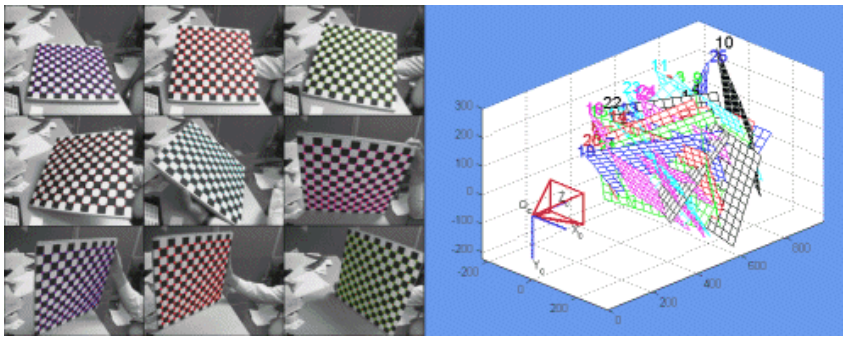
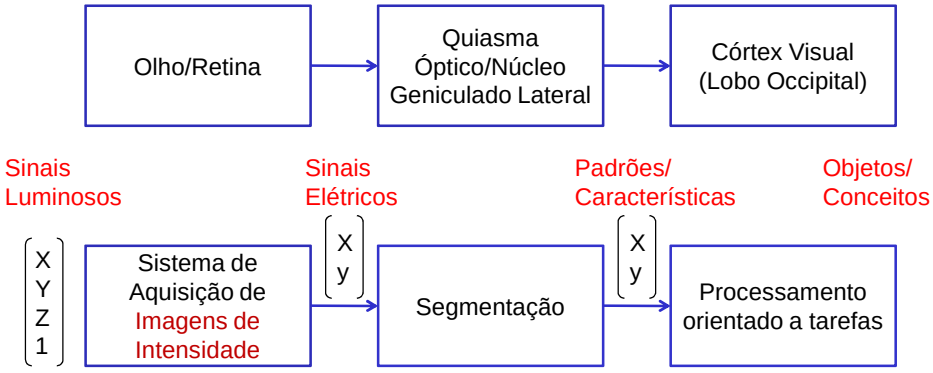
Calibração da Câmera

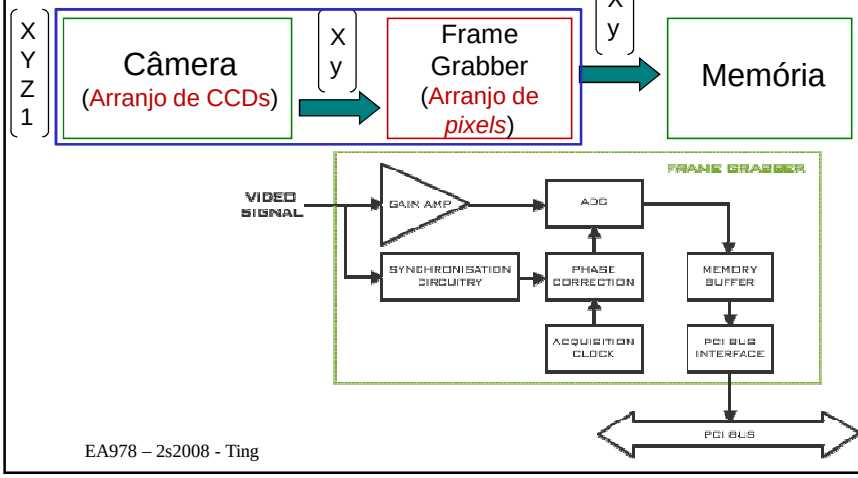


Quais foram os parâmetros intrínsecos e extrínsecos de cada câmera?

