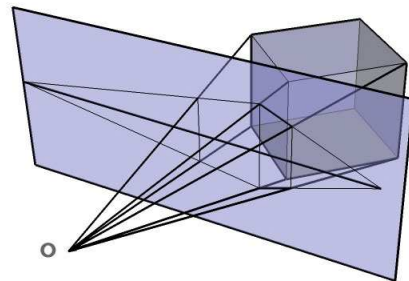
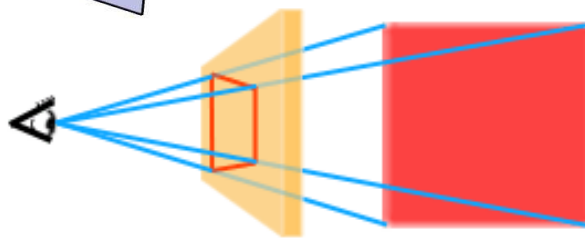


# Transformação de Visualização

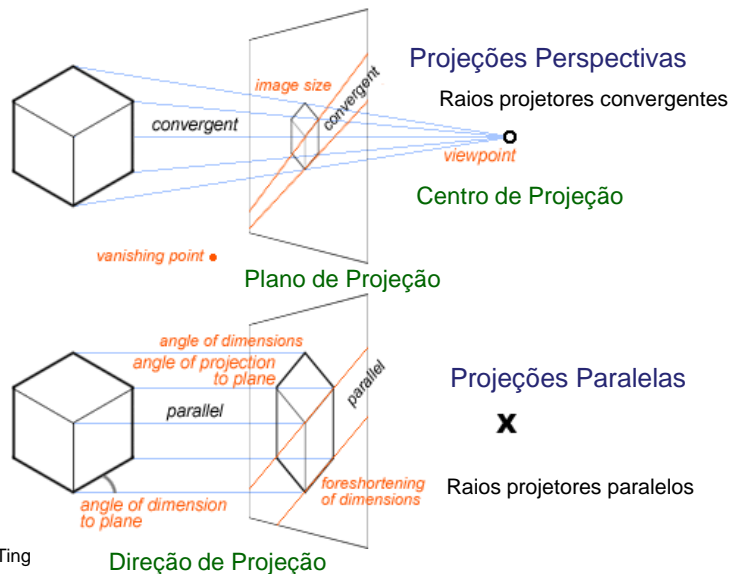


Projetar modelos geométricos 3D numa imagem 2D, exibível em dispositivos de saída 2D, a fim de produzir imagens sintéticas.



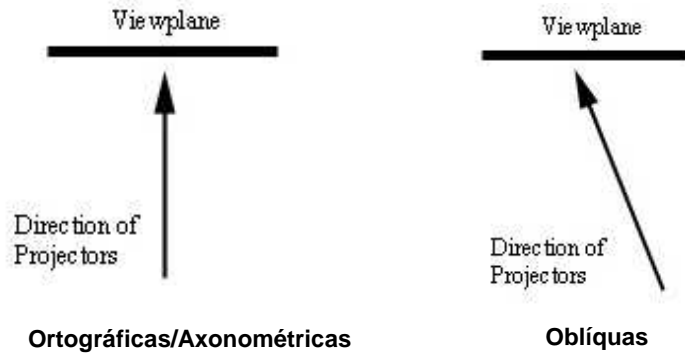
EA978 – 2s2008 - Ting

# Uma Visão Clássica



EA978 – 2s2008 - Ting

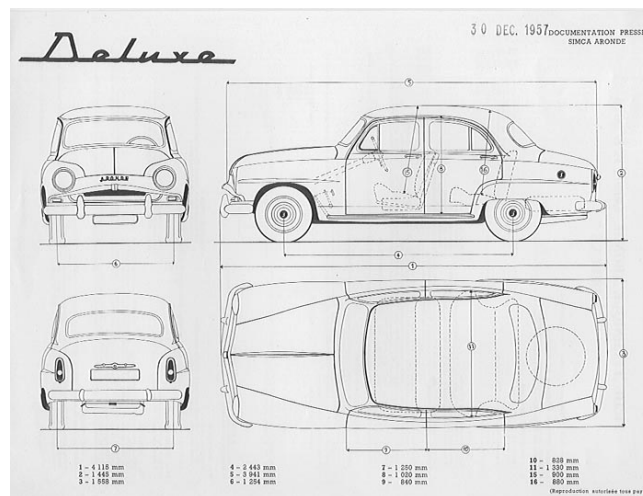
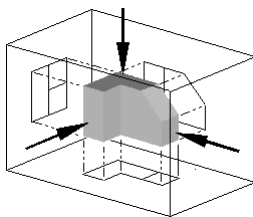
## Uma Visão Clássica Projeções Paralelas



EA978 – 2s2008 - Ting

## Projeções Ortográficas

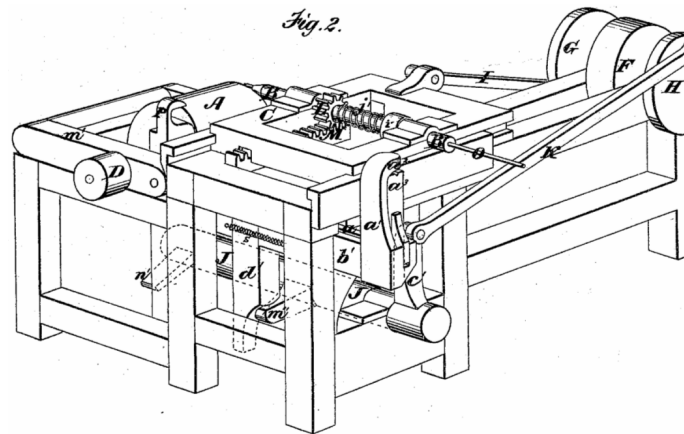
Desenhos técnicos: preserva a relação das medidas



EA978 – 2s2008 - Ting

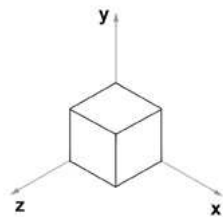
## Projeções Axonométricas

Provêm melhor percepção de profundidade

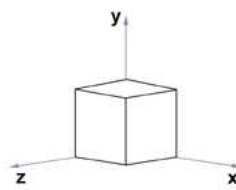


EA978 – 2s2008 - Ting

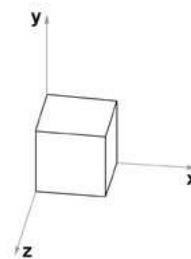
## Projeções Axonométricas



Isométricas  
(mesmo fator de  
redução em 3  
direções)



Dimétricas  
(mesmo fator de  
redução em 2  
direções)



