatrix}7\\6\end{bmatrix}$$

Multiplicidade de Nós

- Curvas uniformes: domínios com intervalos igualmente espaçados.
- Curvas não-uniformes: domínios com intervalos diferentes.
- Multiplicidade r altera a suavidade das curvas e pode gerar cúspides.



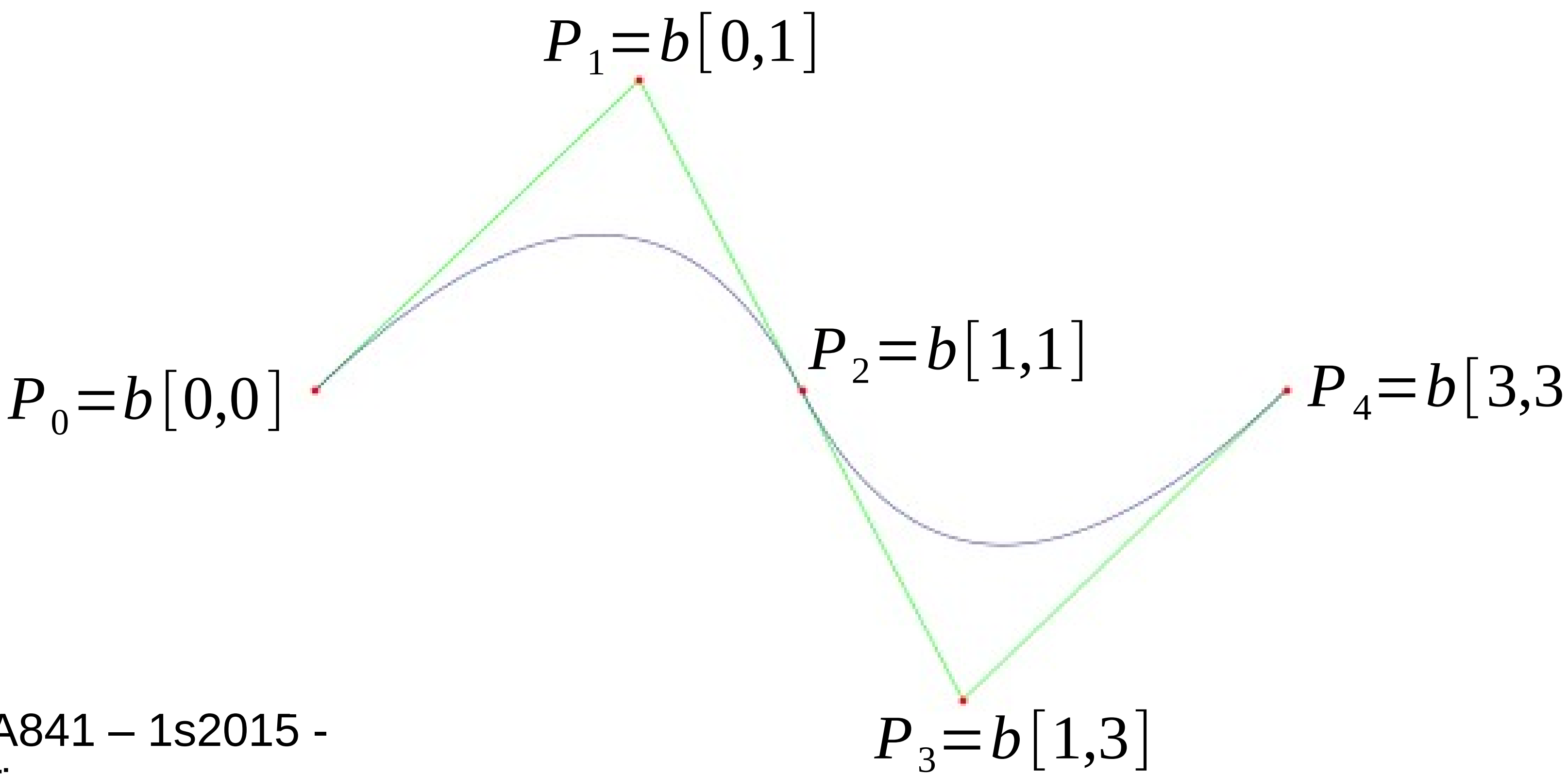
Exemplo: n=2



Intervalos efetivos para algoritmo de Boor

$$u \in [u_{2,} u_3) : P_0 ; P_1 ; P_2$$

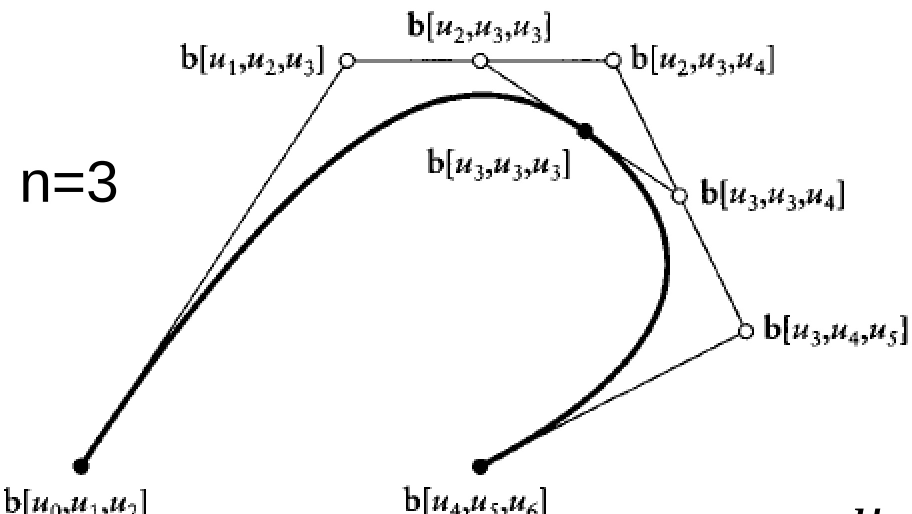
$$u \in [u_{4,} u_5) : P_2 ; P_3 ; P_4$$



Suavidade

- Continuidade $C^k \leftrightarrow$ derivabilidade até ordem k.
- Curva de grau n tem continuidade C^{n-1}.
- Continuidade C^{n-r} em nós de multiplicidade r (derivável até ordem *n-r*).

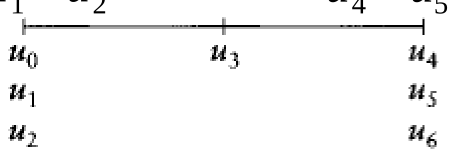
n=3



Convenção: $\frac{0}{0} = 0$

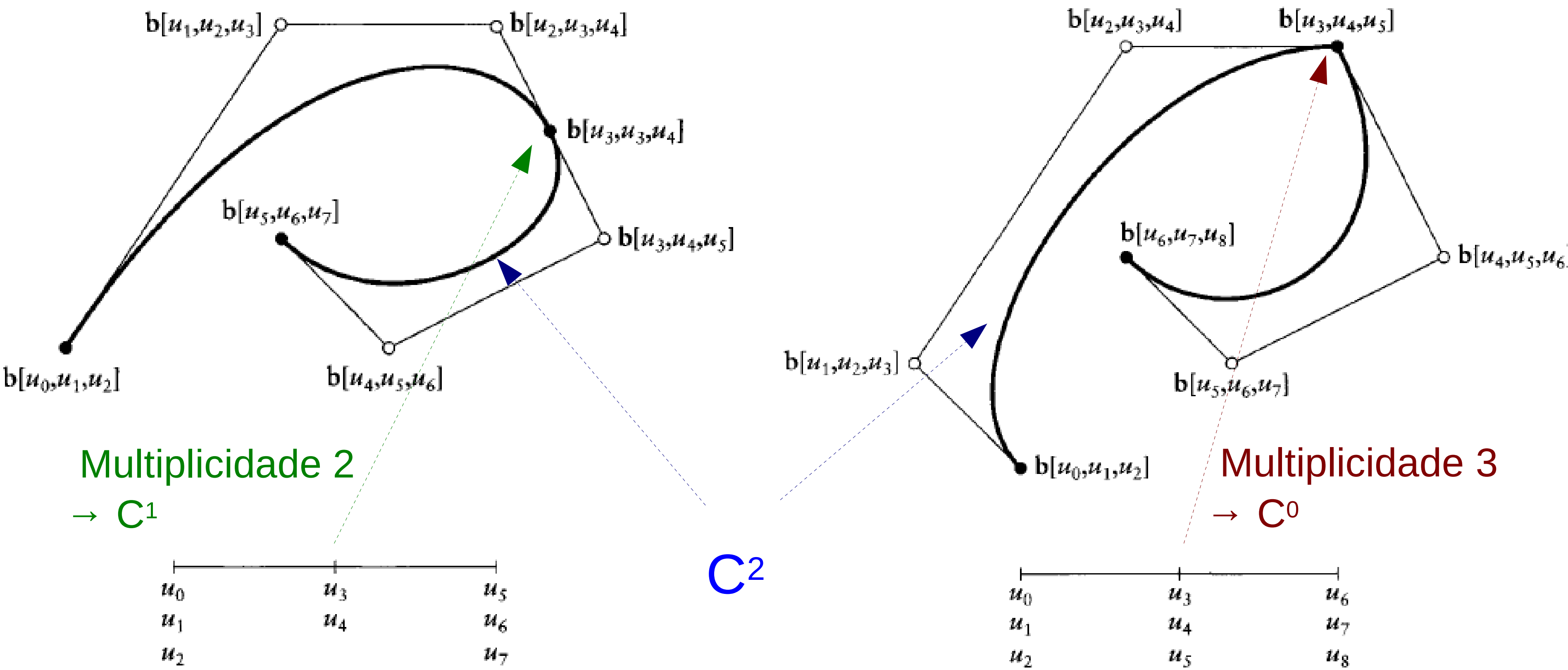
$$D_i^k(u) = \frac{u_{I+i} - u}{u_{I+i} - u_{I+i-k}} D_i^{k-1}(u) + \frac{u - u_{I+i-k}}{u_{I+i} - u_{I+i-k}} D_{i+1}^{k-1}(u)$$