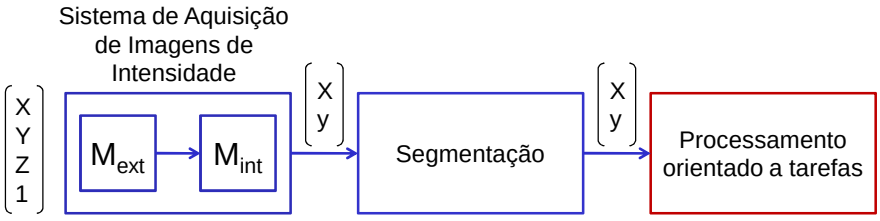
Analogia a Sistema Visual



Aquisição de Imagens de Intensidade

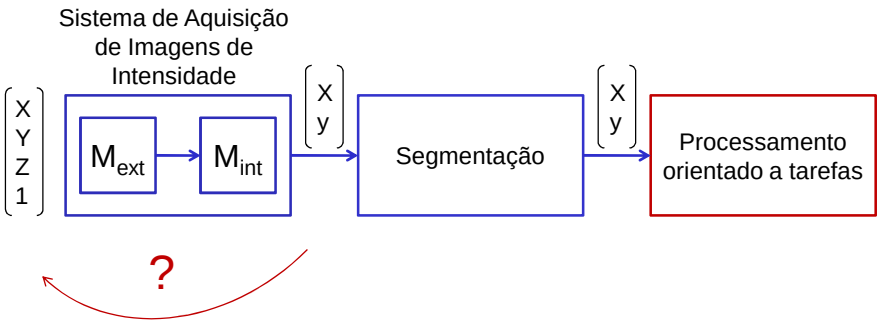


Um Modelo de Câmera



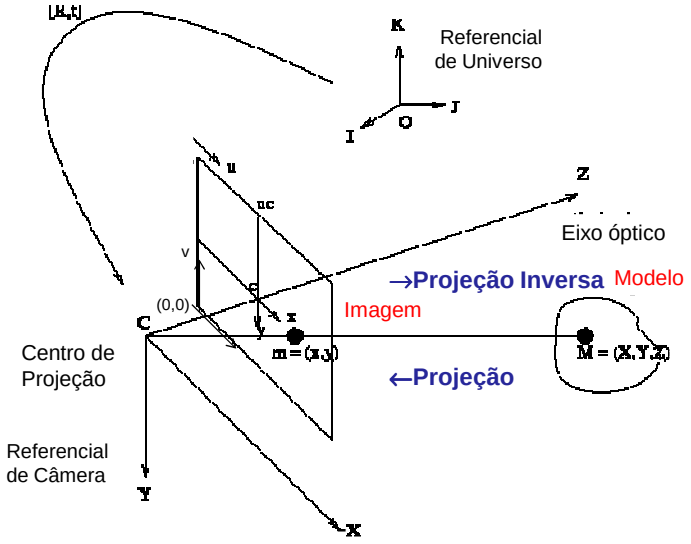
$$M_{int}\, M_{ext} = \begin{pmatrix} -f/s_x & 0 & o_x \\ 0 & -f/s_y & o_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} r_{00} & r_{01} & r_{02} & t_x \\ r_{10} & r_{11} & r_{12} & t_y \\ r_{20} & r_{21} & r_{22} & t_z \end{pmatrix}$$

Transformação Projetiva Inversa



Dada a imagem digital, qual é o modelo 3D correspondente?

Analogia a Síntese de Imagens



Analogia a Síntese de Imagens

Síntese de Imagens

Transformação projetiva

$$\begin{pmatrix} x \\ y \\ w \end{pmatrix} = M_c \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

Incógnitas

Valores conhecidos

Transformação Projetiva Inversa

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = M_c^{-1} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

?