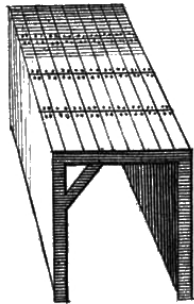
Trimétricas

EA978 – 2s2008 - Ting

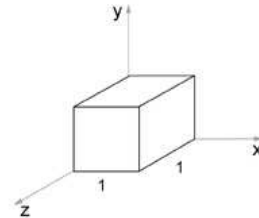
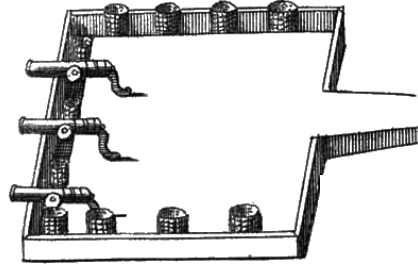
## Projeções Oblíquas Cavalier

“Vista Aérea”: ângulo 45°

*Gallery*



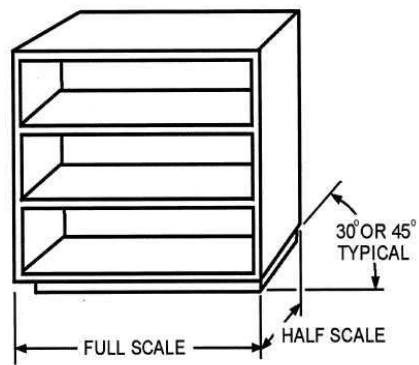
*Battery*



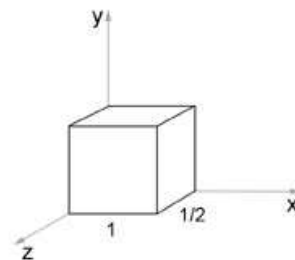
EA978 – 2s2008 - Ting

## Projeções Oblíquas Cabinet

Vista “inclinada” de estantes: ângulo 63.4°



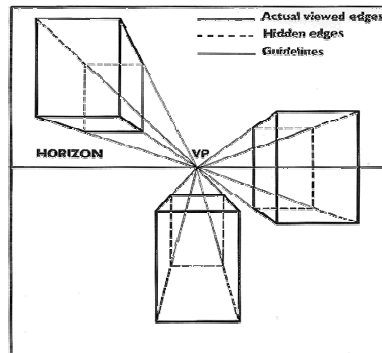
DMV2Ch06f06



EA978 – 2s2008 - Ting

# Projeções Perspectivas

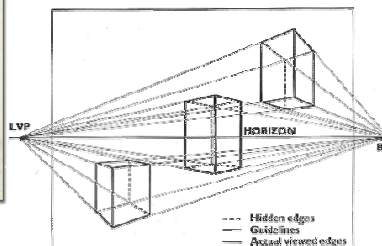
Um ponto de fuga



EA978 – 2s2008 - Ting

# Projeções Perspectivas

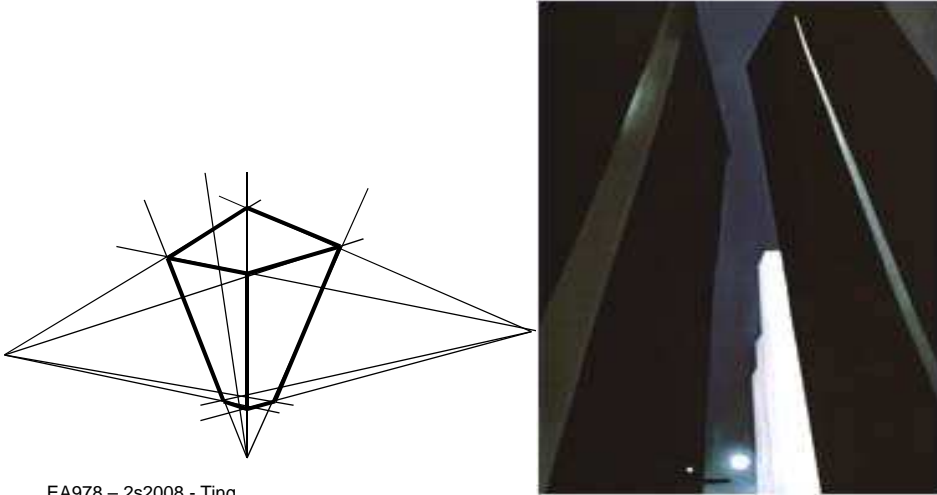
Dois pontos de fuga



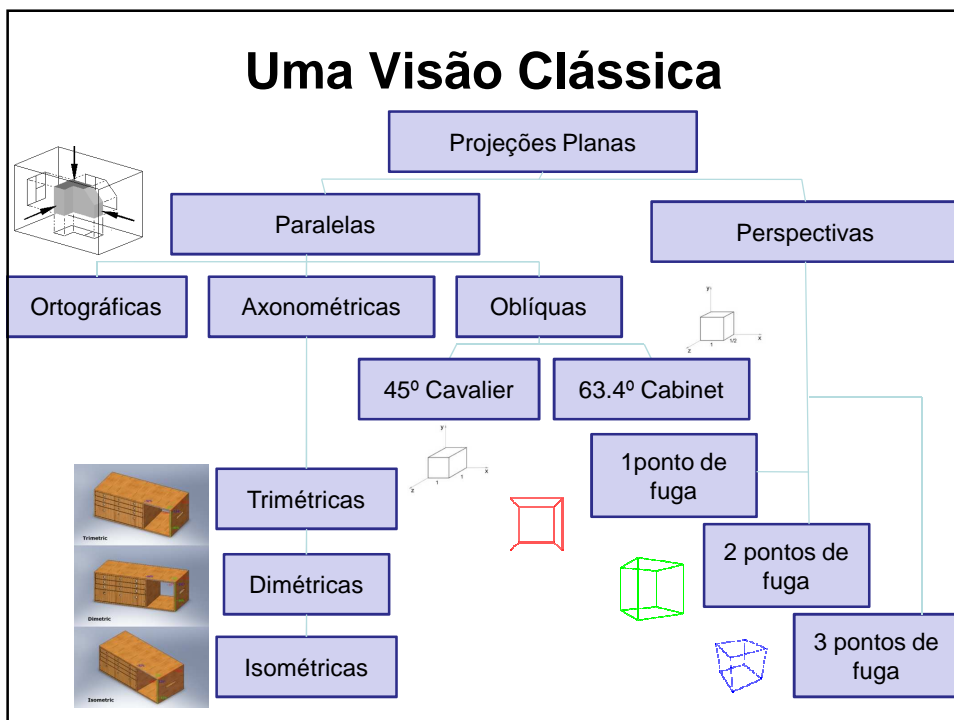
EA978 – 2s2008 - Ting

## Projeções Perspectivas

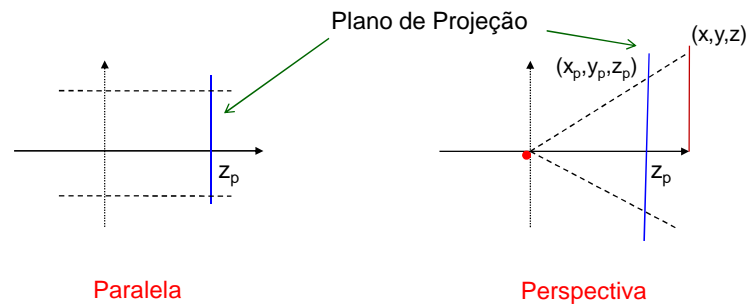