$u_0 = u_1 = u_2$ $\quad$ $u_4 = u_5 = u_6$



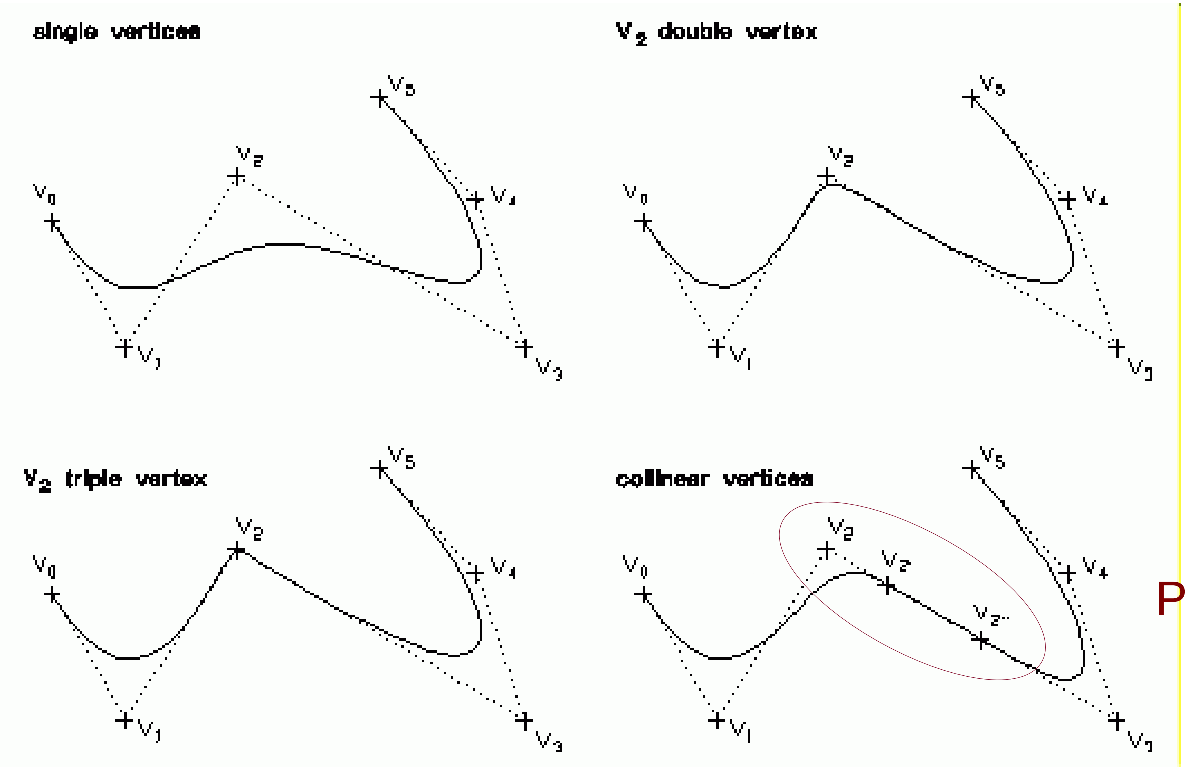
Cúspides

Curvas cúbicas (k=4 e n=3)



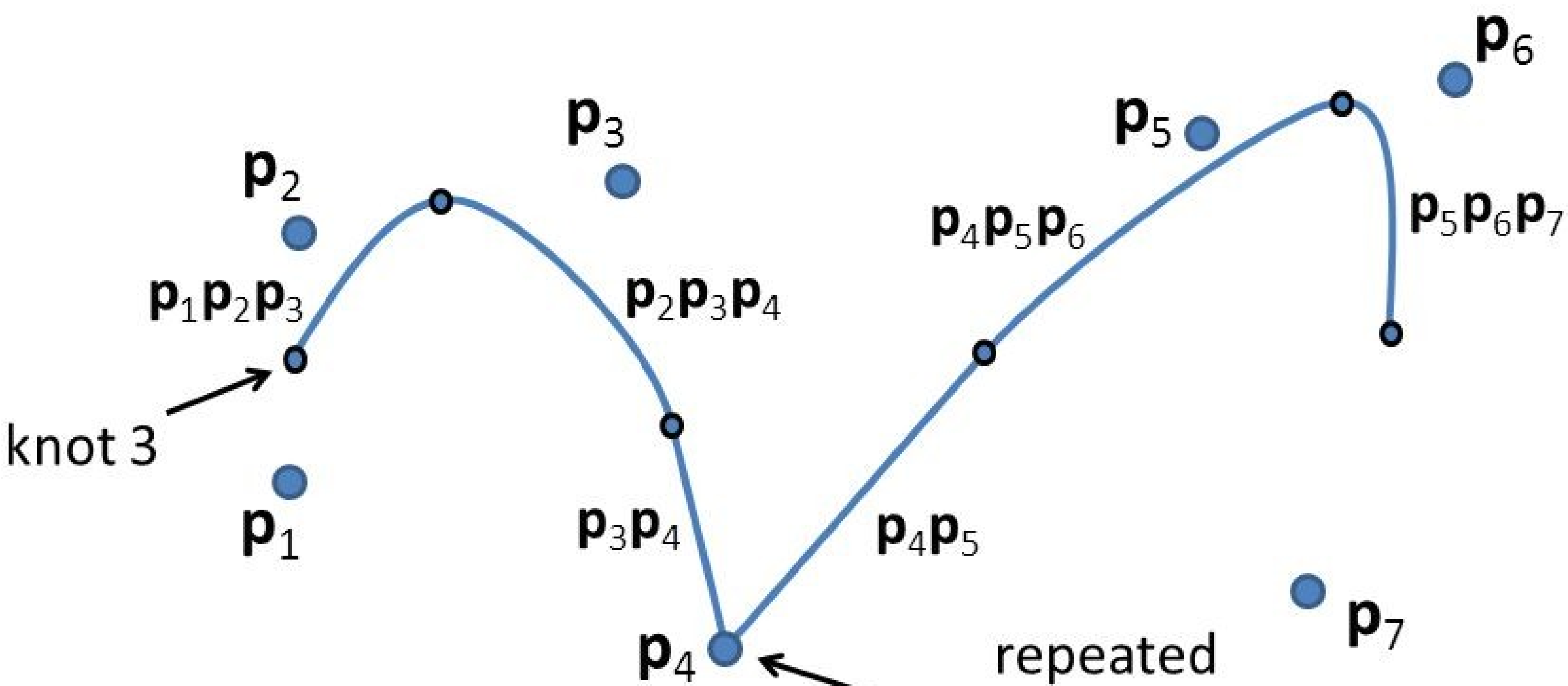
Multiplicidade de pontos de controle

- Curva se aproxima mais dos pontos de controle múltiplos. Quando a multiplicidade é *n*, a curva de grau *n* passa pelo ponto.



Precisão linear

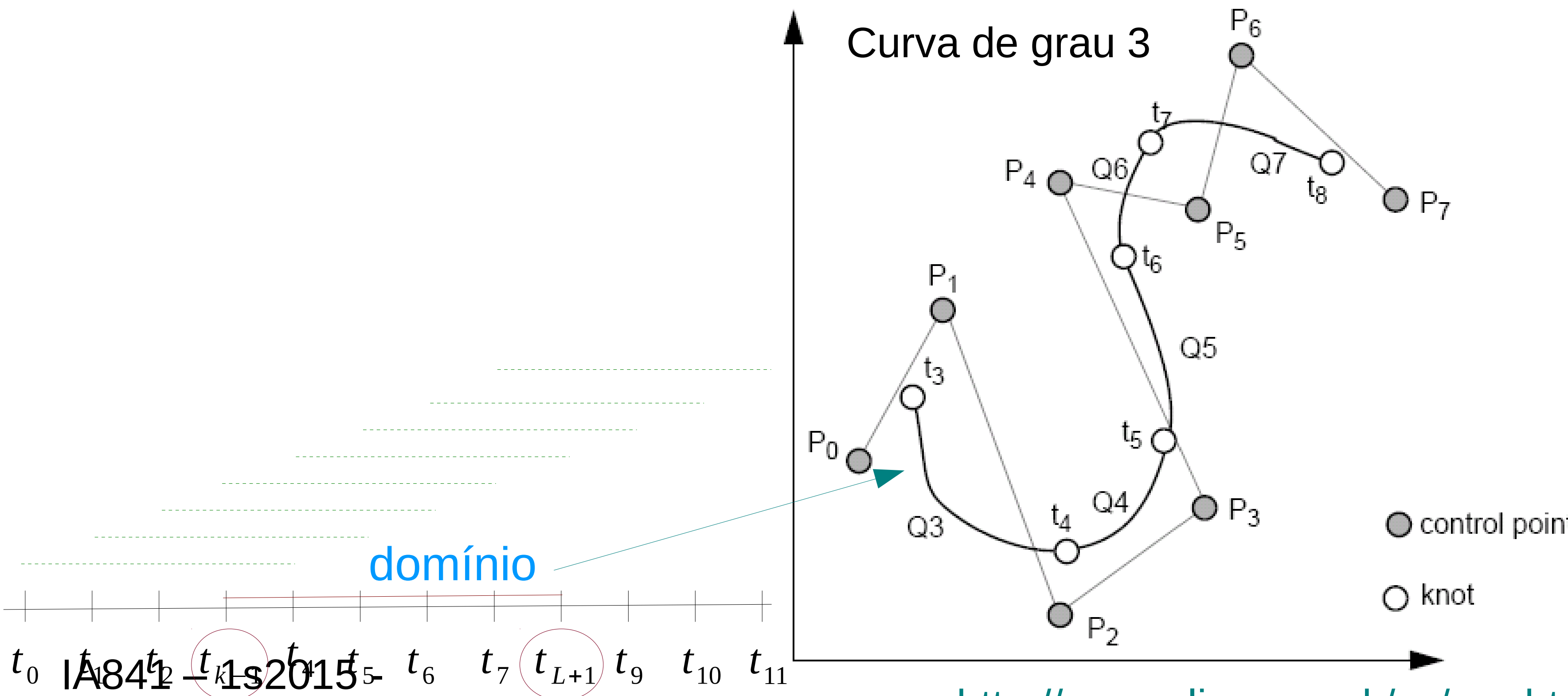
Repeating control points in B-spline curves



quadratic B-spline curve
with repeated control point

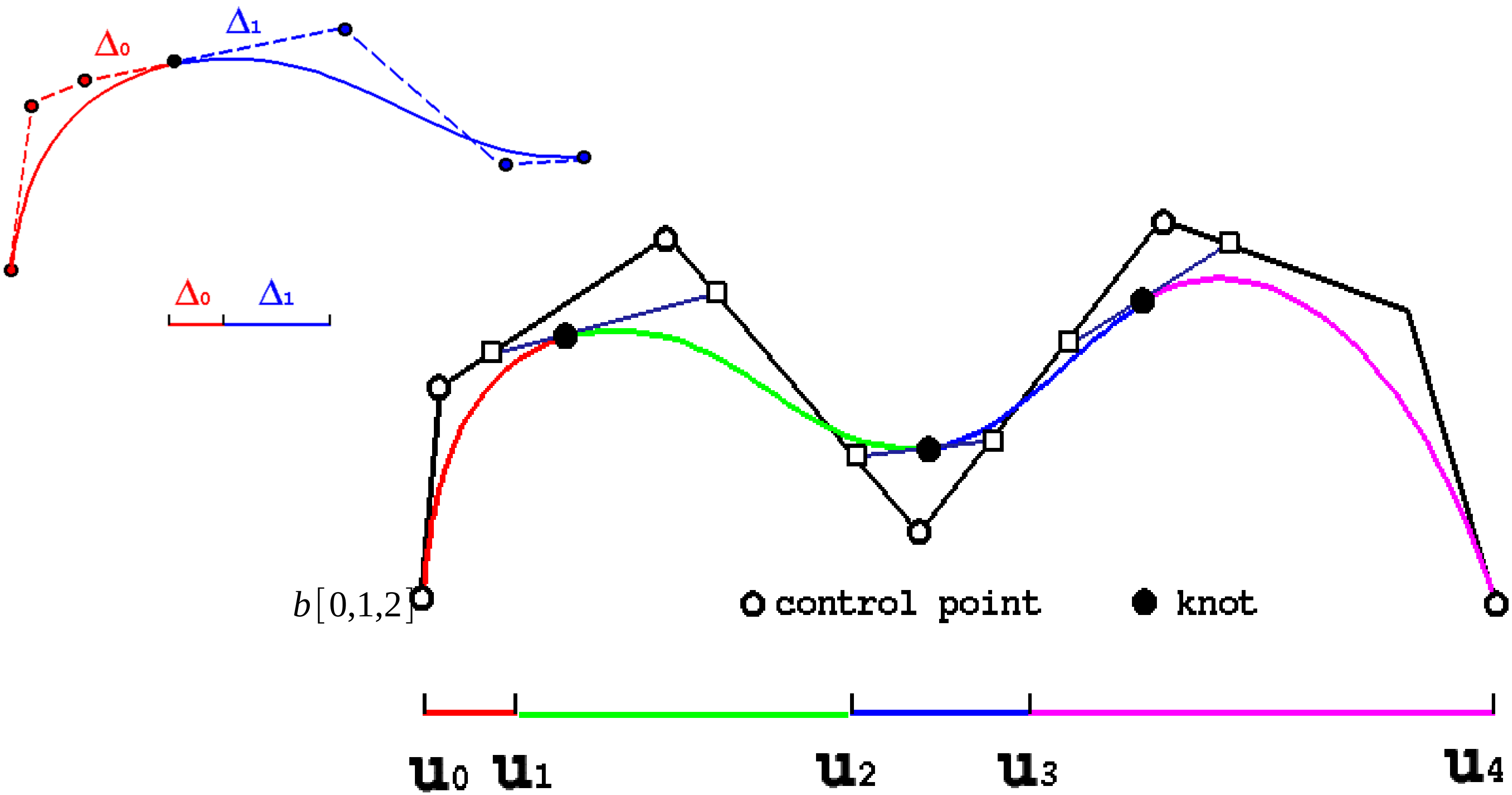
B-Splines

- Segmentos de curvas de grau n → Ordem: k=n+1
- Número de pontos de controle: L+1 = K-n+1
- Número de nós: K+1



