

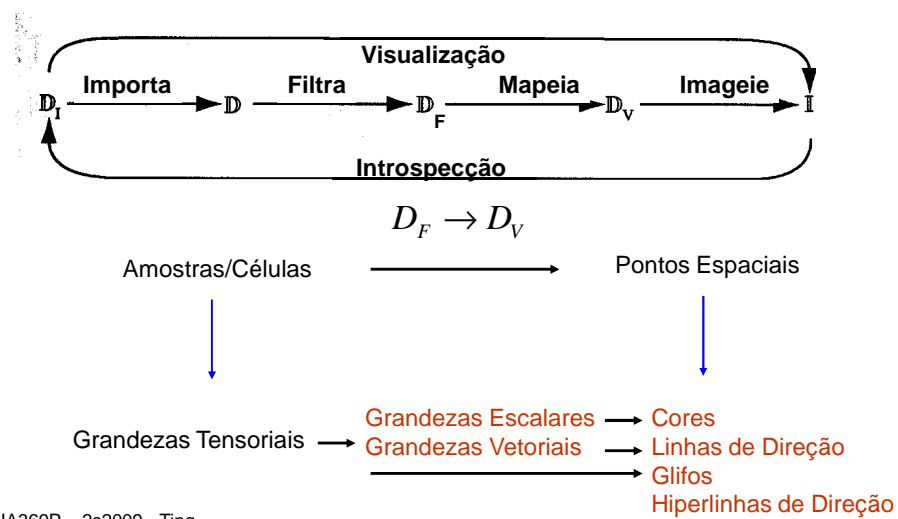
IA369P – Tópicos em Engenharia de Computação VI

Visualização de Informação: Algoritmos

Visualização de Tensores

Capítulo 7 do livro-texto Telea

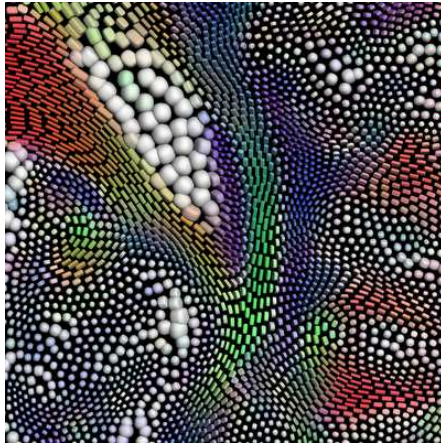
Modelo Conceitual



Tipos de Dados

Tensores

Generalização de “quantidades geométricas” em R^d



Escalares:

$$f(X) \subset R$$

Vetores:

$$f(X) \subset R^d$$

Tensores:

Escalares (tensores de rank 0) + vetores (tensores de rank 1) + outros arranjos matriciais de escalares

Cores

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Difusão de líquido em distintas direções

Tensores Covariantes

Superfície parametrizada em (u,v)

$$ds = \frac{\partial r}{\partial u} du + \frac{\partial r}{\partial v} dv$$

$$E = \frac{\partial r}{\partial u} \frac{\partial r}{\partial u}$$

$$F = \frac{\partial r}{\partial u} \frac{\partial r}{\partial v}$$

$$G = \frac{\partial r}{\partial v} \frac{\partial r}{\partial v}$$

Superfície parametrizada em (u',v')

$$ds = \frac{\partial r}{\partial u'} du' + \frac{\partial r}{\partial v'} dv'$$

$$E' = \frac{\partial r}{\partial u'} \frac{\partial r}{\partial u'}$$

$$F' = \frac{\partial r}{\partial u'} \frac{\partial r}{\partial v'}$$

$$G' = \frac{\partial r}{\partial v'} \frac{\partial r}{\partial v'}$$

$$\begin{bmatrix} E' & F' \\ F' & G' \end{bmatrix} = \begin{bmatrix} \frac{\partial u}{\partial u'} & \frac{\partial u}{\partial v'} \\ \frac{\partial v}{\partial u'} & \frac{\partial v}{\partial v'} \end{bmatrix}^t \begin{bmatrix} E & F \\ F & G \end{bmatrix} \begin{bmatrix} \frac{\partial u}{\partial u'} & \frac{\partial u}{\partial v'} \\ \frac{\partial v}{\partial u'} & \frac{\partial v}{\partial v'} \end{bmatrix}$$

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$$\bar{T}_{ij} = \sum_{j=1}^d \sum_{i=1}^d T_{rs} \frac{\partial x^r}{\partial \bar{x}^i} \frac{\partial x^s}{\partial \bar{x}^j}$$