Três pontos de fuga



EA978 – 2s2008 - Ting



## Uma Visão Computacional: Casos Triviais



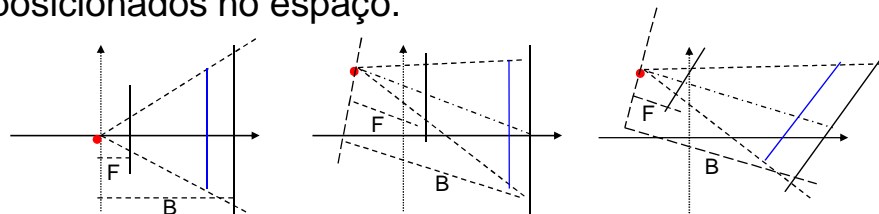
Basta substituir a coordenada  $z$  de cada ponto por  $z_p$

Coordenadas  $x$ ,  $x_p$ ,  $y$  e  $y_p$  coincidem com as "alturas" dos triângulos e as coordenadas  $z$  com as "bases" dos triângulos. Problema se reduz a obter relação entre estas coordenadas pela semelhança de triângulos

EA978 – 2s2008 - Ting

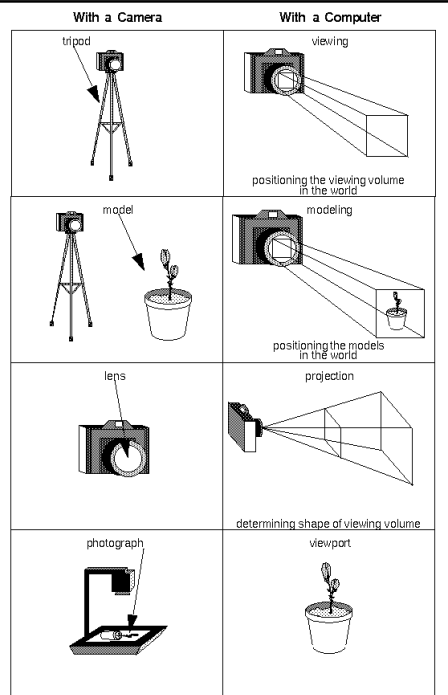
## Uma Visão Computacional: Diversidade de Casos

- Plano de projeção tem o vetor normal na direção do eixo  $z$  e o centro de projeção sobre o eixo  $z$ .
- Plano de projeção tem o vetor normal na direção do eixo  $z$  e o centro de projeção arbitrariamente posicionado.
- Tanto o plano quanto o centro são arbitrariamente posicionados no espaço.