http://www.pling.org.uk/cs/cgv.html

Splines de curvas de Bézier



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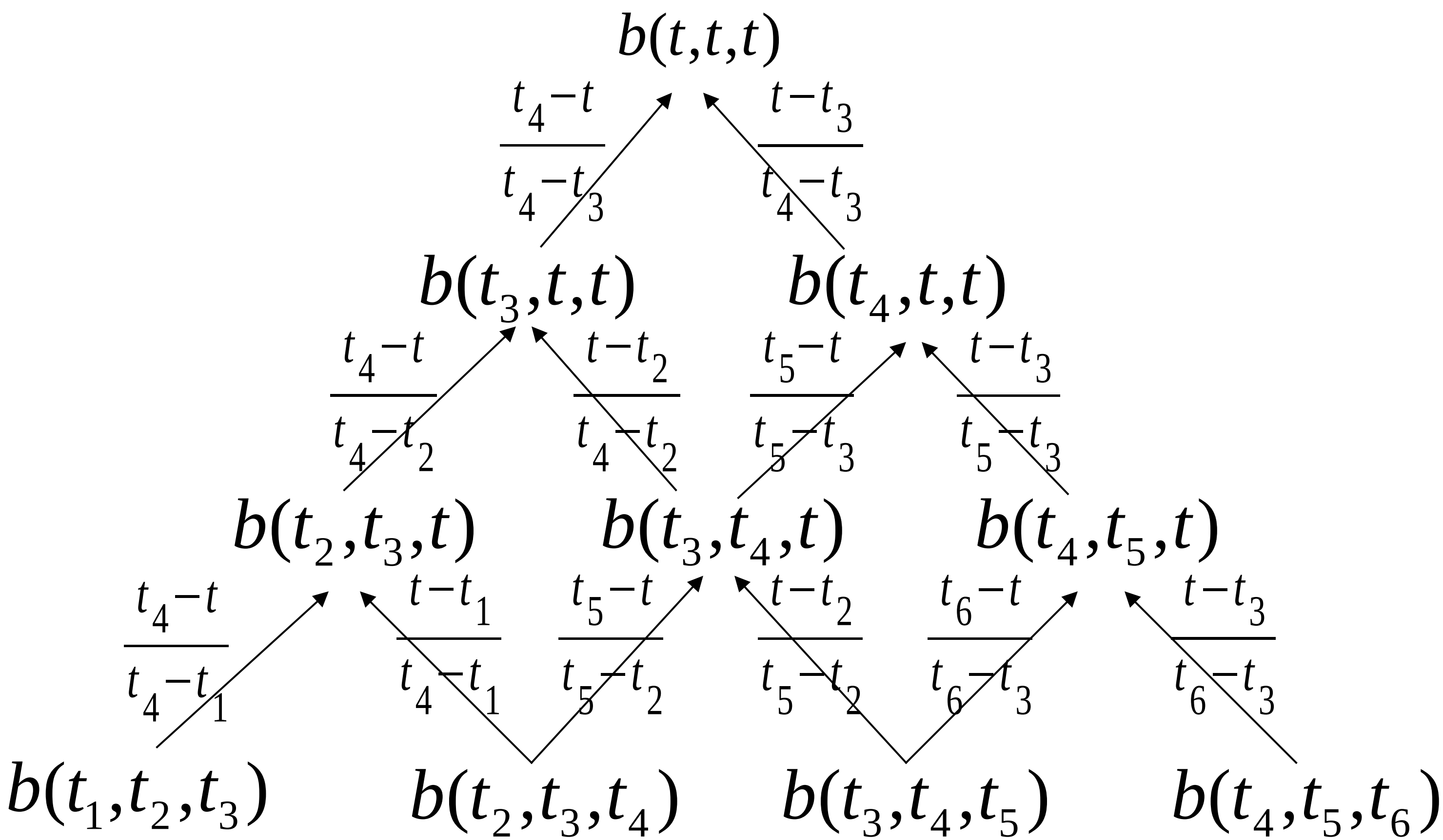
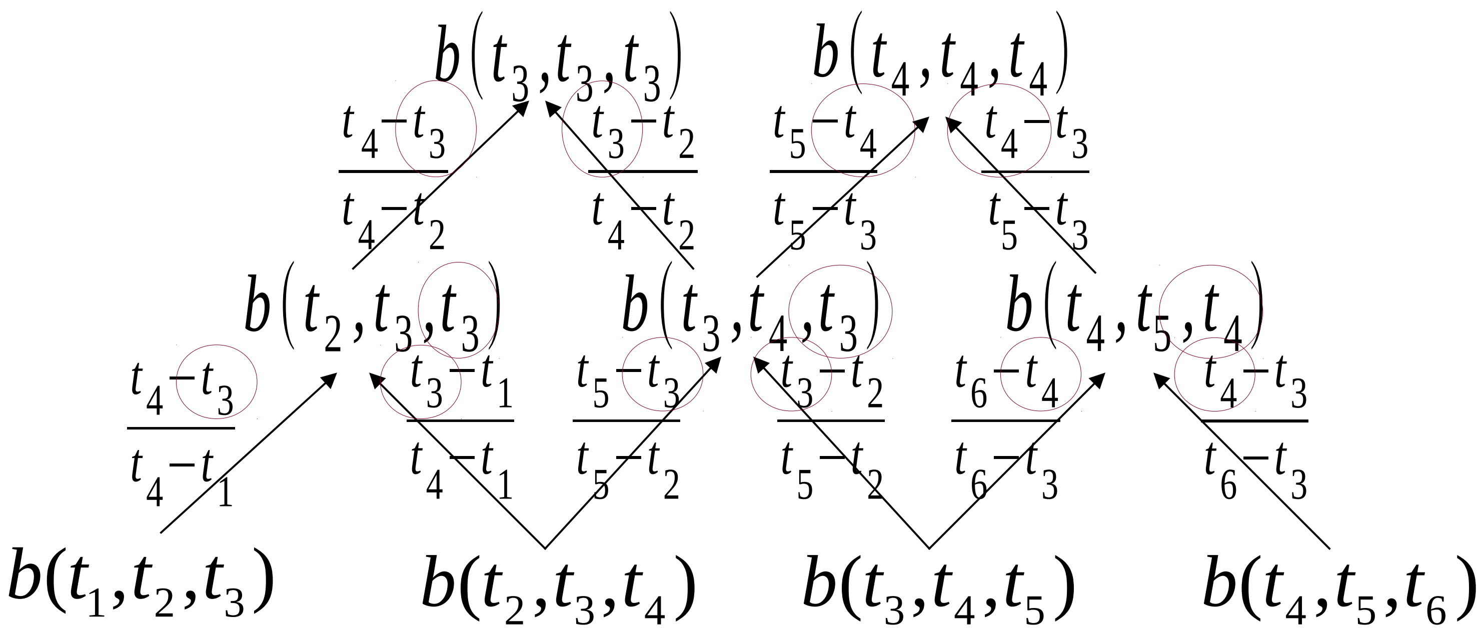
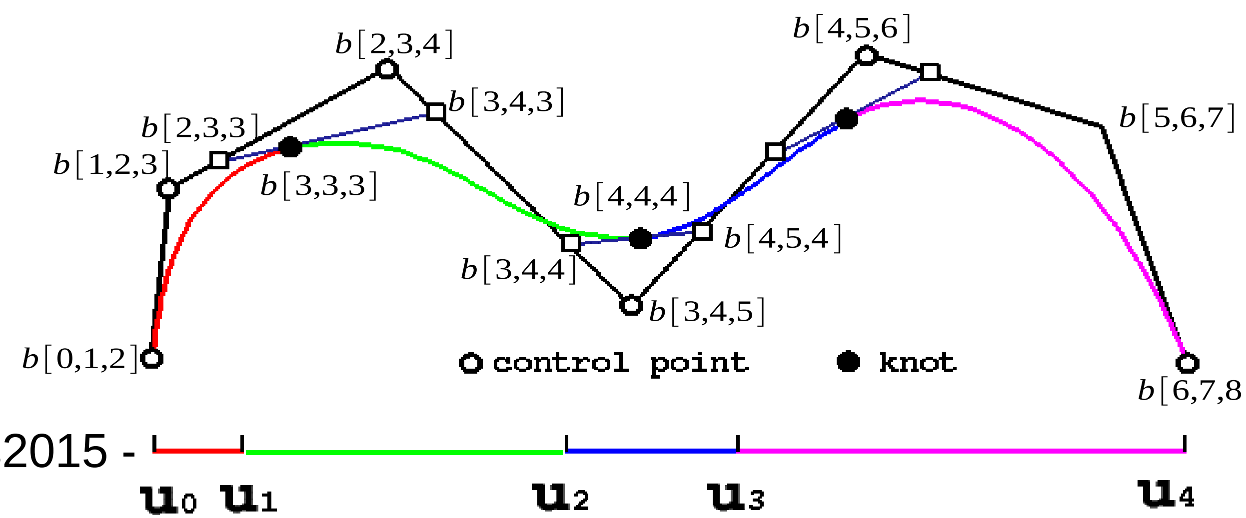
http://groups.csail.mit.edu/graphics/classes/6.838/S98/meetings/m15/