Incógnitas

Valores conhecidos

Transformação M_c

$$\begin{pmatrix} x \\ y \\ w \end{pmatrix} = \begin{pmatrix} a_{00} & a_{01} & a_{02} & a_{03} \\ a_{10} & a_{11} & a_{12} & a_{13} \\ a_{20} & a_{21} & a_{22} & a_{23} \\ a_{30} & a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

$$u = \frac{x}{w} = \frac{a_{00}X + a_{01}Y + a_{02}Z + a_{03}}{a_{30}X + a_{31}Y + a_{32}Z + a_{33}}$$

$$v = \frac{y}{w} = \frac{a_{10}X + a_{11}Y + a_{12}Z + a_{13}}{a_{30}X + a_{31}Y + a_{32}Z + a_{33}}$$

$$a_{00}X + a_{01}Y + a_{02}Z + a_{03} = a_{30}X\,u + a_{31}Y\,u + a_{32}Z\,u + a_{33}\,u$$
$$a_{10}X + a_{11}Y + a_{12}Z + a_{13} = a_{30}X\,v + a_{31}Y\,v + a_{32}Z\,v + a_{33}\,v$$

Transformação M_c

$$a_{00}X + a_{01}Y + a_{02}Z + a_{03} - a_{30}Xu - a_{31}Yu - a_{32}Zu - a_{33}u = 0$$
$$a_{10}X + a_{11}Y + a_{12}Z + a_{13} - a_{30}Xv - a_{31}Yv - a_{32}Zv - a_{33}v = 0$$

Supondo a correspondência $(u,v) \leftrightarrow (X,Y,Z)$ conhecida

$$a_{00}X + a_{01}Y + a_{02}Z + a_{03} \qquad - a_{30}Xu - a_{31}Yu - a_{32}Zu - a_{33}u = 0$$
$$a_{10}X + a_{11}Y + a_{12}Z + a_{13} - a_{30}Xv - a_{31}Yv - a_{32}Zv - a_{33}v = 0$$

12 incógnitas

↓

12 equações

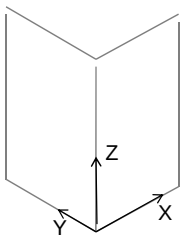
↓

6 pares de correspondência

Padrão de Calibração

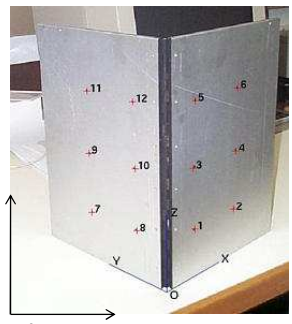
Sistema de Aquisição de Imagens de Intensidade → Segmentação do Padrão de Calibração

Objeto



+

Imagem



→

$(u_1,v_1) \leftrightarrow (X_1,Y_1,Z_1)$
$(u_2,v_2) \leftrightarrow (X_2,Y_2,Z_2)$
$(u_3,v_3) \leftrightarrow (X_3,Y_3,Z_3)$
$(u_4,v_4) \leftrightarrow (X_4,Y_4,Z_4)$
$(u_5,v_5) \leftrightarrow (X_5,Y_5,Z_5)$
$(u_6,v_6) \leftrightarrow (X_6,Y_6,Z_6)$
$(u_7,v_7) \leftrightarrow (X_7,Y_7,Z_7)$
$(u_8,v_8) \leftrightarrow (X_8,Y_8,Z_8)$
$(u_9,v_9) \leftrightarrow (X_9,Y_9,Z_9)$
$(u_{10},v_{10}) \leftrightarrow (X_{10},Y_{10},Z_{10})$
$(u_{11},v_{11}) \leftrightarrow (X_{11},Y_{11},Z_{11})$
$(u_{12},v_{12}) \leftrightarrow (X_{12},Y_{12},Z_{12})$

Cena com Padrão de Calibração

$a_{00}X_1 + a_{01}Y_1 + a_{02}Z_1 + a_{03} - a_{30}X_1 u_1 - a_{31}Y_1 u_1 - a_{32}Z_1 u_1 - a_{33}u_1 = 0$

$a_{10}X_1 + a_{11}Y_1 + a_{12}Z_1 + a_{13} - a_{30}X_1 v_1 - a_{31}Y_1 v_1 - a_{32}Z_1 v_1 - a_{33}v_1 = 0$