EA978 – 2s2008 - Ting

# Uma Visão Computacional dividir para conquistar

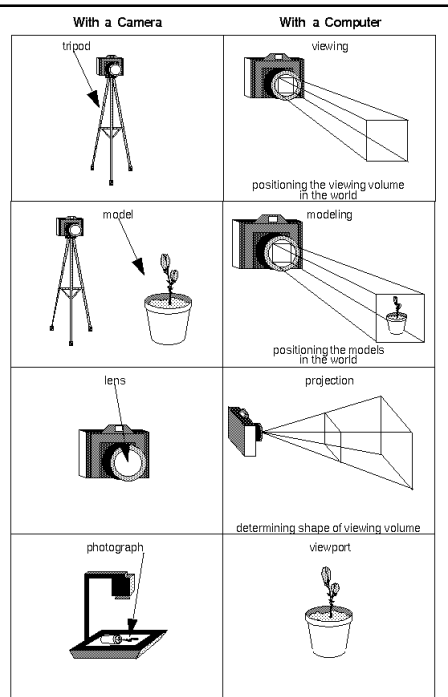


# Distintos Espaços

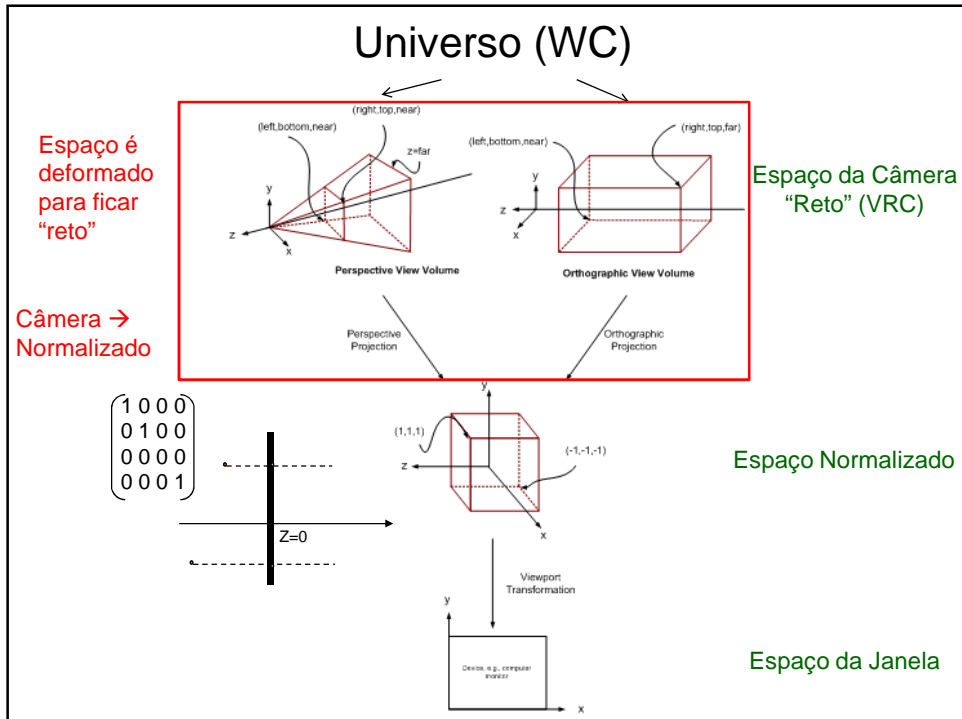
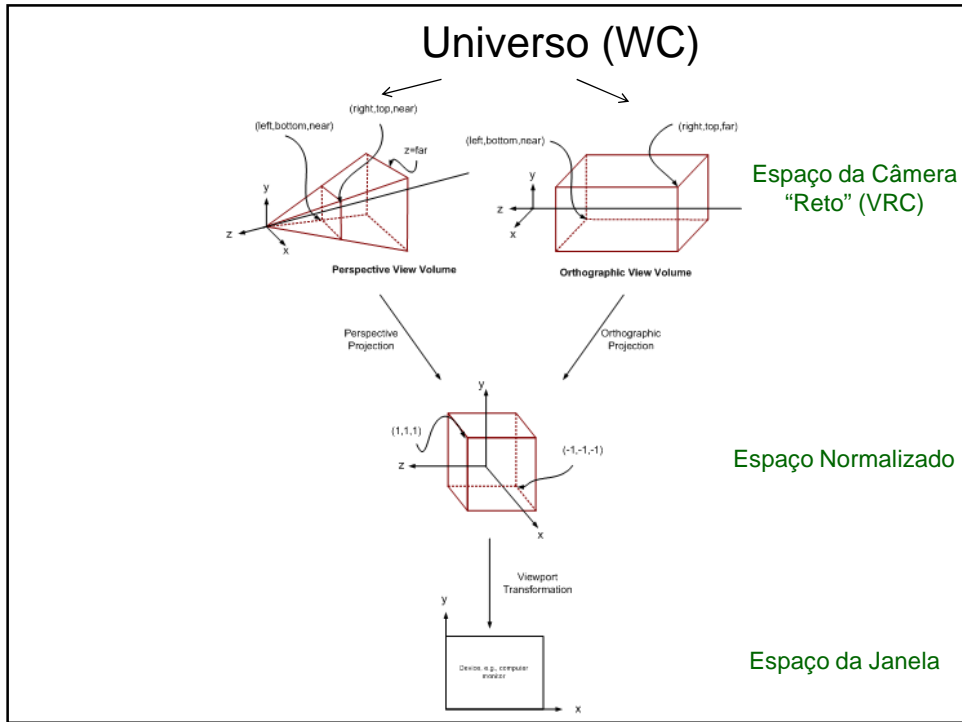
Referencial da Câmera

Referencial Normalizado

Referencial da Janela

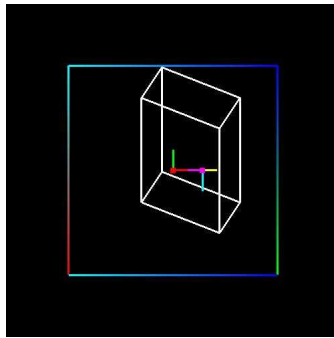




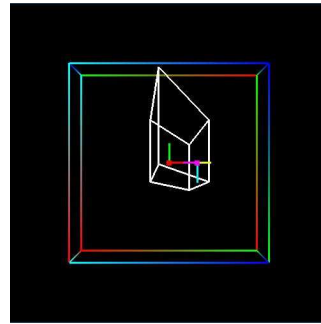


## Projeção Passo a Passo

Nas próximas transparências será ilustrado cada passo com as imagens do programa [projecao.c](#). O programa gera a projeção paralela e perspectiva de um cubo (em branco). São mostrados os referenciais WC (magenta,ciano,amarelo) e VRC (vermelho, verde,azul). O volume de visão é desenhado por linhas coloridas.



Paralela

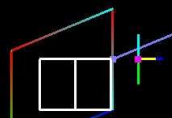


Perspectiva

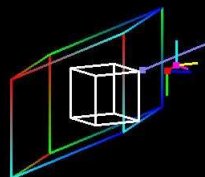
EA978 – 2s2008 - Ting

## WC (Universo)

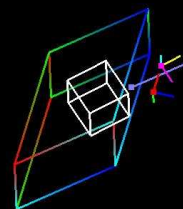
Observe que o referencial WC está centrado no centro da janela



Observe que o volume de visão é um paralelepípedo



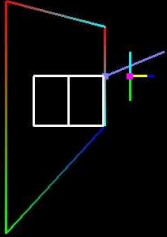
Observe que os referenciais WC e VRC são distintos



EA978 – 2s2008 - Ting

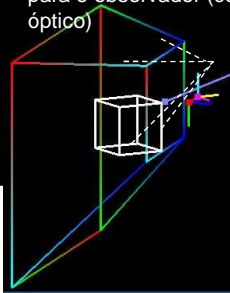
## WC (Universo)

Observe que o referencial WC está no centro da janela

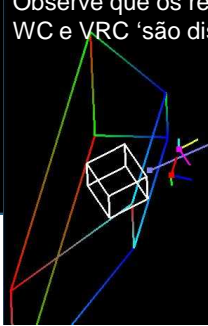


Projeção Perspectiva  
(Várias Vistas)

Observe que o volume de visão é agora um trapezóide com as arestas convergindo para o observador (centro óptico)



Observe que os referenciais WC e VRC 'são distintos



EA978 – 2s2008 - Ting

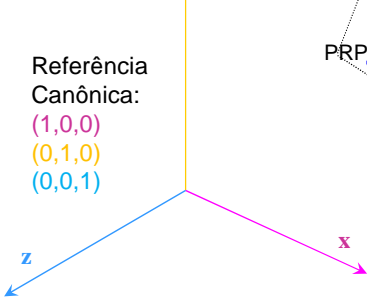
## WC → VRC

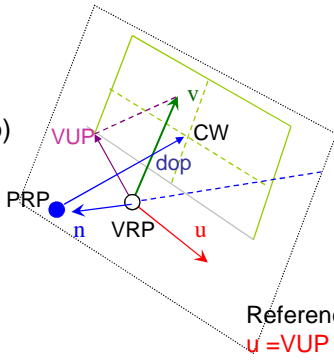
### Referencial da Câmera

VUP (up): view up vector  
 VRP: view reference point  
 VPN (n): view plane normal  
 PRP: eye  
 CW: centro da janela  
 dop: CW-PRP (direção de projeção)

$P' = B_{WC} B_{VRC}^{-1} P$

Referência Canônica:  
 $(1,0,0)$   
 $(0,1,0)$   
 $(0,0,1)$





Referencial da Câmera:  
 $u = \frac{VUP \times n}{|VUP \times n|}$   
 $v = \frac{n \times u}{|n \times u|}$   
 $n = \frac{VPN}{|VPN|}$