Na Forma de *Blossom*

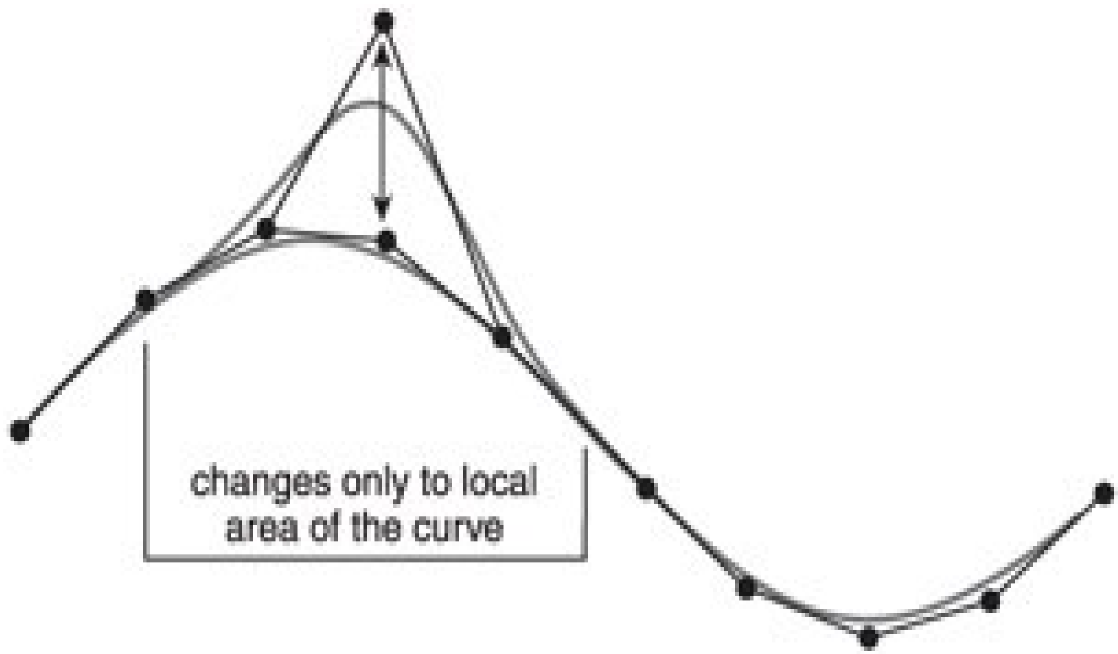


Na Forma de *Blossom*



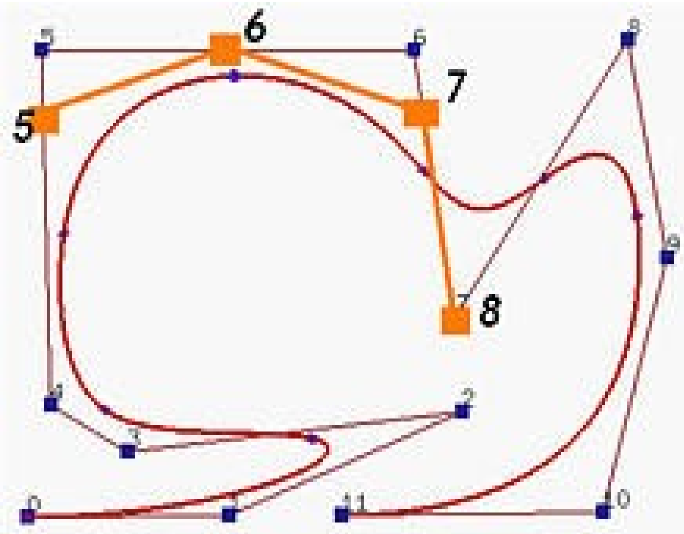


Controle Local



http://flylib.com/books/en/2.258.1.37/1/

Inserção de Nós



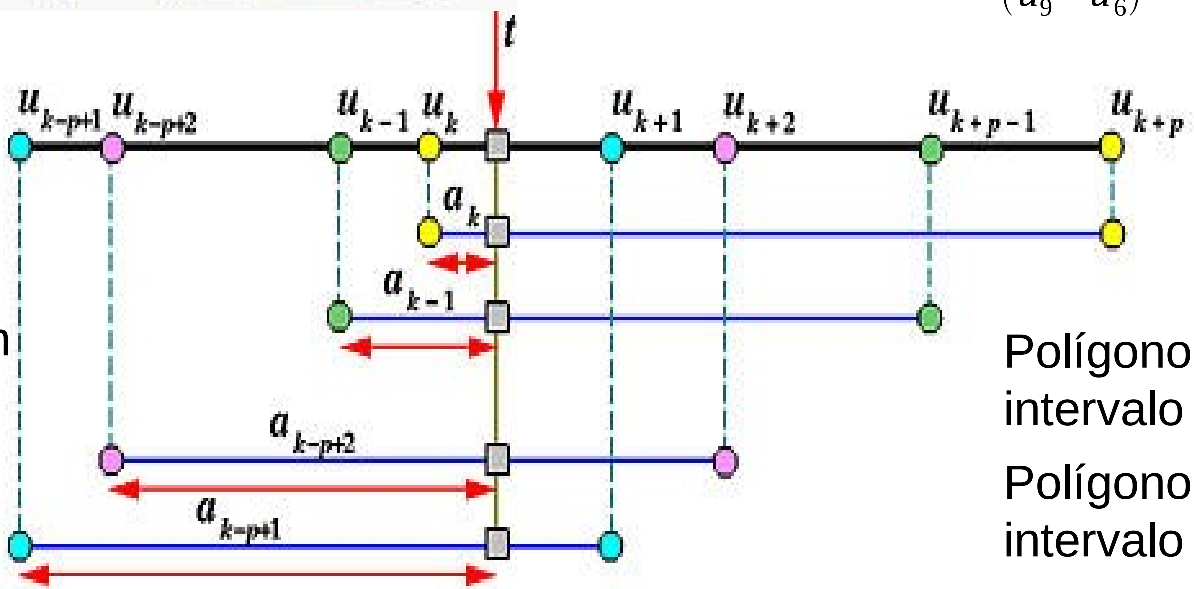
$$(4) = b[4,5,6]$$

$$(5) = b[5,6,7] \qquad N5 = \frac{(u_6 - t)}{(u_6 - u_3)} b[4,5,6] + \frac{(t - u_3)}{(u_6 - u_3)} b[5,6,7]$$

$$(6) = b[6,7,8] \qquad N6 = \frac{(u_7 - t)}{(u_7 - u_4)} b[5,6,7] + \frac{(t - u_4)}{(u_7 - u_4)} b[6,7,8]$$

$$(7) = b[7,8,9] \qquad N7 = \frac{(u_8 - t)}{(u_8 - u_5)} b[6,7,8] + \frac{(t - u_5)}{(u_8 - u_5)} b[7,8,9]$$

$$(8) = b[8,9,10] \qquad N8 = \frac{(u_9 - t)}{(u_9 - u_6)} b[7,8,9] + \frac{(t - u_6)}{(u_9 - u_6)} b[8,9,10]$$



p: ordem

Polígono de controle para o intervalo $[u_k, t)$: (4)N5N6N7

Polígono de controle para o intervalo $[t, u_{k+1})$:N5N6N7N8

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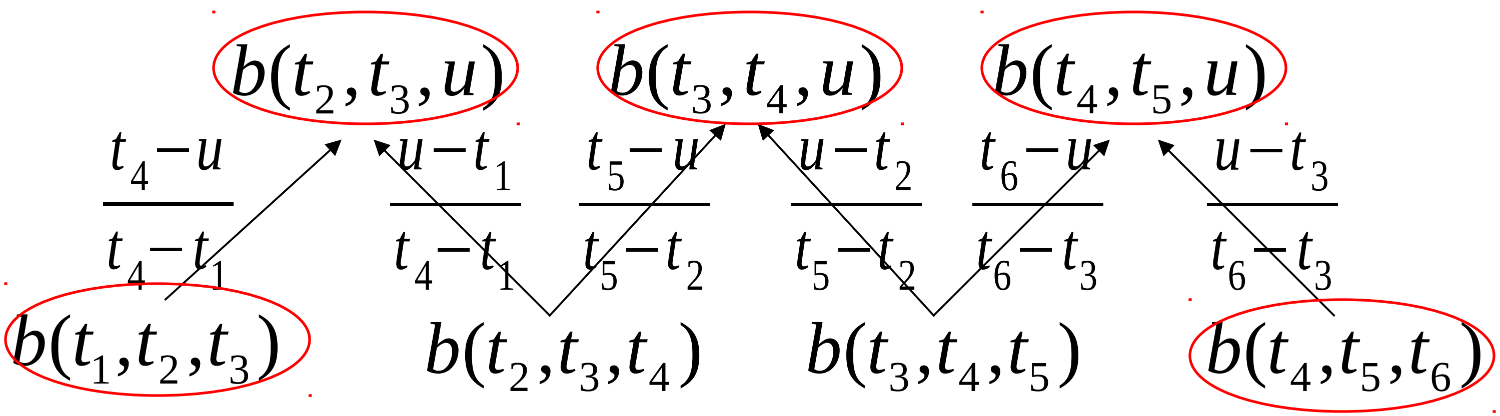
http://www.cs.mtu.edu/~shene/COURSES/cs3621/NOTES/spline/B-spline/single-insertion.html

Algoritmo de Boehm

$$t_3 \leqslant u \leqslant t_4$$



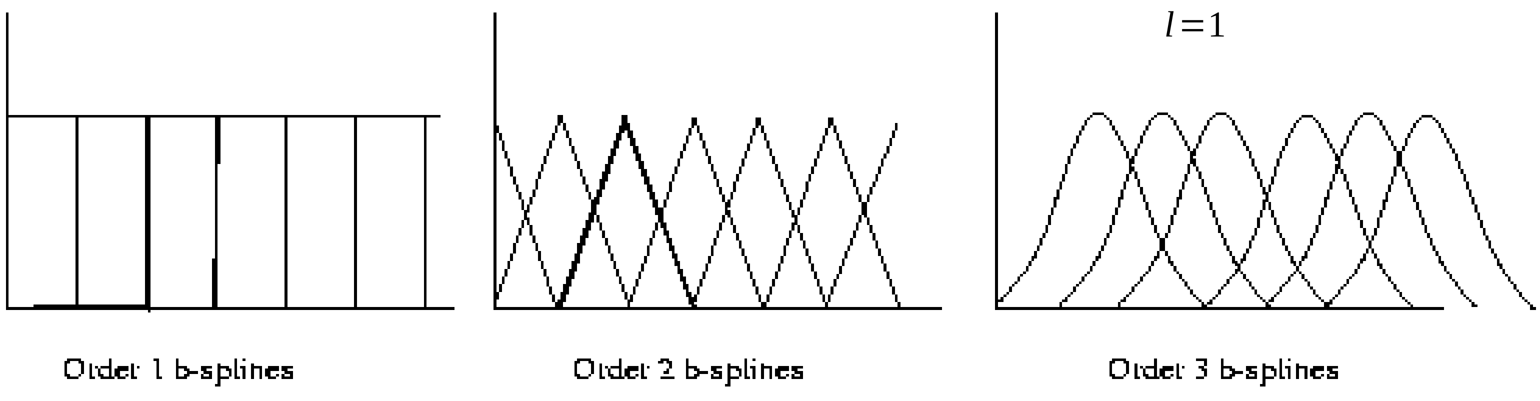
Funções de Base

- Grau: n ; *Order*: n+1
- Vetor de nós: {u_0, u_1, u_2 , …, n_K}
- Intervalo de suporte mínimo: [u_{i-1}, u_{i+n})

$$N_l^n(u) = \frac{u - u_l}{u_{l+n} - u_l} N_l^{n-1}(u) + \frac{u_{l+n+1} - u}{u_{l+n+1} - u_{l+1}} N_{l+1}^{n-1}(u)$$

$$N_i^0(u) = \begin{cases} 1, se\, u_i \leqslant u < u_{i+1} \\ 0, caso\ contrário \end{cases}$$

$$\sum_{l=1}^{n+1} N_l^n(u) = 1$$



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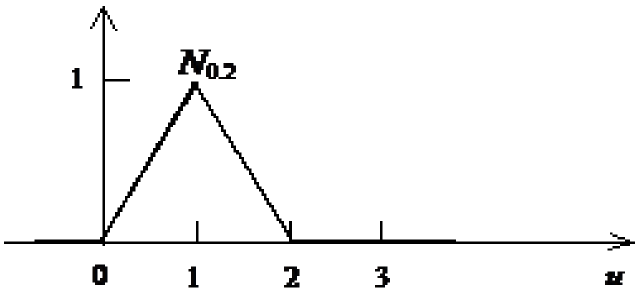
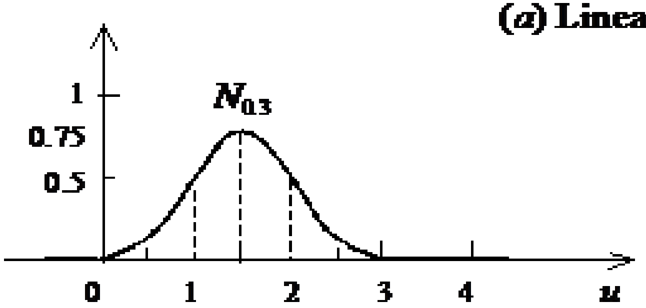
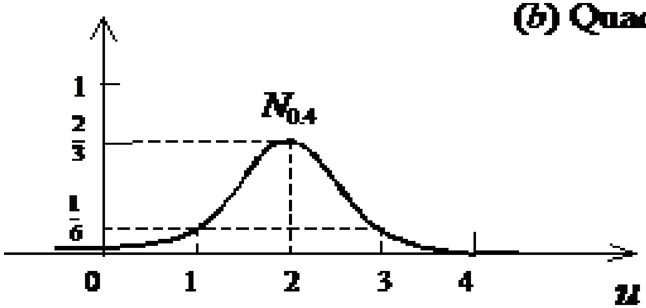
Funções de Base Uniforme