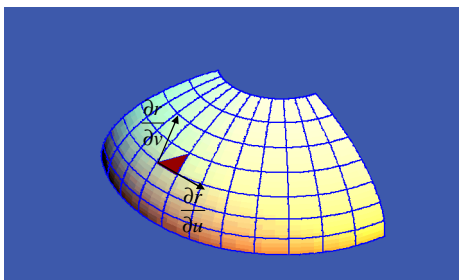
Tensores Contravariantes e Mistos

$$\bar{T}^{ij} = \sum_{j=1}^d \sum_{i=1}^d T^{rs} \frac{\partial \bar{x}^i}{\partial x^r} \frac{\partial \bar{x}^j}{\partial x^s}$$

$$\bar{T}_j^i = \sum_{j=1}^d \sum_{i=1}^d T_s^r \frac{\partial \bar{x}^i}{\partial x^r} \frac{\partial x^s}{\partial \bar{x}^j}$$

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Tensor Métrico



$$ds = \frac{\partial r}{\partial u} du + \frac{\partial r}{\partial v} dv$$

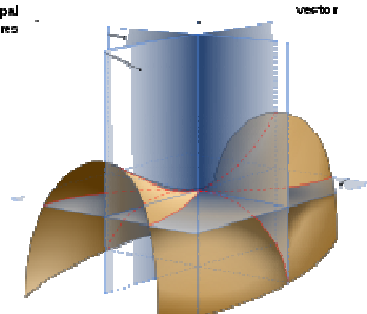
$$ds \cdot ds = \left(\frac{\partial r}{\partial u} du + \frac{\partial r}{\partial v} dv \right) \cdot \left(\frac{\partial r}{\partial u} du + \frac{\partial r}{\partial v} dv \right)$$

$$ds^2 = \underbrace{\frac{\partial r}{\partial u} \frac{\partial r}{\partial u}}_E du^2 + 2 \underbrace{\frac{\partial r}{\partial u} \frac{\partial r}{\partial v}}_F dudv + \underbrace{\frac{\partial r}{\partial v} \frac{\partial r}{\partial v}}_G dv^2$$

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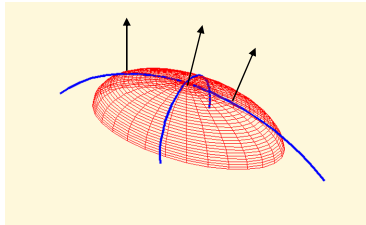
Tensor de Curvatura

planes of principal curvatures



normal vector

tangent plane



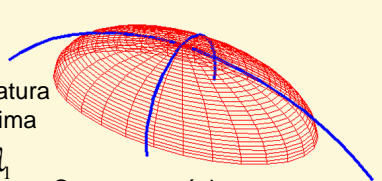
$$-ds \cdot dn = -\left(\frac{\partial r}{\partial u} du + \frac{\partial r}{\partial v} dv\right) \cdot \left(\frac{\partial n}{\partial u} du + \frac{\partial n}{\partial v} dv\right)$$

$$-ds \cdot dn = -\frac{\partial r}{\partial u} \frac{\partial n}{\partial u} du^2 - \frac{\partial r}{\partial u} \frac{\partial n}{\partial v} dudv - \frac{\partial n}{\partial u} \frac{\partial r}{\partial v} dudv - \frac{\partial r}{\partial v} \frac{\partial n}{\partial v} dv^2$$

$$-ds \cdot dn = -\underbrace{\frac{\partial r}{\partial u} \frac{\partial n}{\partial u}}_e du^2 - 2 \underbrace{\frac{\partial r}{\partial u} \frac{\partial n}{\partial v}}_f dudv - \underbrace{\frac{\partial r}{\partial v} \frac{\partial n}{\partial v}}_g dv^2$$

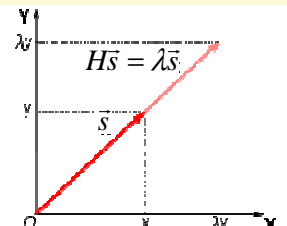
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Curvaturas



Curvatura mínima λ_1

Curvatura máxima λ_2



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$$\begin{bmatrix} h_{11} & h_{21} \\ h_{12} & h_{22} \end{bmatrix} \begin{bmatrix} E & F \\ F & G \end{bmatrix} = - \begin{bmatrix} e & f \\ f & g \end{bmatrix}$$

$$H\vec{s} = \lambda\vec{s}$$

$$\det\left(\begin{bmatrix} h_{11} & h_{21} \\ h_{12} & h_{22} \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right) = 0$$

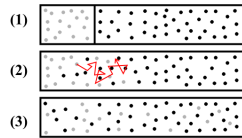
$$(h_{11} - \lambda)(h_{22} - \lambda) - h_{12}h_{21} = 0$$

Soluções λ_1, λ_2 : **autovalores**

$\lambda_i \rightarrow$ um **autovetor** \vec{s}_i

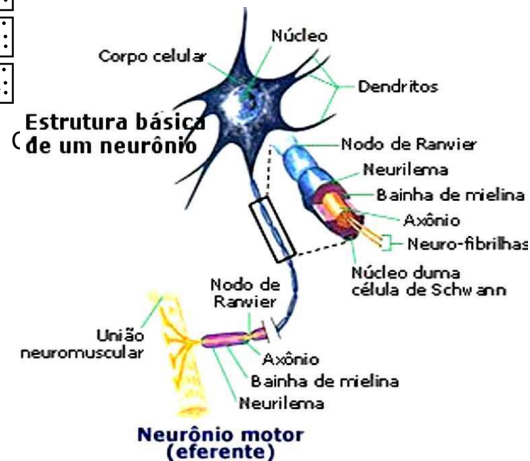
Tensor de Difusão

Difusão é o deslocamento de soluto por diferencial na concentração dos meios.