EA978 – 2s2008 - Ting

## WC → VRC

(1) VRP na origem

$$Tr = \begin{bmatrix} 1 & 0 & 0 & -VRP_x \\ 0 & 1 & 0 & -VRP_y \\ 0 & 0 & 1 & -VRP_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(2) (u,v,n) em vetores base ortonormais

$$\vec{n} = \frac{\vec{VPN}}{|\vec{VPN}|}$$

$$\vec{u} = \frac{\vec{VUP} \times \vec{n}}{|\vec{VUP} \times \vec{n}|}$$

$$\vec{v} = \vec{n} \times \vec{u}$$

$$R = \begin{bmatrix} u_x & u_y & u_z & 0 \\ v_x & v_y & v_z & 0 \\ n_x & n_y & n_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

EA978 – 2s2008 - Ting

## VRC

Projeção Paralela  
(Várias Vistas)

Observe que a origem do referencial VRC ficou no centro da janela

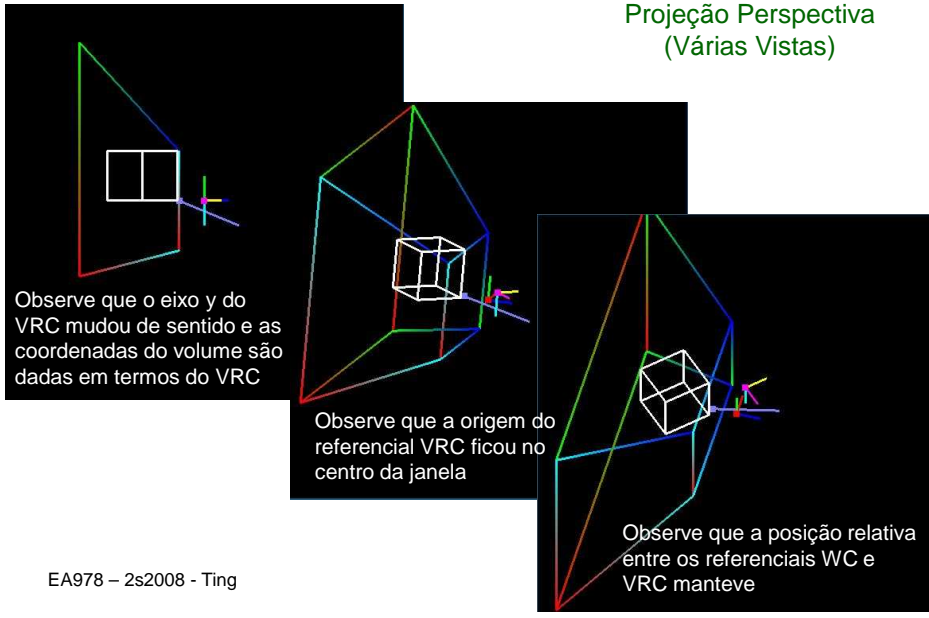
Observe que o eixo y do VRC mudou de sentido e as coordenadas da cena são dadas em termos do VRC

Observe que a posição relativa entre os referenciais WC e VRC manteve

EA978 – 2s2008 - Ting

## VRC

Projeção Perspectiva  
(Várias Vistas)



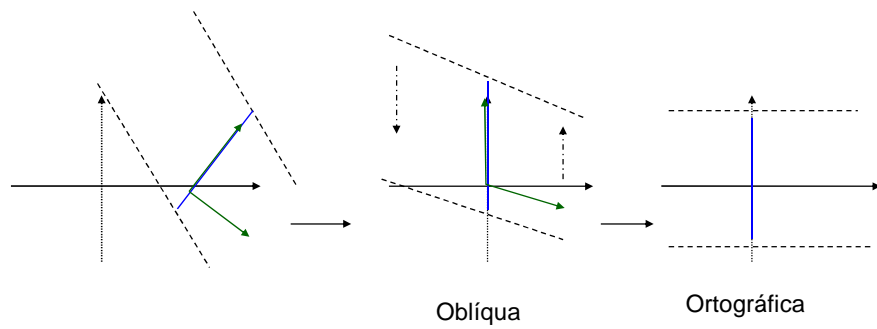
## PRP na origem de VRC

$$PRP' = R \ Tr \ PRP$$

$$Tr = \begin{bmatrix} 1 & 0 & 0 & -PRP_x \\ 0 & 1 & 0 & -PRP_y \\ 0 & 0 & 1 & -PRP_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

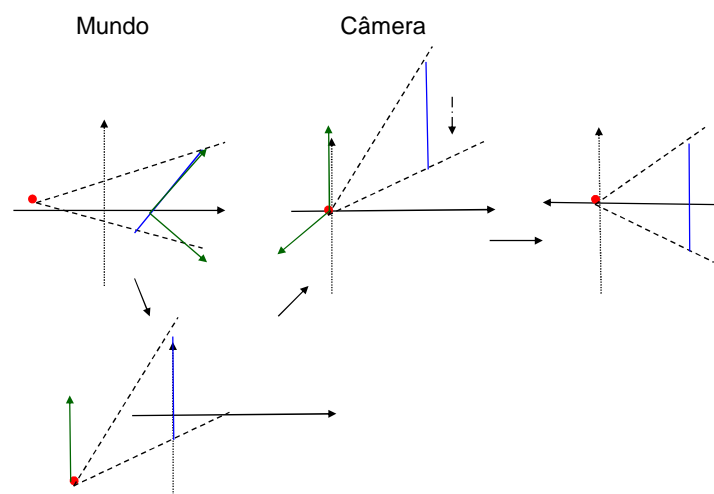
EA978 – 2s2008 - Ting

## Deformação do Volume Paralelo em VRC Cisalhamento



EA978 – 2s2008 - Ting

## Deformação do Volume Perspectivo



EA978 – 2s2008 - Ting

## Deformação do Volume em VRC Cisalhamento

$$\begin{bmatrix} 1 & 0 & -dop_x & 0 \\ 0 & 1 & -dop_y & 0 \\ 0 & 0 & -dop_z & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$S_h = \begin{bmatrix} 1 & 0 & -(dop_x/dop_z) & 0 \\ 0 & 1 & -(dop_y/dop_z) & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

VRP': view reference point in VRC  
 PRP': projection reference point in VRC  
 dop: direction of projection  
 CW: center of window

EA978 – 2s2008 - Ting

## VRC Cisalhado

Projeção Paralela  
(Várias Vistas)

Observe que dop está na mesma direção do eixo z do VRC

Observe que PRP está na origem do VRC

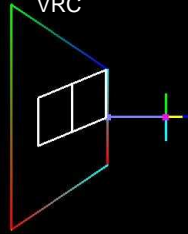
Observe que a posição relativa entre os referenciais é mantida