(a) Linear function (k = 2)



(b) Quadratic function (k = 3)



(c) Cubic function (k = 4)

Vetor de nós igualmente espaçados {0,1,2,3,4,5,6,...}

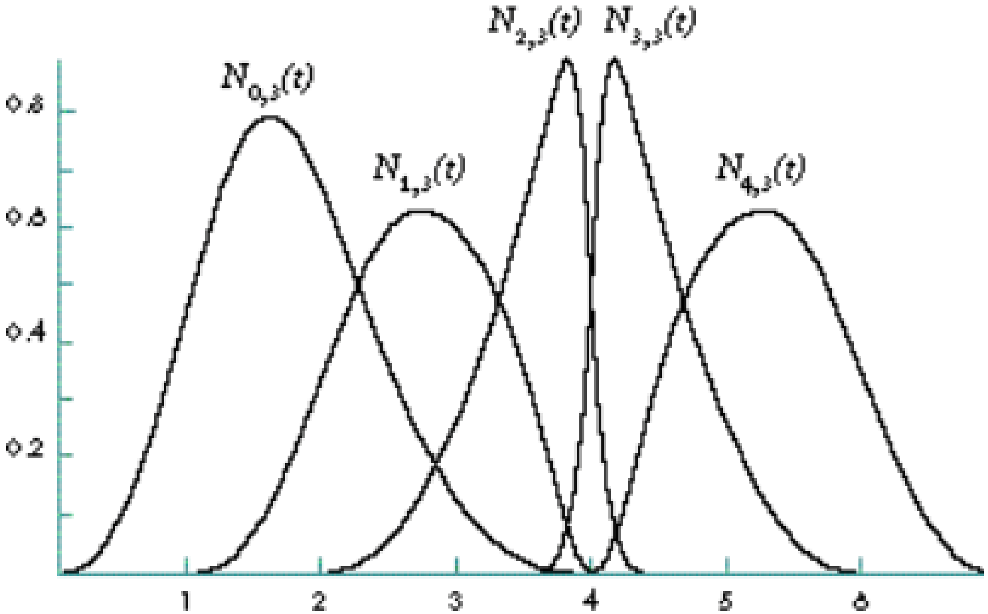
$N_i^n(t)$ definidas por partes!

$$N_0^1(u) = \frac{u - u_0}{u_1 - u_0} N_0^0(u) + \frac{u_1 - u}{u_2 - u_1} N_1^0(u)$$

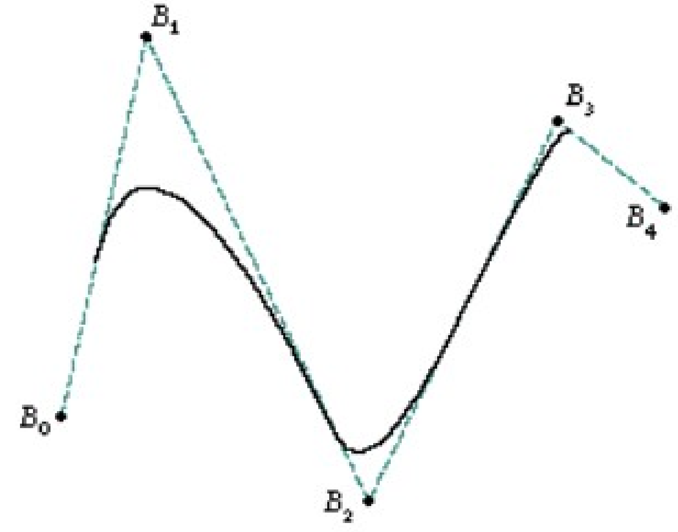
$$N_0^1(u) = \left\{ \begin{array}{l} \frac{u - u_0}{u_1 - u_0} N_0^0(u), u_0 \leqslant u < u_1 \\ \frac{u_2 - u}{u_2 - u_1} N_0^1(u), u_1 \leqslant u < u_2 \\ 0, caso\ contrário \end{array} \right\}$$

Funções de Base Não-Uniformes

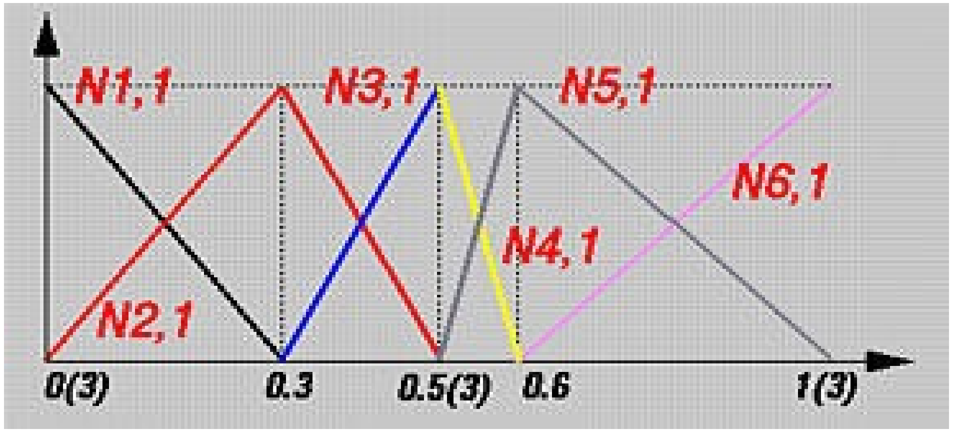
Invervalos não-nulos e não-uniformes



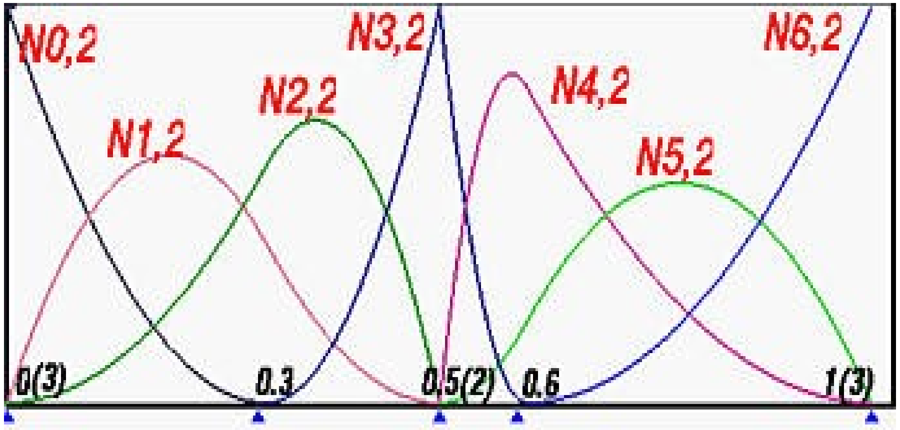
{0,1.0, 2.0, 3.75, 4.0, 4.25, 6.0. 7.0}



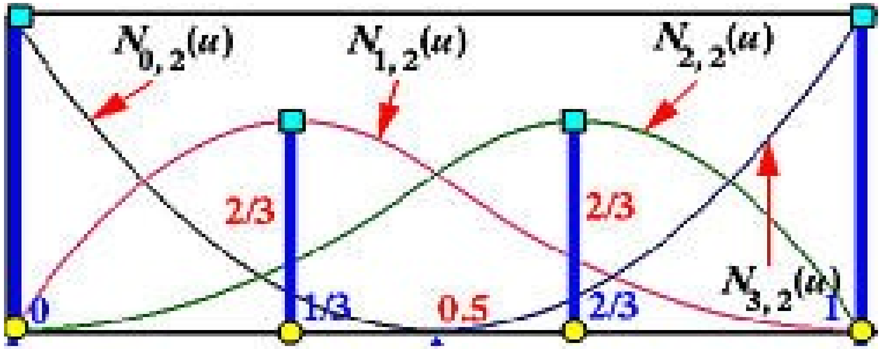
Funções de Base Não-Uniformes



{0,0,0,0.3,0.5,0.5,0.6,1,1,1}



Intervalos nulos → multiplicidade de nós



{0,0,0,0.5,1,1,1}

http://www.cs.mtu.edu/~shene/COURSES/cs3621/NOTES/spline/B-spline/bspline-ex-1.html