Feixe de fibras nervosas na **substância branca** do tecido nervoso dá uma direção preferencial para o movimento microscópico das moléculas de água sob variações de campos magnéticos. **Coefficientes de difusão H anisotrópicos:**

- Alta mobilidade ao longo das fibras;
- Baixa mobilidade na direção perpendicular.



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Tensor de Difusão

- **Coefficientes de difusividade** permite estimar a direção das fibras nervosas.
- Estes coeficientes são quantidades que não dependem do referencial.

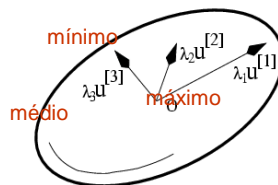
$$H = \begin{bmatrix} h_{xx} & h_{xy} & h_{xz} \\ h_{yx} & h_{yy} & h_{yz} \\ h_{zx} & h_{zy} & h_{zz} \end{bmatrix}$$

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Tensor de Difusão

$$\lambda_1 \geq \lambda_2 \geq \lambda_3 \rightarrow \text{difusividade}$$

$$H = \begin{bmatrix} h_{xx} & h_{xy} & h_{xz} \\ h_{yx} & h_{yy} & h_{yz} \\ h_{zx} & h_{zy} & h_{zz} \end{bmatrix}$$



3 x 3 diffusion tensor

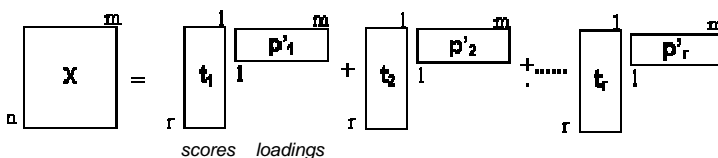
$$\det \left(\begin{bmatrix} h_{xx} & h_{xy} & h_{xz} \\ h_{yx} & h_{yy} & h_{yz} \\ h_{zx} & h_{zy} & h_{zz} \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) = 0$$

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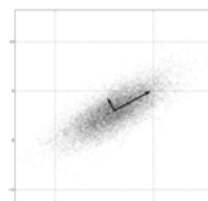
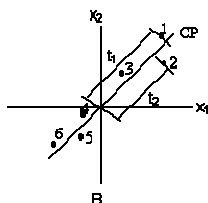
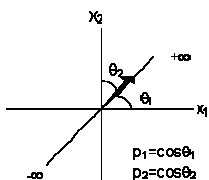
Análise de Componentes Principais

- Decomposição de uma matriz de dados X de posto r como soma de matrizes de posto igual a 1

$$X = t_1 p'_1 + t_2 p'_2 + \dots + t_h p'_h$$



Scores: projeções das amostras na direção da componente principal
Loadings: cossenos dos ângulos formados entre a componente principal e cada variável



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Exercícios

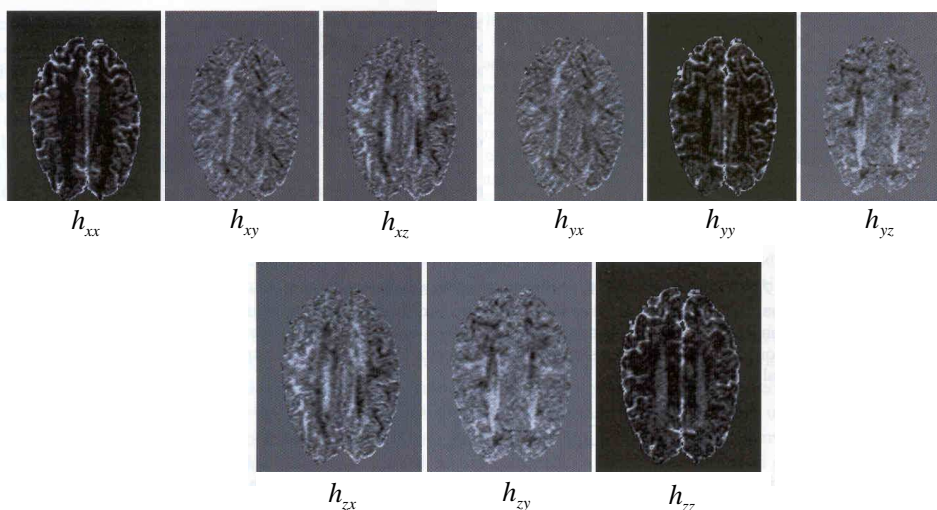
1. Determina os autovalores e autovetores das seguintes matrizes