EA978 – 2s2008 - Ting

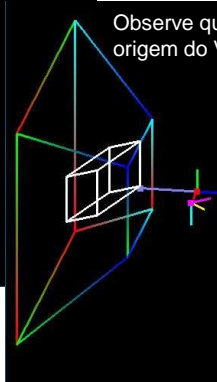
## VRC Cisalhado

Projeção Perspectiva  
(Várias Vistas)

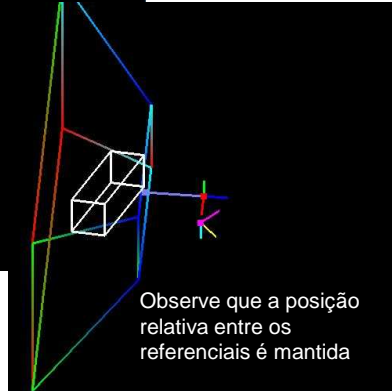
Observe que dop está na  
mesma direção do eixo z do  
VRC



Observe que PRP está na  
origem do VRC

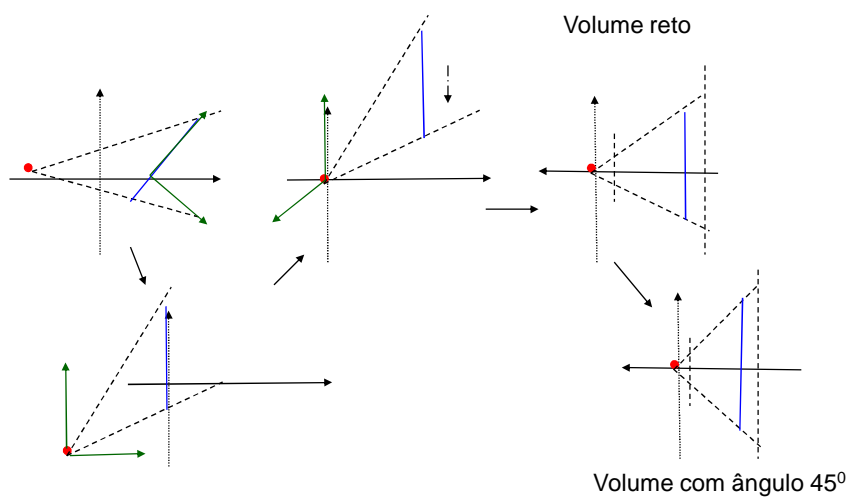


Observe que a posição  
relativa entre os  
referenciais é mantida



EA978 – 2s2008 - Ting

## Deformação do Volume Perspectivo Mudança de Escala

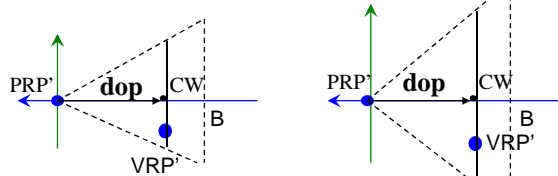
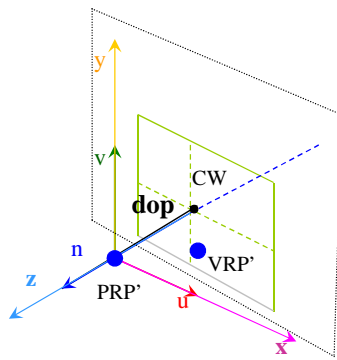


EA978 – 2s2008 - Ting



## Deformação do Volume Perspectivo Abertura: 45°

$(u_{min}, u_{max}, v_{min}, v_{max}, F, B)$



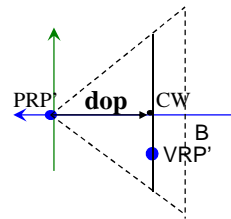
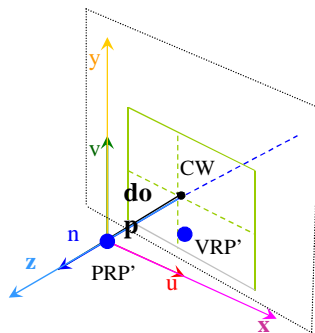
$$\begin{bmatrix} (u_{max}-u_{min})/2 & 0 & 0 \\ 0 & (v_{max}-v_{min})/2 & 0 \\ 0 & 0 & -VRP'_z \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -VRP'_z & 0 & 0 \\ 0 & -VRP'_z & 0 \\ 0 & 0 & -VRP'_z \\ 0 & 0 & 0 \end{bmatrix}$$

EA978 - 2s2008 - Ting

## Deformação do Volume Perspectivo

$(u_{min}, u_{max}, v_{min}, v_{max}, F, B)$

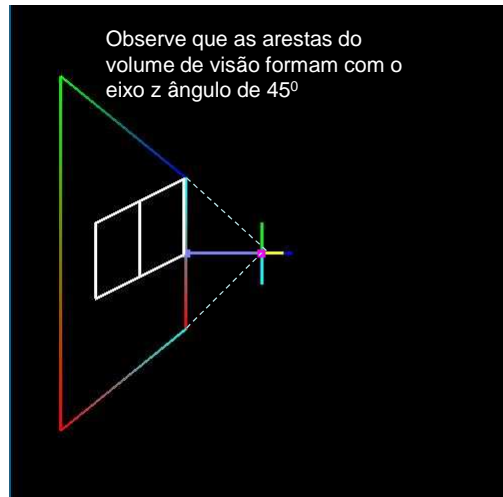


$$S_{per} = \begin{bmatrix} -2VRP'_z/(u_{max}-u_{min}) & 0 & 0 & 0 \\ 0 & -2VRP'_z/(v_{max}-v_{min}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

EA978 - 2s2008 - Ting

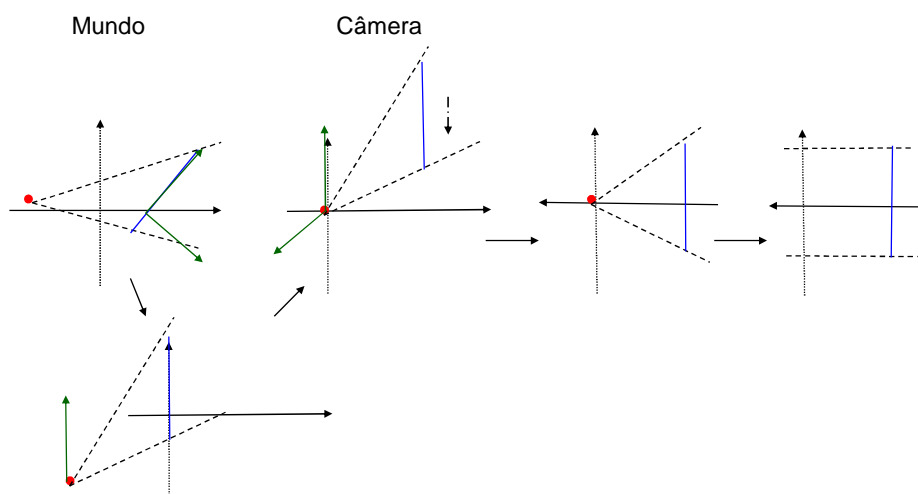
## VRC com abertura $45^\circ$

### Projeção Perspectiva



EA978 – 2s2008 - Ting

## VRC $\rightarrow$ Normalizado



EA978 – 2s2008 - Ting

## VRC → Normalizado

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2/(r-l) & 0 & 0 & 0 \\ 0 & 2/(t-b) & 0 & 0 \\ 0 & 0 & 2/(n-f) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & -(r+l)/2 \\ 0 & 1 & 0 & -(t+b)/2 \\ 0 & 0 & 1 & -(-n-f)/2 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$(l,r,b,t,-n,-f) \rightarrow (-1,1,-1,1,-1,1)$