$a_{00}X_2 + a_{01}Y_2 + a_{02}Z_2 + a_{03} - a_{30}X_2 u_2 - a_{31}Y_2 u_2 - a_{32}Z_2 u_2 - a_{33}u_2 = 0$

$a_{10}X_2 + a_{11}Y_2 + a_{12}Z_2 + a_{13} - a_{30}X_2 v_2 - a_{31}Y_2 v_2 - a_{32}Z_2 v_2 - a_{33}v_2 = 0$

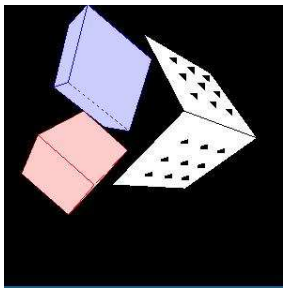
$\vdots$

$a_{00}X_n + a_{01}Y_n + a_{02}Z_n + a_{03} - a_{30}X_n u_n - a_{31}Y_n u_n - a_{32}Z_n u_n - a_{33}u_n = 0$

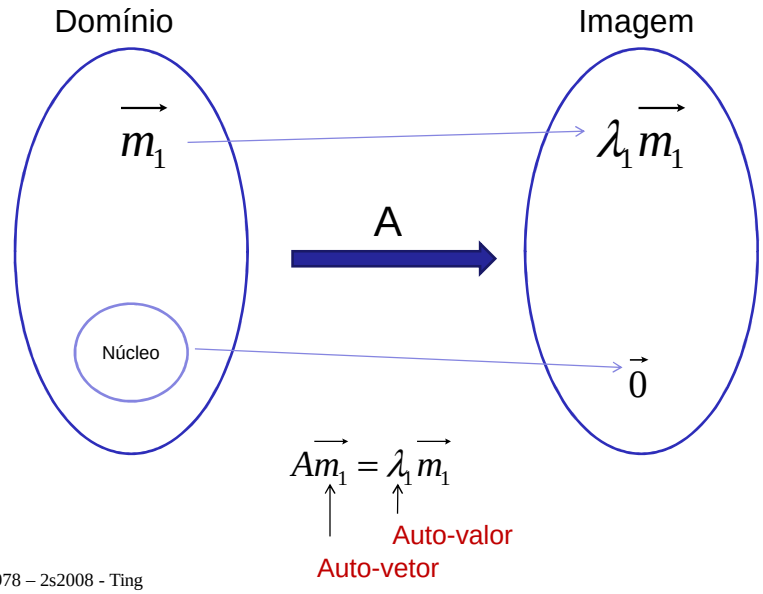
$a_{10}X_n + a_{11}Y_n + a_{12}Z_n + a_{13} - a_{30}X_n v_n - a_{31}Y_n v_n - a_{32}Z_n v_n - a_{33}v_n = 0$

$$A\vec{m} = 0$$

$\vec{m}$: elementos de M_c



Uma Solução



$$A\vec{m_1} = \lambda_1 \vec{m_1}$$

Auto-valor

Auto-vetor

Decomposição em Valores Singulares

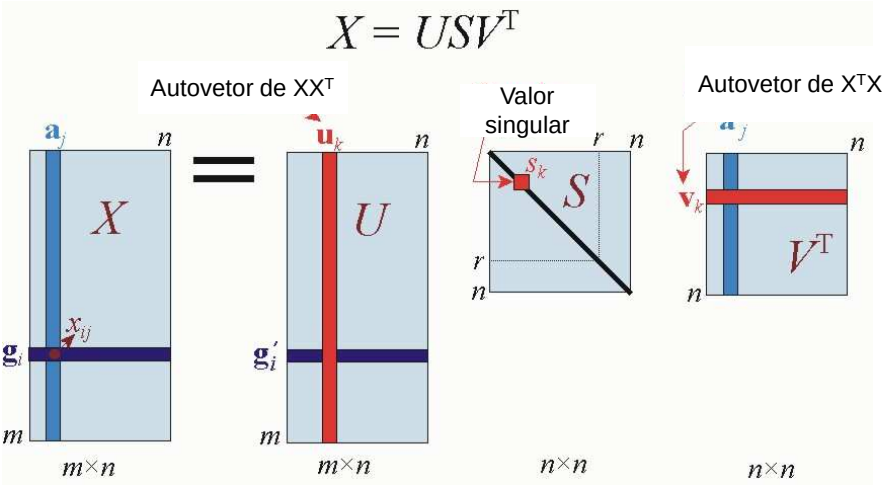
$$X_{(mxn)} = U_{(mxn)}\, S_{(nxn)}\, V_{(nxn)}^T$$