$$\begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

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Redução em Escalares



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Tensor de Difusão

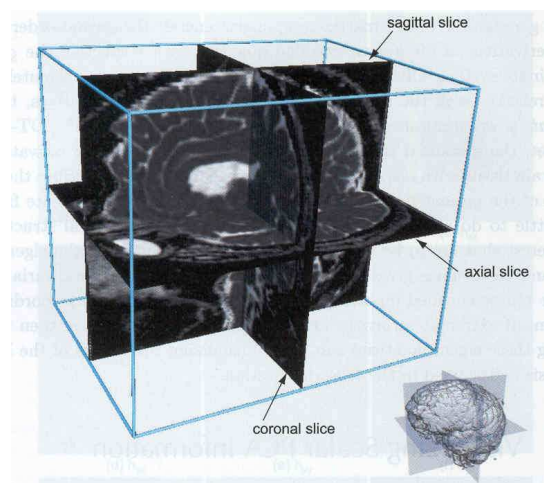
- Difusividade radial: $(\lambda_2 + \lambda_3)/2$
- Difusividade média: $\mu = (\lambda_1 + \lambda_2 + \lambda_3)/3$
- Anisotropia Fracional:

$$FA = \sqrt{\frac{3}{2}} \frac{\sqrt{\sum_{i=1}^3 (\lambda_i - \mu)^2}}{\sqrt{\lambda_1^2 + \lambda_2^2 + \lambda_3^2}}$$
 - Caso linear $\lambda_1 \gg \lambda_2 \approx \lambda_3$ $c_l = \frac{\lambda_1 - \lambda_2}{(\lambda_1 + \lambda_2 + \lambda_3)}$
 - Caso planar $\lambda_1 \approx \lambda_2 \gg \lambda_3$ $c_p = \frac{2(\lambda_2 - \lambda_3)}{(\lambda_1 + \lambda_2 + \lambda_3)}$
 - Caso esférico $\lambda_1 \approx \lambda_2 \approx \lambda_3$ $c_s = \frac{3\lambda_3}{(\lambda_1 + \lambda_2 + \lambda_3)}$
- Anisotropia Relativa:

$$RA = \sqrt{\frac{3}{2}} \frac{\sqrt{\sum_{i=1}^3 (\lambda_i - \mu)^2}}{\lambda_1 + \lambda_2 + \lambda_3}$$

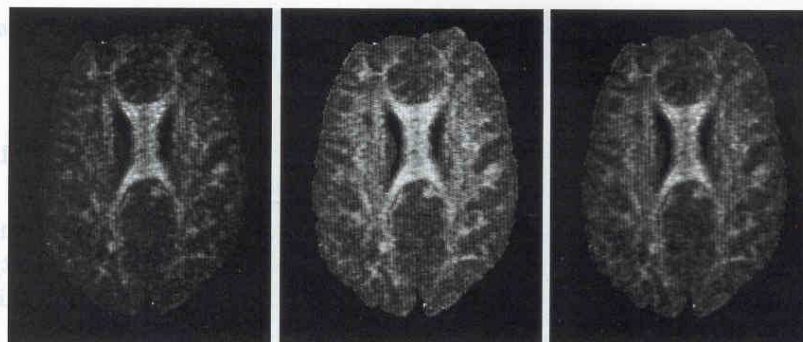
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Redução em Escalares Difusividade Média



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Redução em Escalares Medidas de Anisotropia

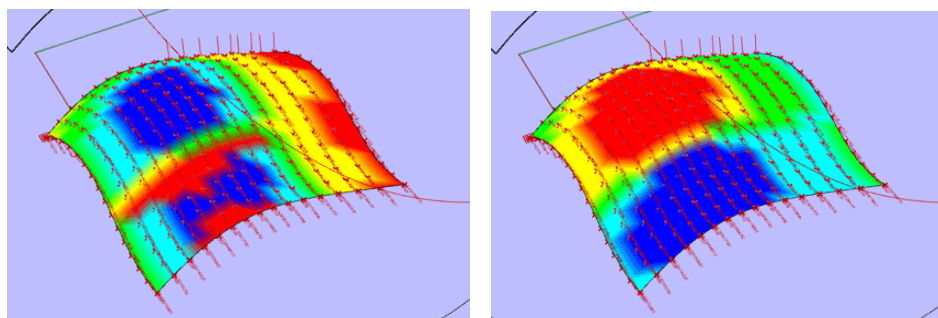
(a) c_1 linear estimator

(b) fractional anisotropy

(c) relative anisotropy

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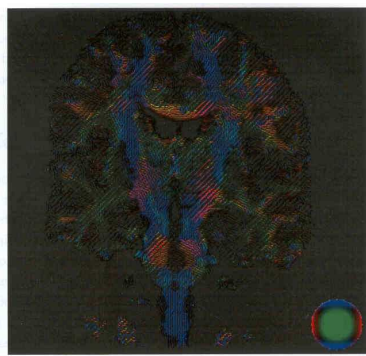
Redução em Escalares Curvaturas Gaussiana e Média