$$\begin{bmatrix} 2/(r-l) & 0 & 0 & -(r+l)/(r-l) \\ 0 & 2/(t-b) & 0 & -(t+b)/(t-b) \\ 0 & 0 & -2/(f-n) & -(f+n)/(f-n) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

EA978 – 2s2008 - Ting

## Normalizado

Observe que a relação do volume com o referencial

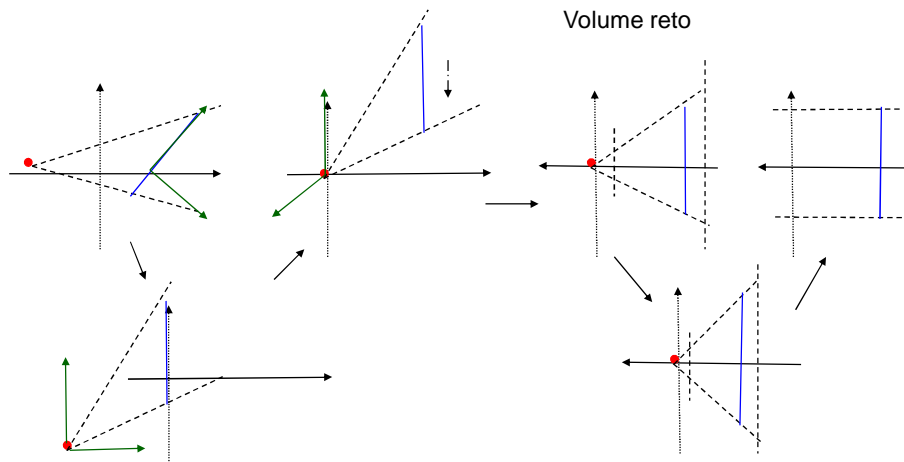
Projeção Paralela  
(Várias Vistas)

Observe que PRP é um ponto da cena

Observe que o volume de visão ficou um cubo centrado na origem do referencial VRC

EA978 – 2s2008 - Ting

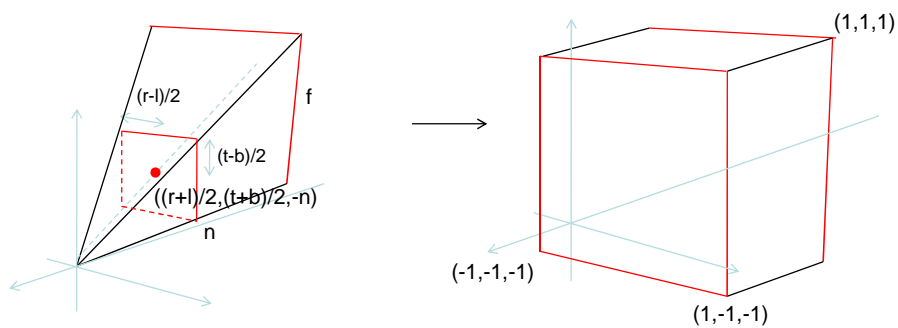
## VRC → Normalizado



EA978 – 2s2008 - Ting

## Volume Perspectivo em VRC → Normalizado

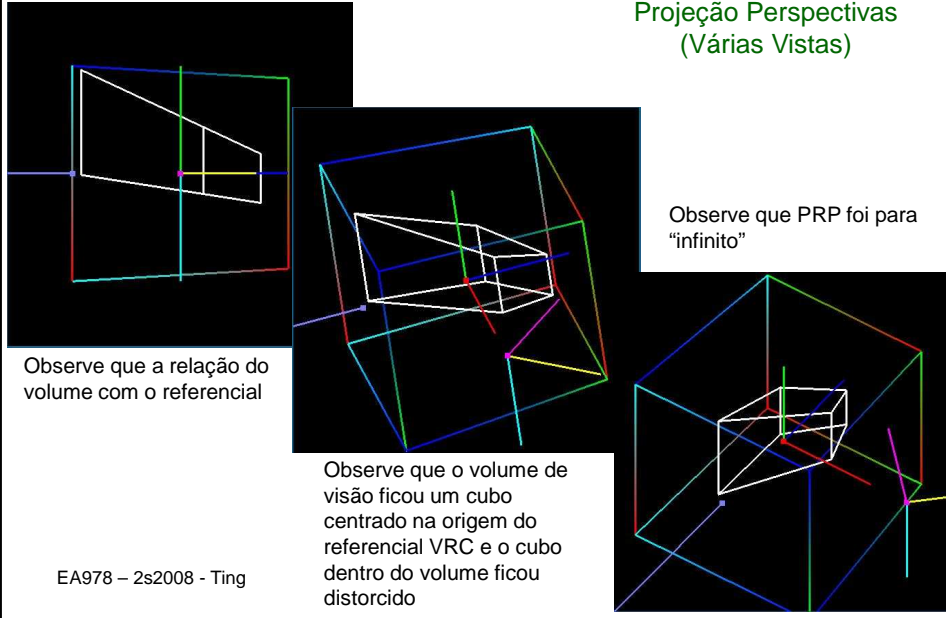
$$\begin{bmatrix} 2n/(r-l) & 0 & (r+l)/(r-l) & 0 \\ 0 & 2n/(t-b) & (t+b)/(t-b) & 0 \\ 0 & 0 & -(f+n)/(f-n) & -2fn/(f-n) \\ 0 & 0 & -1 & 0 \end{bmatrix}$$