onde $UU^T = I$
$VV^T = I$

1. Achar os auto-valores de X^TX.
2. Achar os auto-vetores correspondentes para construir V.
3. Formar S com a raiz dos auto-valores
4. Computar cada coluna de U com uso de um par (auto-valor,auto-vetor) e A

http://www.uwlax.edu/faculty/will/svd/index.html

SVD



SVD
Exemplos

$$\begin{pmatrix} 2 & 0 \\ 0 & -3 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 4 \\ 1 & 3 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0.82 & -0.58 \\ 0.58 & 0.82 \\ 0.0 & 0.0 \\ 0.0 & 0.0 \end{pmatrix} \begin{pmatrix} 5.47 & 0 \\ 0 & 0.37 \end{pmatrix} \begin{pmatrix} 0.40 & -0.91 \\ 0.91 & 0.40 \end{pmatrix}$$

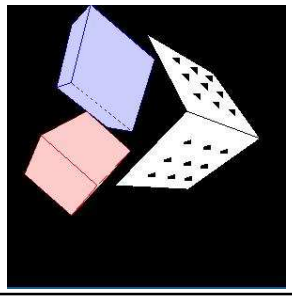
Solução por SVD

$$a_{00}X_1 + a_{01}Y_1 + a_{02}Z_1 + a_{03} - a_{30}X_1 u_1 - a_{31}Y_1 u_1 - a_{32}Z_1 u_1 - a_{33}u_1 = 0$$
$$a_{10}X_1 + a_{11}Y_1 + a_{12}Z_1 + a_{13} - a_{30}X_1 v_1 - a_{31}Y_1 v_1 - a_{32}Z_1 v_1 - a_{33}v_1 = 0$$
$$a_{00}X_2 + a_{01}Y_2 + a_{02}Z_2 + a_{03} - a_{30}X_2 u_2 - a_{31}Y_2 u_2 - a_{32}Z_2 u_2 - a_{33}u_2 = 0$$
$$a_{10}X_2 + a_{11}Y_2 + a_{12}Z_2 + a_{13} - a_{30}X_2 v_2 - a_{31}Y_2 v_2 - a_{32}Z_2 v_2 - a_{33}v_2 = 0$$

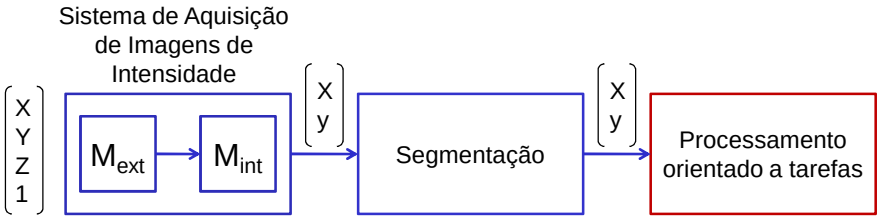
$$a_{00}X_n + a_{01}Y_n + a_{02}Z_n + a_{03} - a_{30}X_n u_n - a_{31}Y_n u_n - a_{32}Z_n u_n - a_{33}u_n = 0$$
$$a_{10}X_n + a_{11}Y_n + a_{12}Z_n + a_{13} - a_{30}X_n v_n - a_{31}Y_n v_n - a_{32}Z_n v_n - a_{33}v_n = 0$$

$$A = USV^T$$

Solução: coluna de V^T que tiver menor valor singular associado



Um Modelo de Câmera



$$M_{int}\, M_{ext} = \begin{pmatrix} -f/s_x & 0 & o_x \\ 0 & -f/s_y & o_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} r_{00} & r_{01} & r_{02} & t_x \\ r_{10} & r_{11} & r_{12} & t_y \\ r_{20} & r_{21} & r_{22} & t_z \end{pmatrix}$$