Curvatura Gaussiana ($K=k_1k_2$)Curvatura Média ($H=(k_1+k_2)/2$)

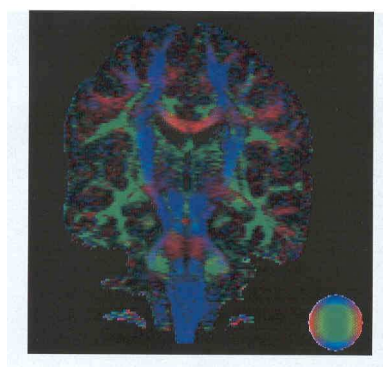
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Redução em Vetores

Direção de λ_1



Mapear em glifos "coloridos"

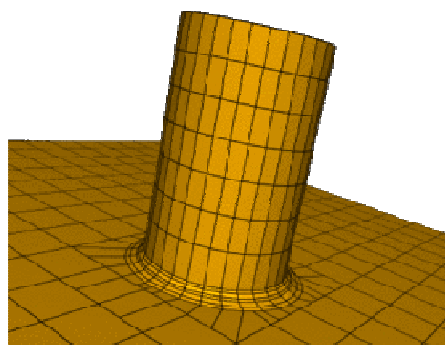
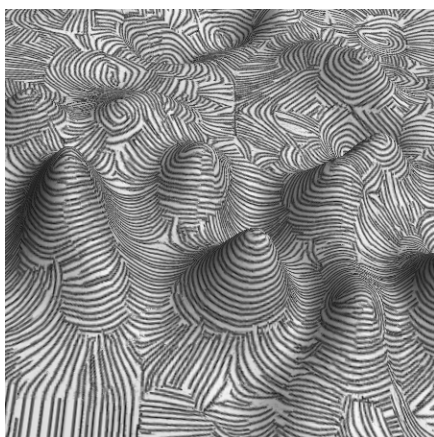


x-->R; y → G; z → B

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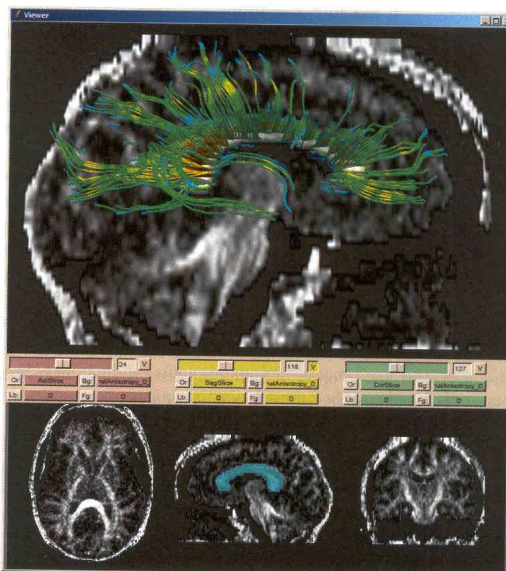
Redução em Vetores

Linhas de Direção



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Redução em Vetores Linhas de Direção

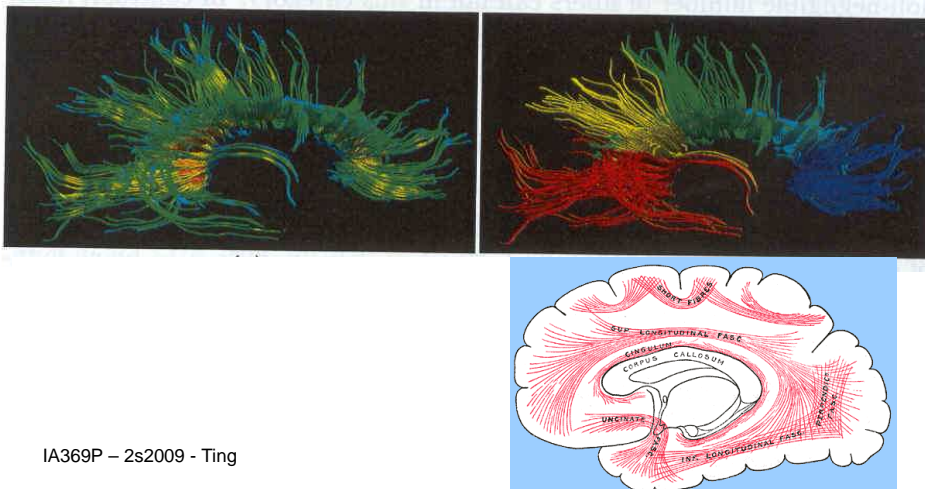


Interação para
escolha de
semente

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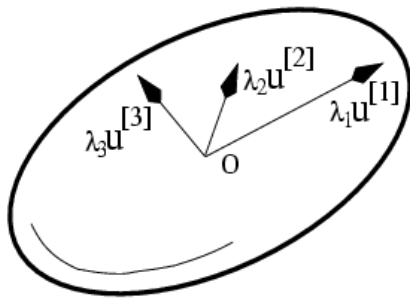
Redução em Vetores Linhas de Direção