Representação Algébrica de *B-Splines*

$$P(u)=\sum_{j=0}^{L} d_j N_j^n(u)$$

$$P(t)=\sum_{j=0}^{L} d_j N_j^n(t)$$

$$P(u-2)=P(t)$$

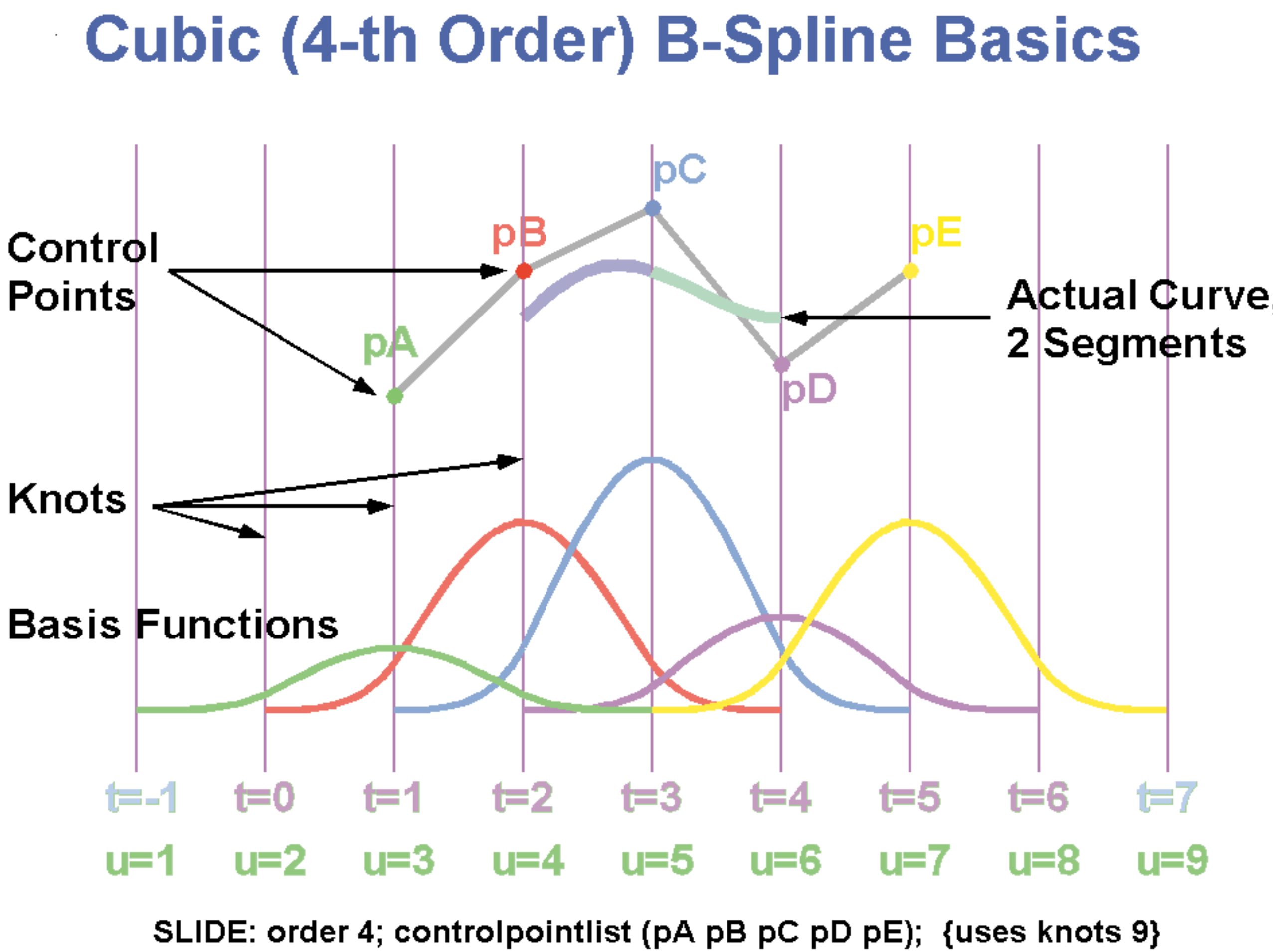
Grau: n

Ordem: k = n+1

Número de pontos de controle: L+1

Qtde de nós:

K+1=L+n+2



http://www.cs.berkeley.edu/~sequin/CS184/GuestLecture_05/SplineLect2.html

Exemplo Numérico

$$P(u)=\begin{bmatrix}0\\0\end{bmatrix}N_0^3(u)+\begin{bmatrix}8\\8\end{bmatrix}N_1^3(u)+\begin{bmatrix}8\\0\end{bmatrix}N_2^3(u)$$

$$P(2.5)=?$$

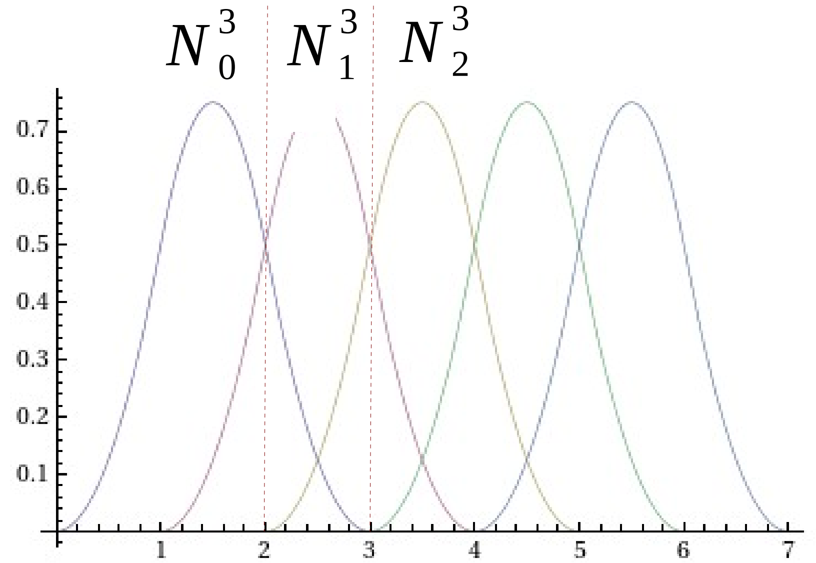
$$N_0^3(u)=\frac{(3-u)^2}{2} \quad N_1^3(u)=\frac{(u-1)(3-u)}{2}+\frac{(4-u)(u-2)}{2} \quad N_2^3(u)=\frac{(u-2)^2}{2}$$

$$N_0^3(2.5)=0.125$$

$$N_1^3(2.5)=0.75$$

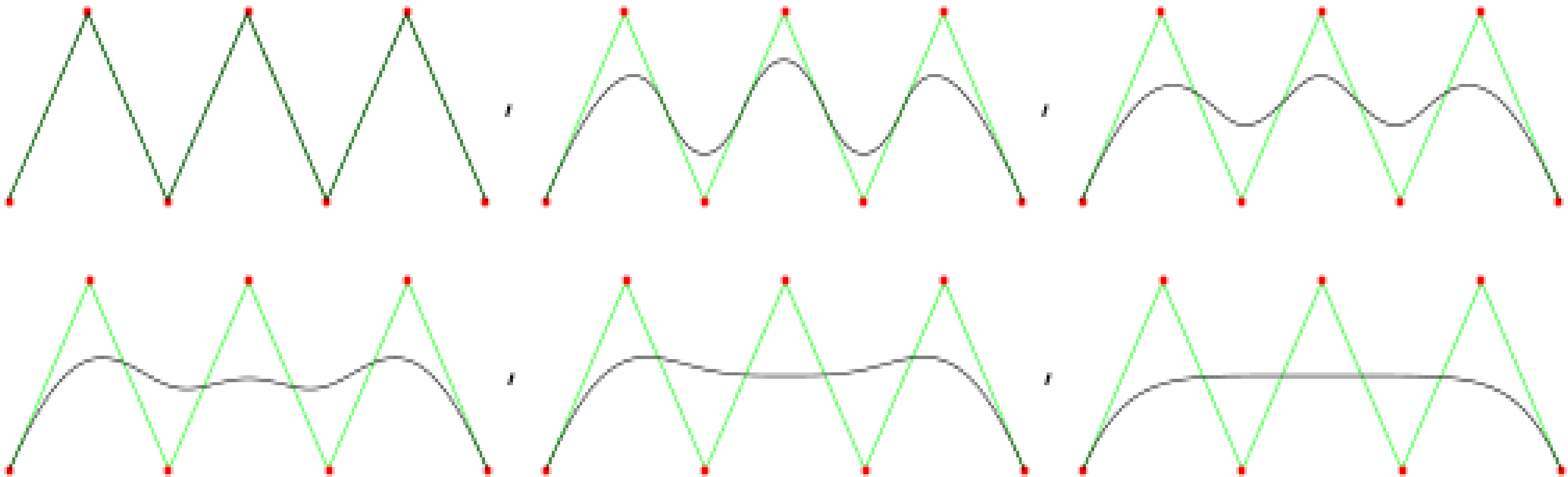
$$N_2^3(2.5)=0.125$$

$$P(2.5)=\begin{bmatrix}7\\6\end{bmatrix}$$



Influência do Grau nas curvas

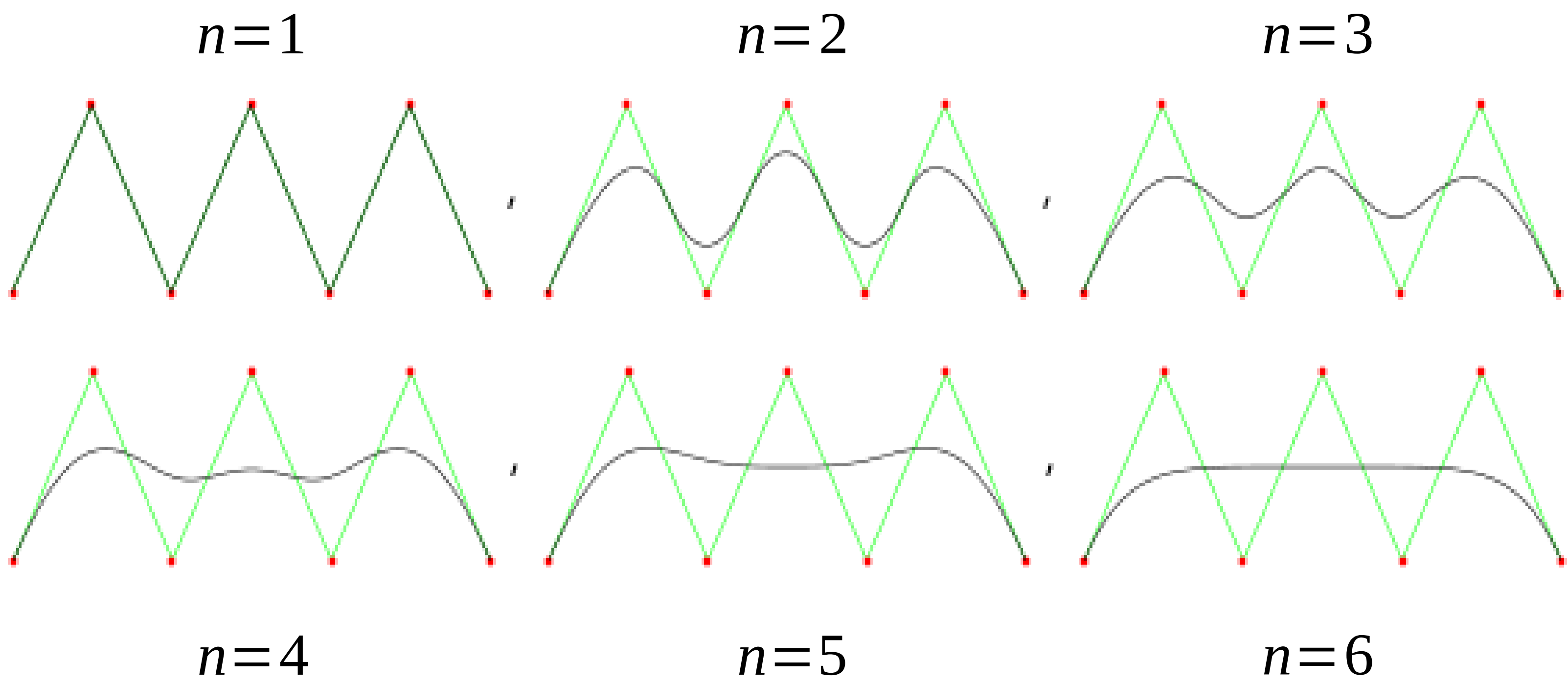
$n=1$ $n=2$ $n=3$



$n=4$ $n=5$ $n=6$

Qual é a multiplicidade dos nós extremos em cada *BSpline*?