Parâmetros Extrínsecos e Intrínsecos

$$\begin{pmatrix} a_{00} & a_{01} & a_{02} & a_{03} \\ a_{10} & a_{11} & a_{12} & a_{13} \\ a_{30} & a_{31} & a_{32} & a_{33} \end{pmatrix} = \gamma M_{int}\, M_{ext} = \begin{pmatrix} -f/s_x & 0 & o_x \\ 0 & -f/s_y & o_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} r_{00} & r_{01} & r_{02} & t_x \\ r_{10} & r_{11} & r_{12} & t_y \\ r_{20} & r_{21} & r_{22} & t_z \end{pmatrix}$$

$$\begin{pmatrix} -f/s_x r_{00}+o_x r_{20} & -f/s_x r_{01}+o_x r_{21} & -f/s_x r_{02}+o_x r_{22} & -f/s_x t_x+o_x t_z \\ -f/s_y r_{00}+o_y r_{20} & -f/s_y r_{01}+o_y r_{21} & -f/s_y r_{02}+o_y r_{22} & -f/s_y t_y+o_y t_z \\ r_{20} & r_{21} & r_{22} & t_z \end{pmatrix}$$

Parâmetros Extrínsecos e Intrínsecos

$$\begin{pmatrix} a_{00} & a_{01} & a_{02} & a_{03} \\ a_{10} & a_{11} & a_{12} & a_{13} \\ a_{30} & a_{31} & a_{32} & a_{33} \end{pmatrix} = \gamma M_{int}\, M_{ext} =$$

$$\begin{pmatrix} -f/s_x r_{00}+o_x r_{20} & -f/s_x r_{01}+o_x r_{21} & -f/s_x r_{02}+o_x r_{22} & -f/s_x t_x+o_x t_z \\ -f/s_y r_{00}+o_y r_{20} & -f/s_y r_{01}+o_y r_{21} & -f/s_y r_{02}+o_y r_{22} & -f/s_y t_y+o_y t_z \\ r_{20} & r_{21} & r_{22} & t_z \end{pmatrix}$$

$$\sqrt{a_{30}^{\,2}+a_{31}^{\,2}+a_{32}^{\,2}} = |\gamma|\sqrt{r_{20}^{\,2}+r_{21}^{\,2}+a_{22}^{\,2}} = |\gamma|$$

Parâmetros Extrínsecos e Intrínsecos

$$\begin{pmatrix} a_{00} & a_{01} & a_{02} & a_{03} \\ a_{10} & a_{11} & a_{12} & a_{13} \\ a_{30} & a_{31} & a_{32} & a_{33} \end{pmatrix} = \gamma M_{int}\, M_{ext} =$$

$$\begin{pmatrix} -f/s_x r_{00}+o_x r_{20} & -f/s_x r_{01}+o_x r_{21} & -f/s_x r_{02}+o_x r_{22} & -f/s_x t_x+o_x t_z \\ -f/s_y r_{00}+o_y r_{20} & -f/s_y r_{01}+o_y r_{21} & -f/s_y r_{02}+o_y r_{22} & -f/s_y t_y+o_y t_z \\ r_{20} & r_{21} & r_{22} & t_z \end{pmatrix}$$

$$\frac{a_{33}}{|\gamma|} = \sigma a_{33} = t_z \qquad \frac{a_{31}}{|\gamma|} = r_{21}$$