Agrupamento de fibras paralelas

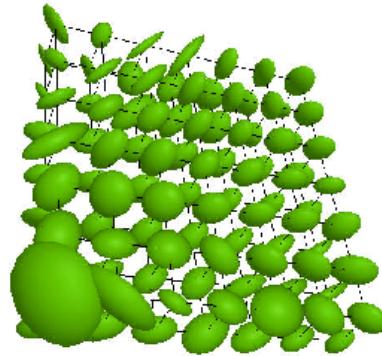


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Mapeamento em Glifos



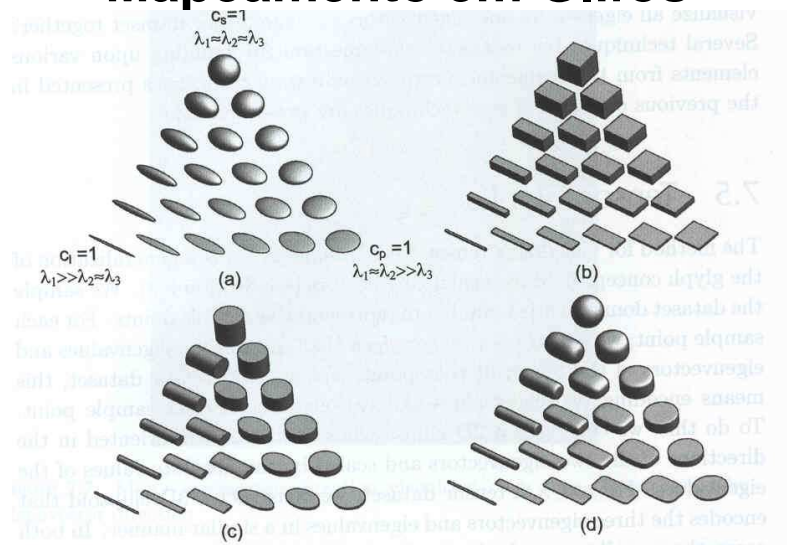
3 × 3 diffusion tensor



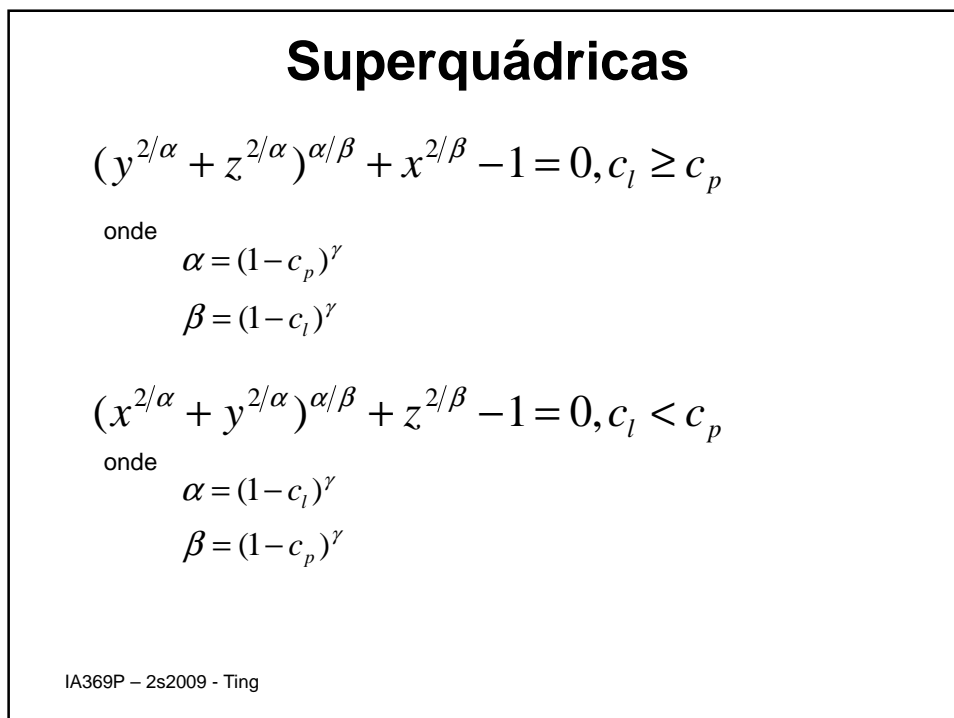
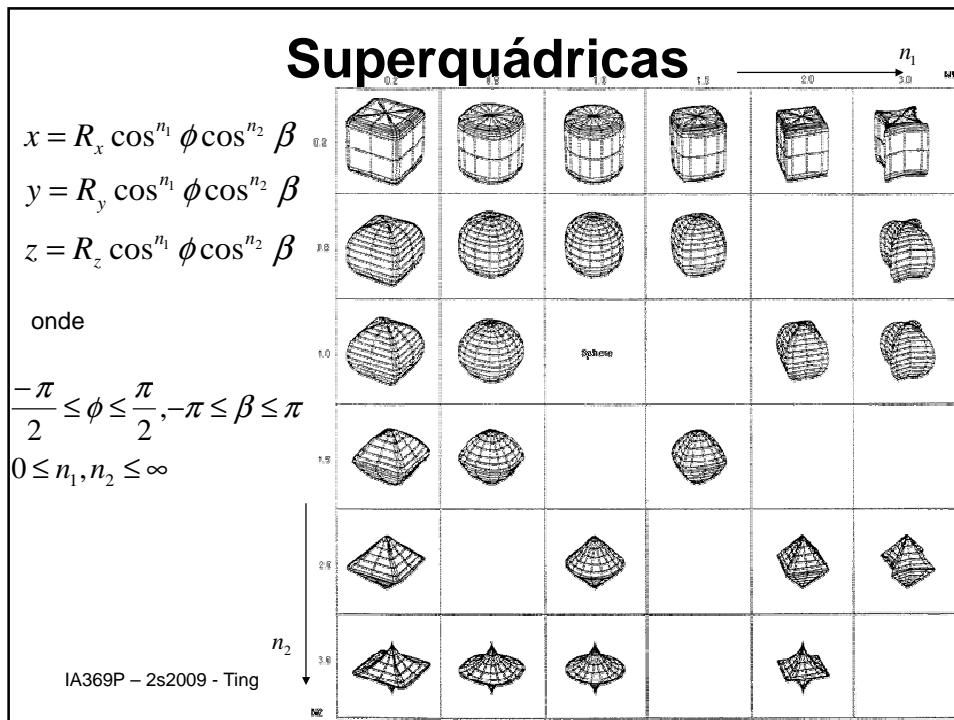
3D Diffusion tensor field