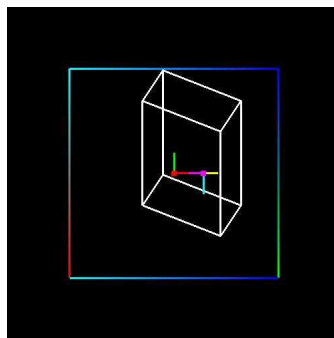
EA978 – 2s2008 - Ting

## Normalizado

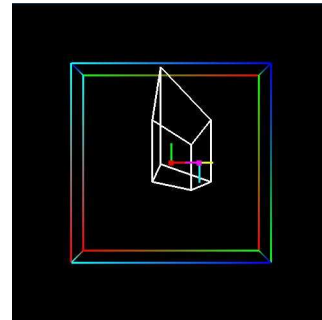
Projeção Perspectivas  
(Várias Vistas)



## Projeções Paralelas em DC

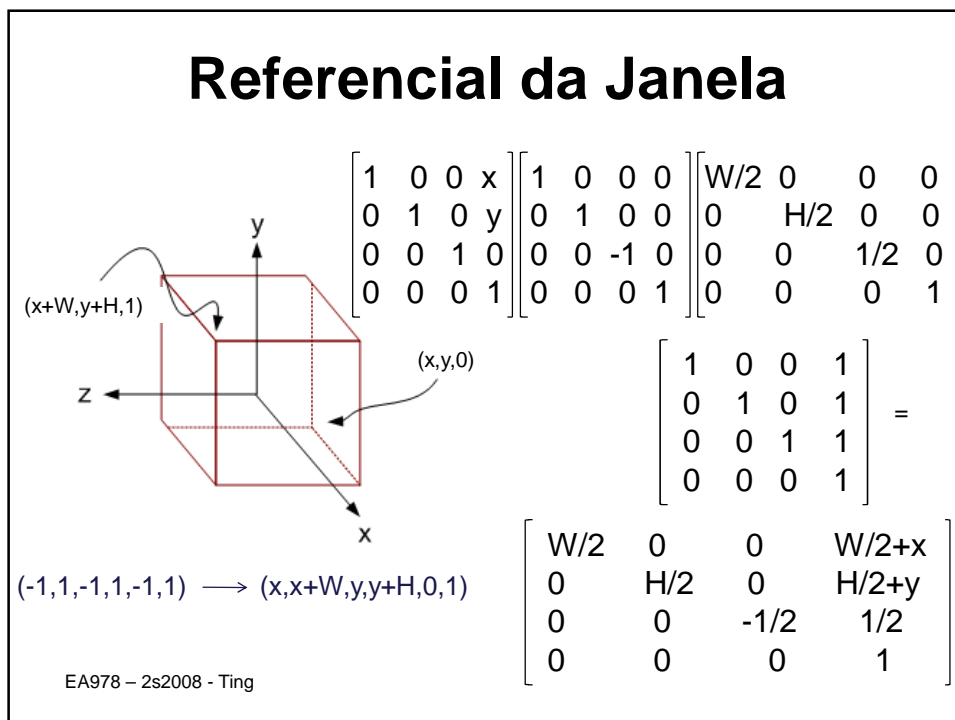
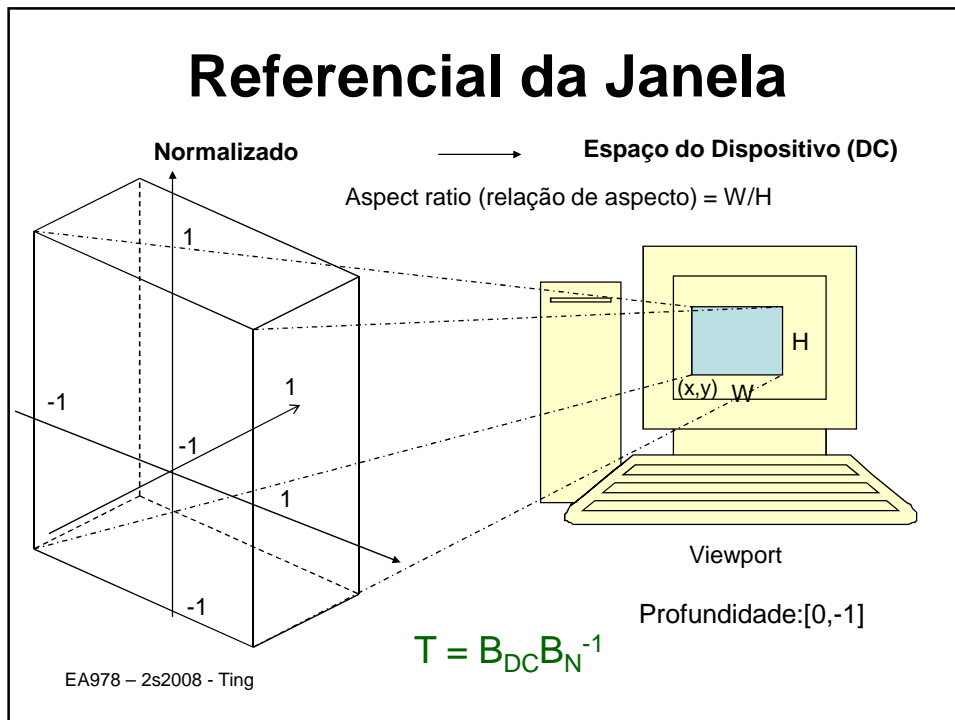


Paralela

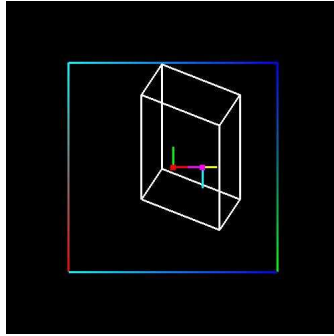


Perspectiva

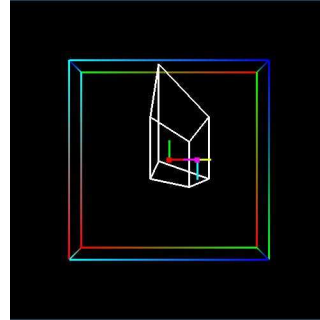
EA978 – 2s2008 - Ting



## Projeções Paralelas em DC



Paralela



Perspectiva

EA978 – 2s2008 - Ting

## Matriz de Transformação

Transformações lineares

Translação

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ p_x & p_y & p_z & 0 \end{pmatrix}$$

Perspectivo

EA978 – 2s2008 - Ting

# OpenGL

Pilha de Matrizes de Transformação

MODELVIEW  
PROJECTION

$$\begin{bmatrix} a_0 & a_4 & a_8 & a_{12} \\ a_1 & a_5 & a_9 & a_{13} \\ a_2 & a_6 & a_{10} & a_{14} \\ a_3 & a_7 & a_{11} & a_{15} \end{bmatrix}$$

glTranslate  
glRotate  
glScale  
glMultMatrix  
glFrustum  
glOrtho  
glViewport

glPushMatrix



glPopMatrix



EA978 – 2s2008 - Ting