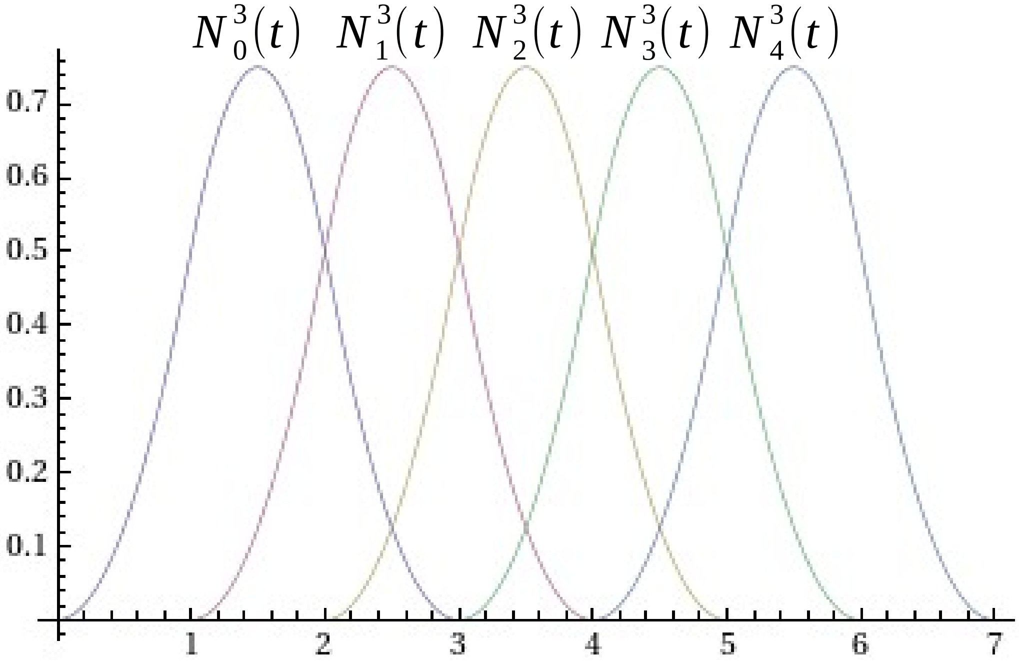
Influência do Grau nas curvas



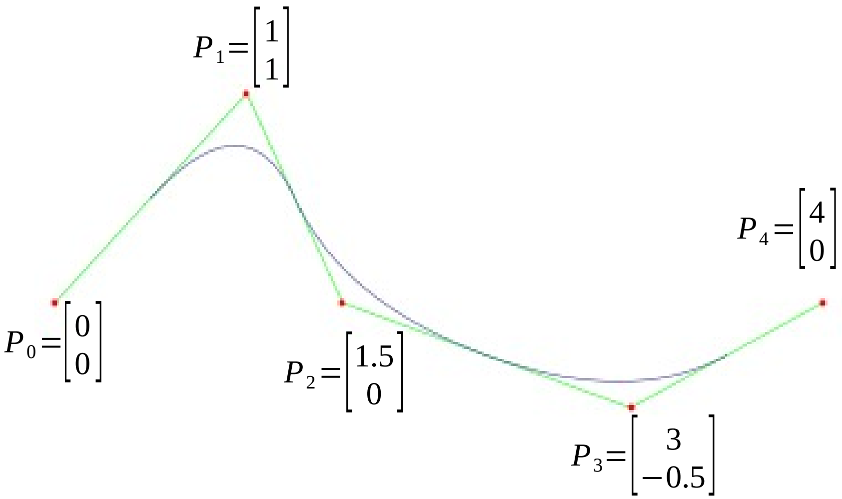
Qual é a multiplicidade dos nós extremos em cada *BSpline*?

k=n+1

Curvas uniformes (abertas)

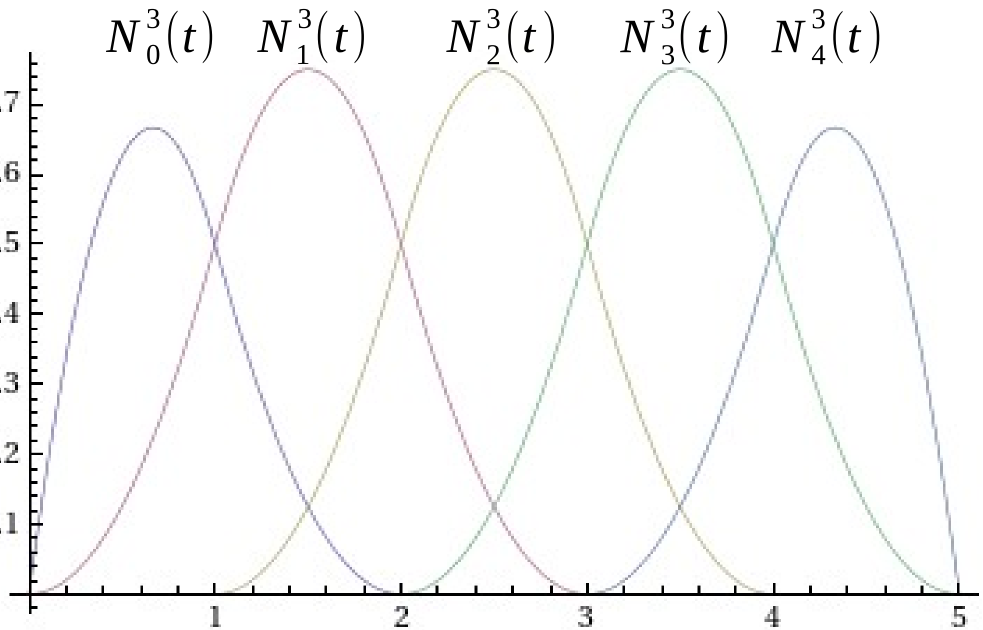


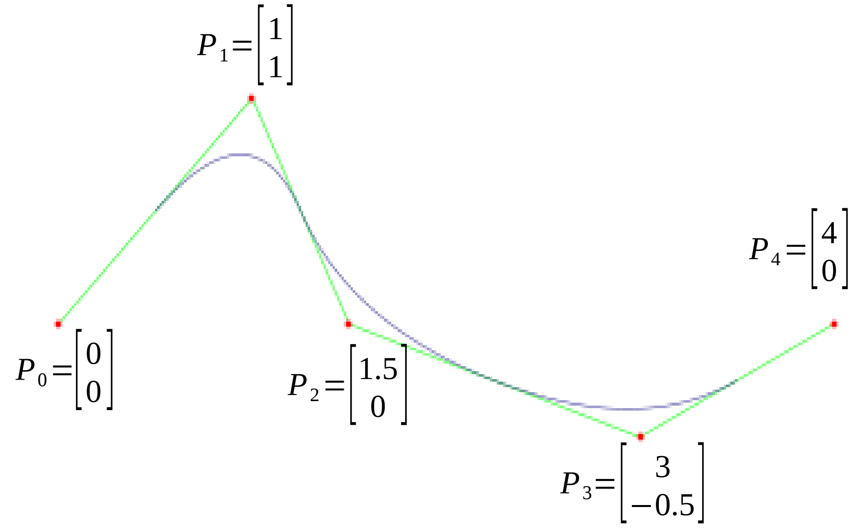
Vetor de Nós: {0,1,2,3,4,5,6,7}



$$P(t)=P_0 N_0^3(t)+P_1 N_1^3(t)+P_2 N_2^3(t)+P_3 N_3^3(t)+P_4 N_4^3(t)$$

Curvas não-uniformes

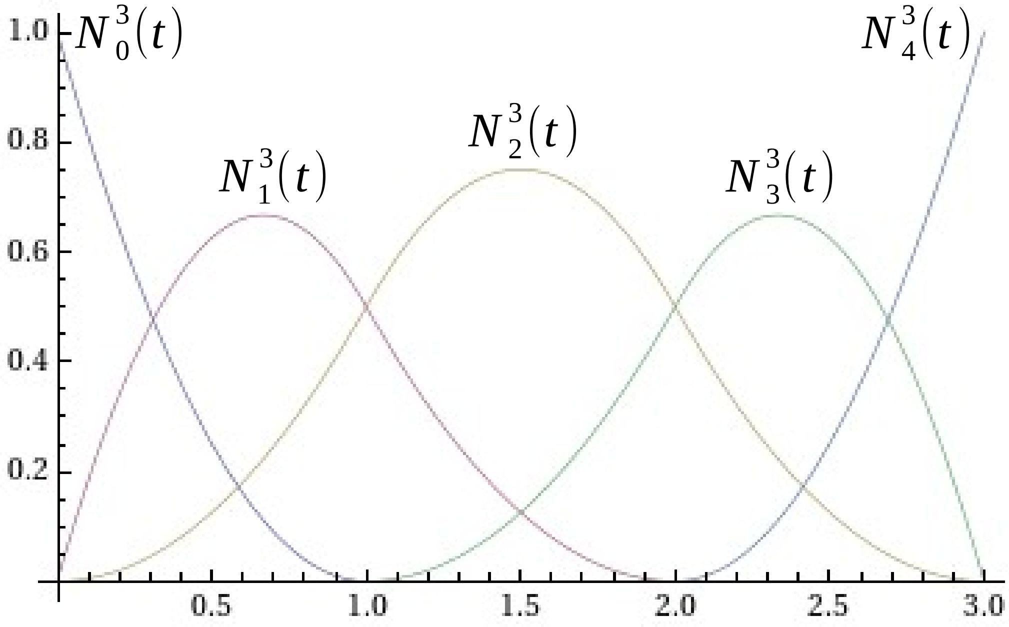


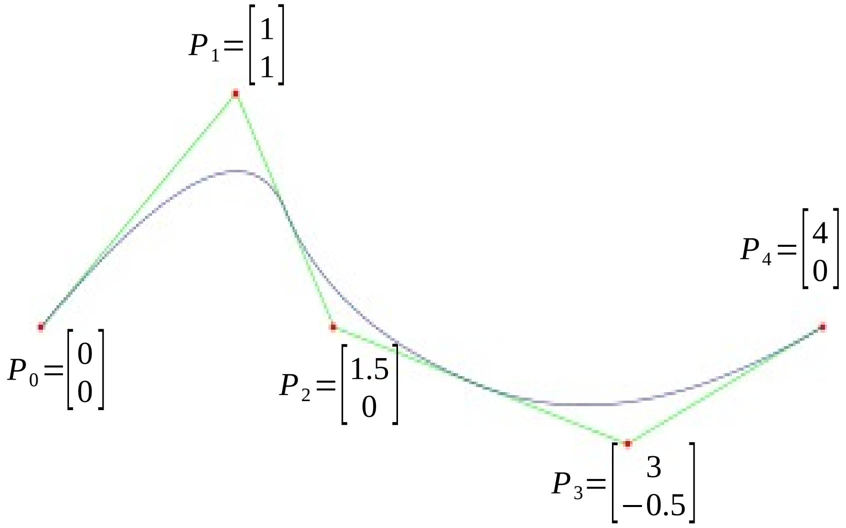


Vetor de Nós: {0,0,1,2,3,4,5,5}

$$P(t)=P_0 N_0^3(t)+P_1 N_1^3(t)+P_2 N_2^3(t)+P_3 N_3^3(t)+P_4 N_4^3(t)$$

Curvas “atachadas”