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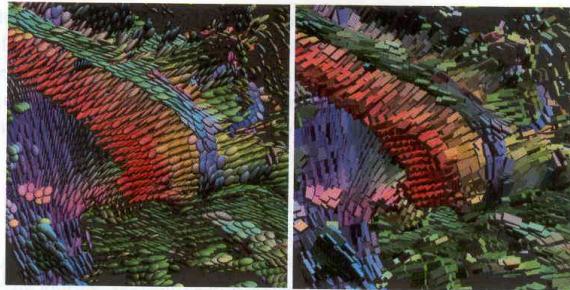
Mapeamento em Glifos



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Mapeamento em Glifos

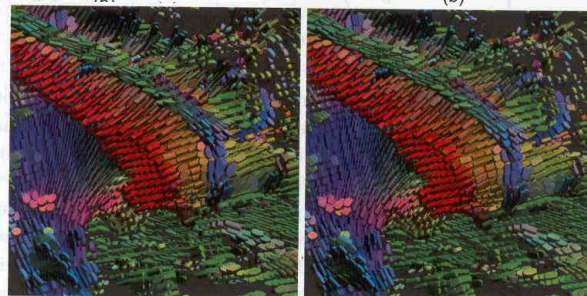


Cubóide

Elipsóide

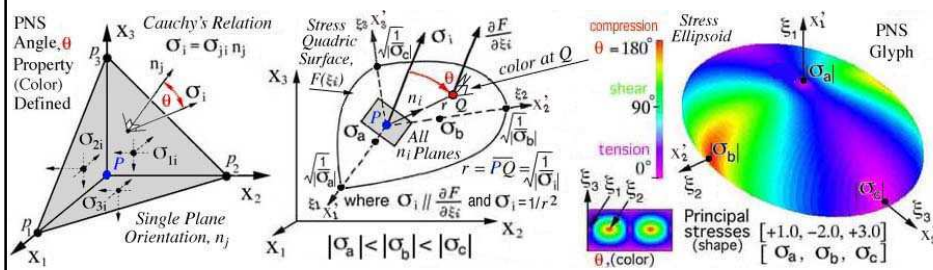
Cilíndrico

Superquádricas



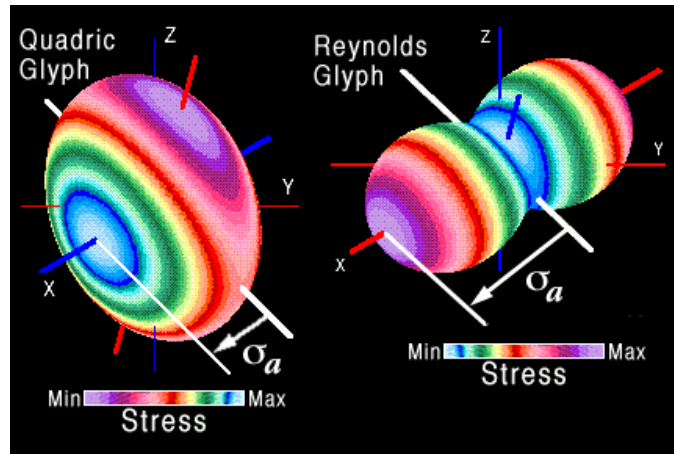
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Mapeamento em Glifos Tensor de Estresse (Mecânico)