$$\frac{a_{30}}{|\gamma|} = r_{20} \qquad \frac{a_{32}}{|\gamma|} = r_{22}$$

Parâmetros Extrínsecos e Intrínsecos

$$\begin{pmatrix} a_{00} & a_{01} & a_{02} & a_{03} \\ a_{10} & a_{11} & a_{12} & a_{13} \\ a_{30} & a_{31} & a_{32} & a_{33} \end{pmatrix} = \gamma M_{int}\, M_{ext} =$$

$$\begin{matrix} q_1^T \\ q_2^T \\ q_3^T \end{matrix} \begin{pmatrix} -f/s_x r_{00}+o_x r_{20} & -f/s_x r_{01}+o_x r_{21} & -f/s_x r_{02}+o_x r_{22} & -f/s_x t_x+o_x t_z \\ -f/s_y r_{00}+o_y r_{20} & -f/s_y r_{01}+o_y r_{21} & -f/s_y r_{02}+o_y r_{22} & -f/s_y t_y+o_y t_z \\ r_{20} & r_{21} & r_{22} & t_z \end{pmatrix}$$

$$o_x = q_1^T q_3$$

$$o_y = q_2^T q_3$$

Parâmetros Extrínsecos e Intrínsecos

$$\begin{pmatrix} a_{00} & a_{01} & a_{02} & a_{03} \\ a_{10} & a_{11} & a_{12} & a_{13} \\ a_{30} & a_{31} & a_{32} & a_{33} \end{pmatrix} = \gamma M_{int}\, M_{ext} =$$

$$\begin{matrix} q_1^T \\ q_2^T \\ q_3^T \end{matrix} \begin{pmatrix} -f/s_x r_{00}+o_x r_{20} & -f/s_x r_{01}+o_x r_{21} & -f/s_x r_{02}+o_x r_{22} & -f/s_x t_x+o_x t_z \\ -f/s_y r_{00}+o_y r_{20} & -f/s_y r_{01}+o_y r_{21} & -f/s_y r_{02}+o_y r_{22} & -f/s_y t_y+o_y t_z \\ r_{20} & r_{21} & r_{22} & t_z \end{pmatrix}$$

$$f_x = \frac{f}{s_x} = \sqrt{q_1^T q_1 - o_x^2}$$

$$f_y = \frac{f}{s_y} = \sqrt{q_2^T q_2 - o_y^2}$$

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Parâmetros Extrínsecos e Intrínsecos

$$\begin{pmatrix} a_{00} & a_{01} & a_{02} & a_{03} \\ a_{10} & a_{11} & a_{12} & a_{13} \\ a_{30} & a_{31} & a_{32} & a_{33} \end{pmatrix} = \gamma M_{int}\, M_{ext} =$$

$$\begin{pmatrix} -f/s_x r_{00}+o_x r_{20} & -f/s_x r_{01}+o_x r_{21} & -f/s_x r_{02}+o_x r_{22} & -f/s_x t_x+o_x t_z \\ -f/s_y r_{00}+o_y r_{20} & -f/s_y r_{01}+o_y r_{21} & -f/s_y r_{02}+o_y r_{22} & -f/s_y t_y+o_y t_z \\ r_{20} & r_{21} & r_{22} & t_z \end{pmatrix}$$

$$r_{0i} = \frac{\sigma(o_x a_{3i} - a_{0i})}{f_x} \qquad t_x = \frac{\sigma(o_x t_z - a_{03})}{f_x}$$

$$r_{1i} = \frac{\sigma(o_y a_{3i} - a_{1i})}{f_x} \qquad t_y = \frac{\sigma(o_y t_z - a_{13})}{f_y}$$

Algoritmo

1. Estimativa do núcleo da matriz de transformação Pa=0
   - ✓ Auto-vetor com menor auto-valor correspondente
2. Estimativa dos parâmetros intrínsicos
   - ✓ Determinar fator de escala γ
   - ✓ Determinar sinal de t_z, σ
   - ✓ Determinar ponto principal (o_x, o_y)
   - ✓ Determinar a distância focal (f_x, f_y)
   - ✓ Determinar outros parâmetros extrínsicos