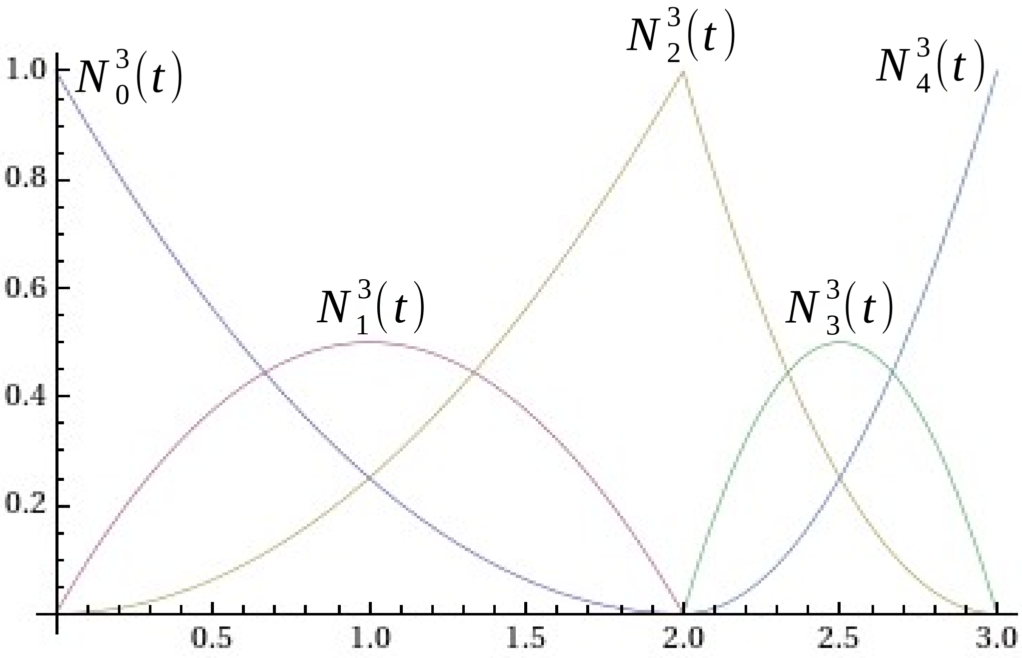


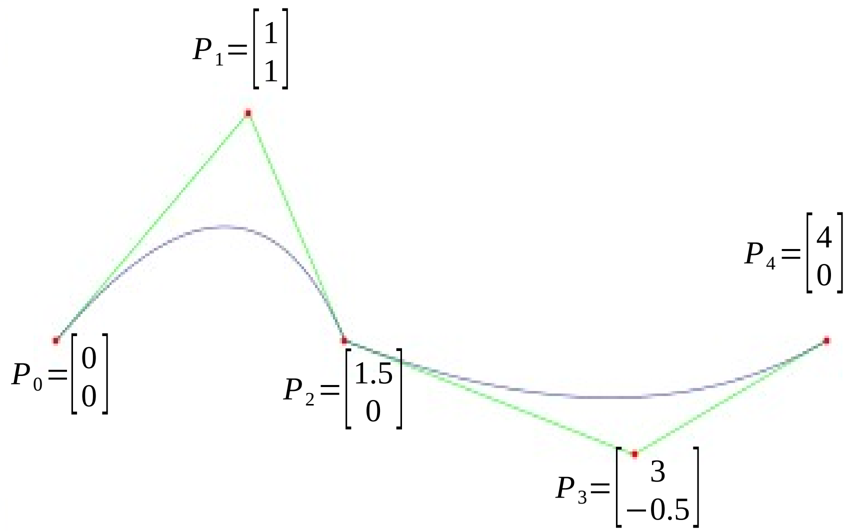


Vetor de Nós: {0,0,0,1,2,3,3,3}

$$P(t)=P_0 N_0^3(t)+P_1 N_1^3(t)+P_2 N_2^3(t)+P_3 N_3^3(t)+P_4 N_4^3(t)$$

Curvas com cúspides

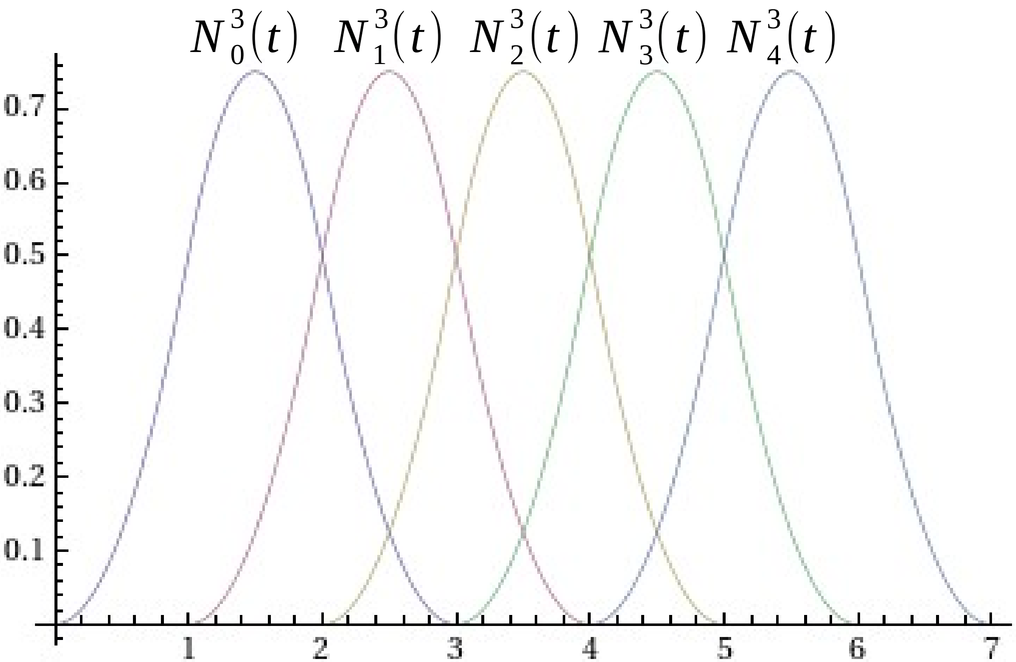


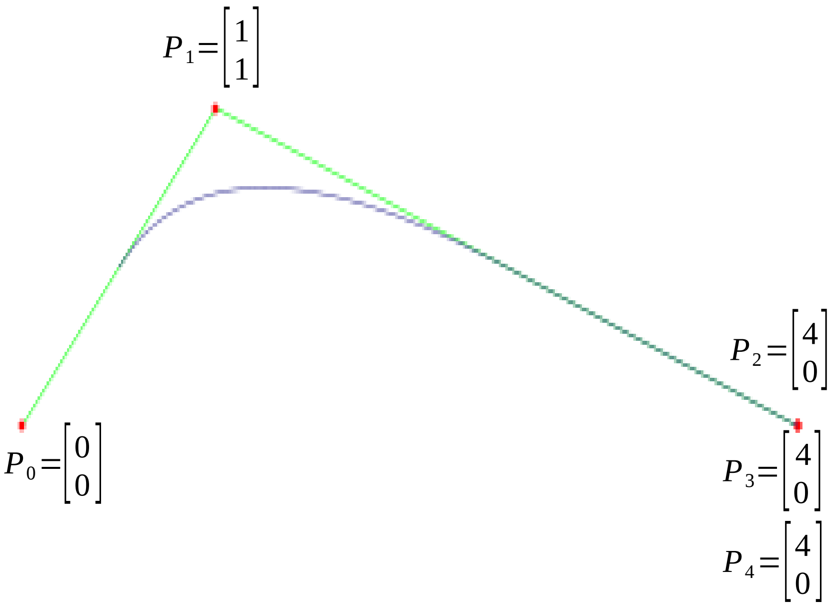


Vetor de Nós: {0,0,0,2,2,3,3,3}

$$P(t)=P_0 N_0^3(t)+P_1 N_1^3(t)+P_2 N_2^3(t)+P_3 N_3^3(t)+P_4 N_4^3(t)$$

Multiplicidade de Pontos de Controle

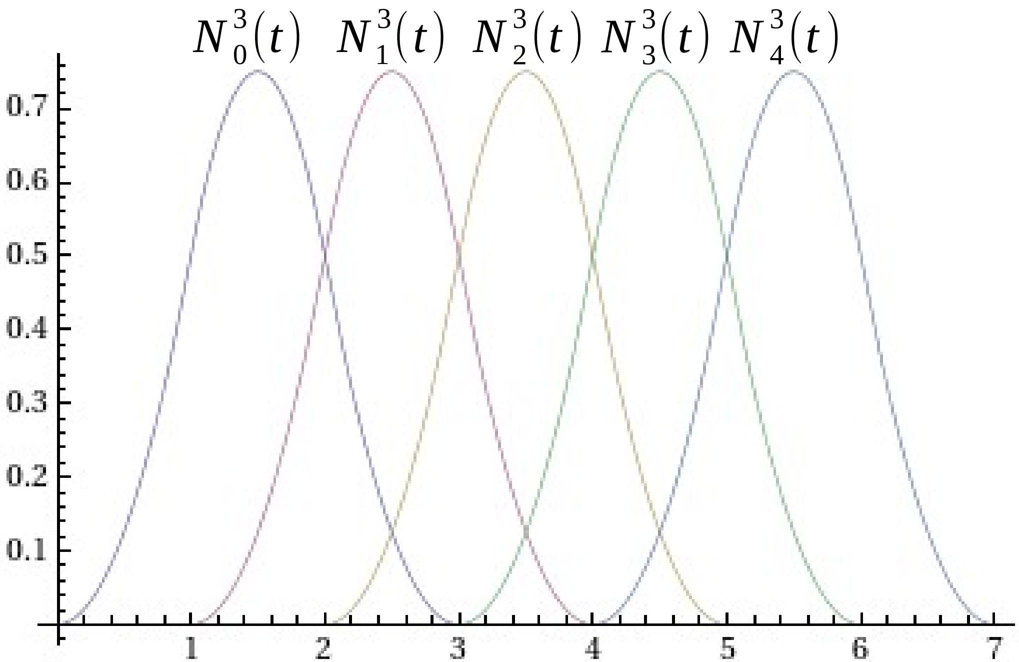


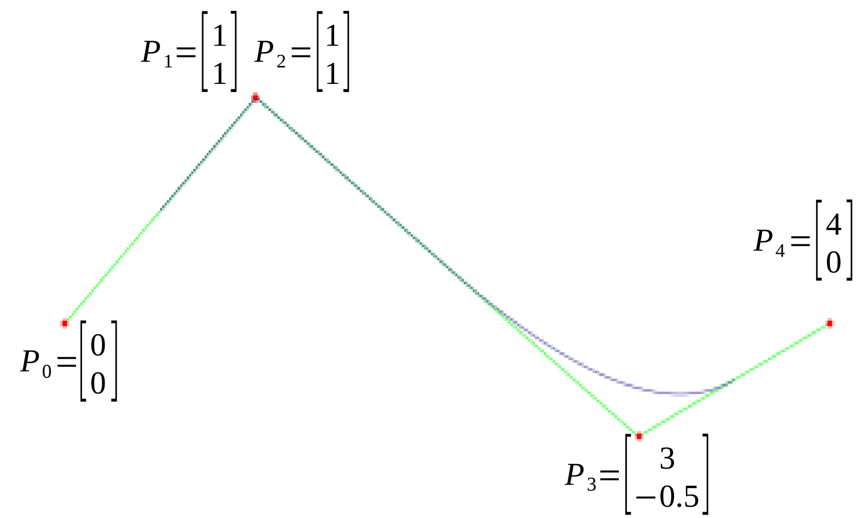


Vetor de Nós: {0,1,2,3,4,5,6,7}

$$P(t)=P_0 N_0^3(t)+P_1 N_1^3(t)+P_2 N_2^3(t)+P_3 N_3^3(t)+P_4 N_4^3(t)$$

Multiplicidade de Pontos de Controle





Vetor de Nós: {0,1,2,3,4,5,6,7}

$$P(t)=P_0 N_0^3(t)+P_1 N_1^3(t)+P_2 N_2^3(t)+P_3 N_3^3(t)+P_4 N_4^3(t)$$

Propriedades de *B-Splines*

- Contidas no fecho convexo do polígono de controle das curvas por parte.
- Possui a propriedade de *variation diminishing*.
- São invariantes sob transformações afins.
- Apresenta a precisão linear.
- Com um vetor de nós de multiplicidade 1, B-splines de grau n tem continuidade C^{n-1}.
- Continuidade em nós de multiplicidade r é C^{n-r}.

NURBS

- Non-Uniform Rational B-Splines

$$P(u)=\frac{\sum_{i=0}^{L}\omega_i d_i N_i^n(u)}{\sum_{i=0}^{n}\omega_i N_i^n(u)}$$