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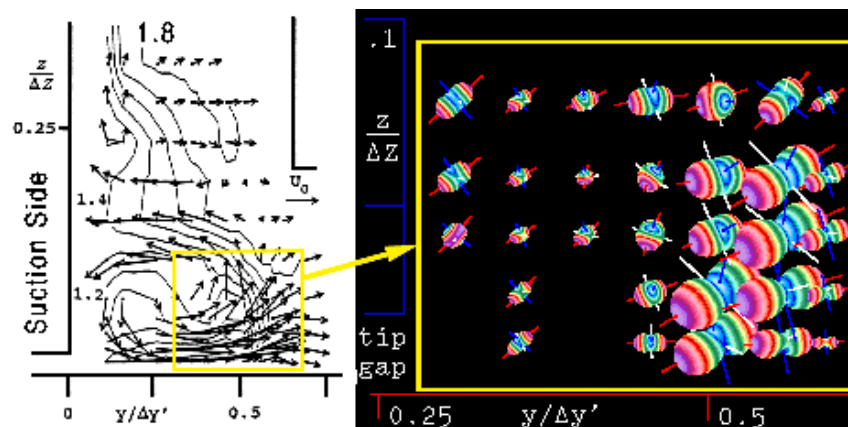
Mapeamento em Glifos Tensor de Estresse (Mecânico)



Fonte: http://www.sv.vt.edu/NCSA_WkShp/kriz/WkShp_kriz.html

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Mapeamento em Glifos Tensor de Estresse (Mecânico)



Problemas: resolução espacial limitada, densidade

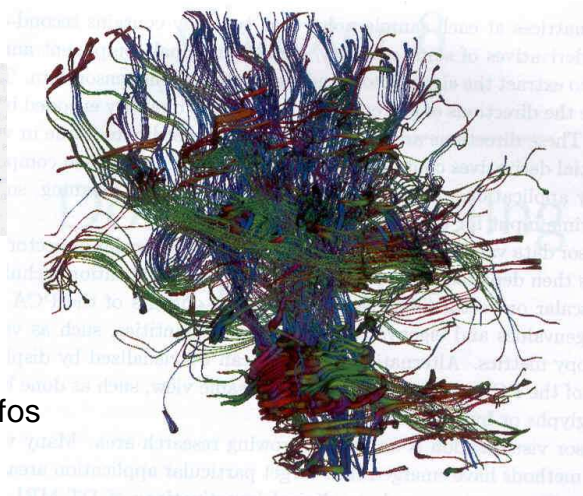
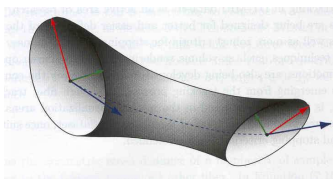
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Exercícios

1. Como se reduz um tensor em um escalar para utilizar a técnica de mapeamento deste último em atributos gráficos? Qual é o problema desta técnica apesar da simplicidade?
2. Como se reduz um tensor em um vetor para utilizar a técnica deste último em atributos gráficos? Quais são os problemas apresentados nesta redução?
3. Compare os quatro tipos de glifos apresentados para visualizar os tensores em termos de prover valor e direção das grandezas de interesse.
4. Compare o mapeamento em linhas de direção como o mapeamento em glifos em termos de prover informação direcional.

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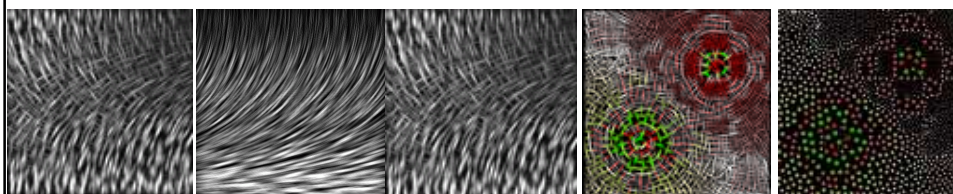
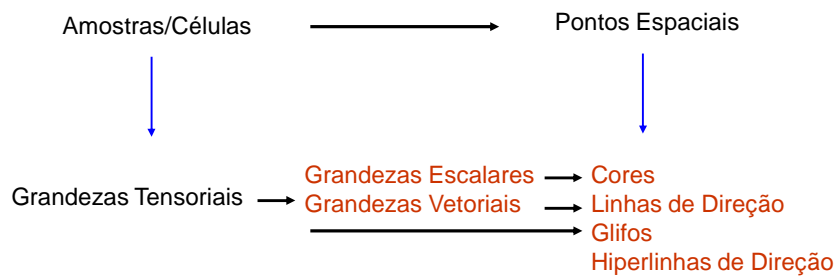
Mapeamento em Hiperlinhas



Linhas de direção + glifos

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Visualização de Tensores



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Fonte:

<http://graphics.idav.ucdavis.edu/~lfeng/research/tensor/index.html>