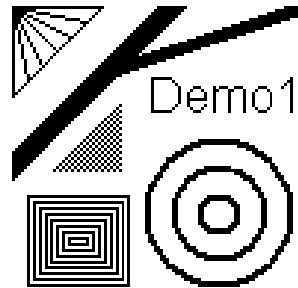
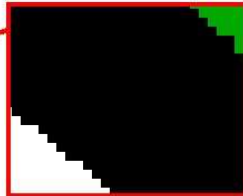


Amostragem

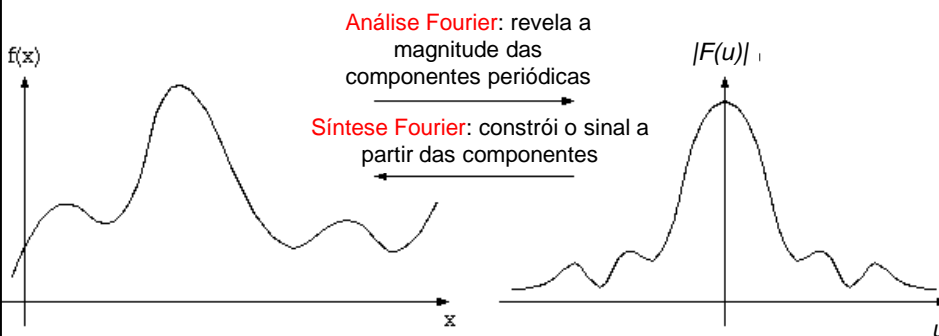
Efeito de Borda Serrilhada



Por quê ocorrem estes efeitos?

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Transformada de Fourier 1D



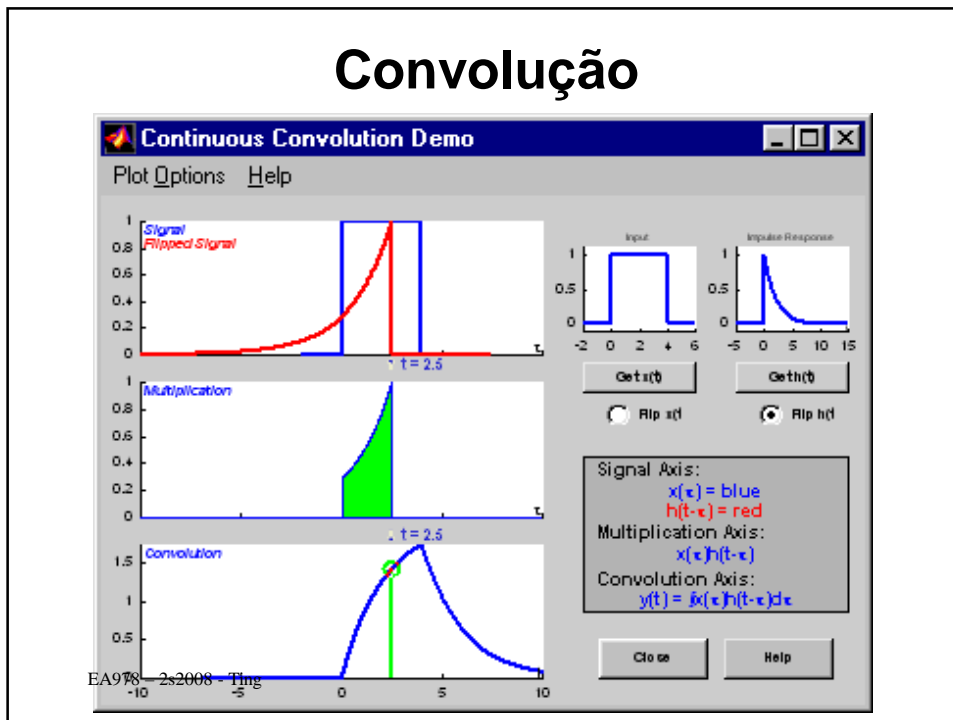
Par de transformadas:

$$F(u) = \int_{-\infty}^{\infty} f(x) e^{-j2\pi ux} dx \quad (\text{Análise})$$

$$f(x) = \int_{-\infty}^{\infty} F(u) e^{j2\pi ux} du \quad (\text{Síntese})$$

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Convolução

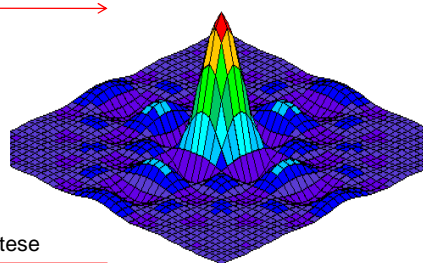
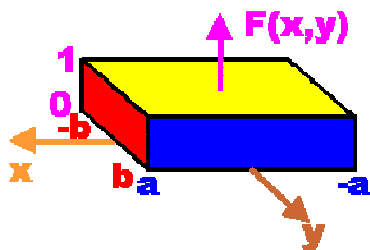


Transformada de Fourier 2D

$$F(u, v) = \int \int_{-\infty}^{\infty} f(x, y) e^{-j2\pi(ux+vy)} dx dy$$

$$F(u, v) = (4ab) \frac{\sin(2\pi ua)}{2\pi ua} \frac{\sin(2\pi vb)}{2\pi vb}$$

Análise



Síntese

$$f(x, y) = \int \int_{-\infty}^{\infty} F(u, v) e^{j2\pi(ux+vy)} du dv$$

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Transformada de Fourier 2D

$$F(u, v) = \int \int_{-\infty}^{\infty} f(x, y) e^{-j2\pi(ux+vy)} dx dy$$

$$P(u, v) = R^2(u, v) + I^2(u, v) \quad \text{Espectro de Potência}$$

$$|F(u, v)| = (R^2(u, v) + I^2(u, v))^{1/2} \quad \text{Espectro de Fourier}$$

$$\varphi(u, v) = \text{tg}^{-1} (I(u, v)/R(u, v)) \quad \text{Ângulo de fase}$$

$$F(u, v) = |F(u, v)| e^{j\varphi(u, v)}$$

$$f(x, y) = \int \int_{-\infty}^{\infty} F(u, v) e^{j2\pi(ux+vy)} du dv$$

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Transformada Discreta de Fourier 1D

$$F(u) = \int_{-\infty}^{\infty} f(x) e^{-j2\pi ux} dx \quad f(x) = \int_{-\infty}^{\infty} F(u) e^{j2\pi ux} du$$

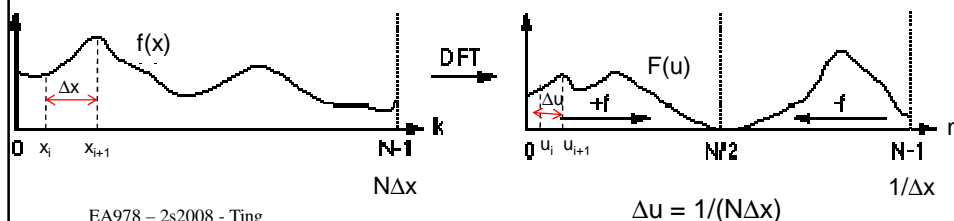
$$e^{-j2\pi ux} = \cos(2\pi ux) - j \text{sen}(2\pi ux)$$

$$f(x_0), f(x_1=x_0+\Delta x), f(x_2=x_0+2\Delta x), \dots, f(x_{N-1}=x_0+(N-1)\Delta x)$$

$$f(x_k) = f(x_{k+N})$$

$$F(u_n) = (1/N) \sum_{k=0}^{N-1} f(x_k) e^{-j2\pi nk/N}$$

$$f(x_k) = \sum_{n=0}^{N-1} F(u_n) e^{j2\pi nk/N}$$



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Transformada Discreta de Fourier 1D

$$F(u_n) = (1/N) \sum_{k=0}^{N-1} f(x_k) e^{-j2\pi nk/N}$$

$$F(0) = (1/N) \sum_{k=0}^{N-1} f(x_k) e^{-j2\pi 0k/N} = 1/(N) \sum_{k=0}^{N-1} f(x_k)$$

A **componente DC** corresponde à média da intensidade da imagem.

$$F(u_{n+N}) = (1/N) \sum_{k=0}^{N-1} f(x_k) e^{-j2\pi(n+N)k/N} = F(u_n)$$

Transformada discreta de Fourier é **periódica**.

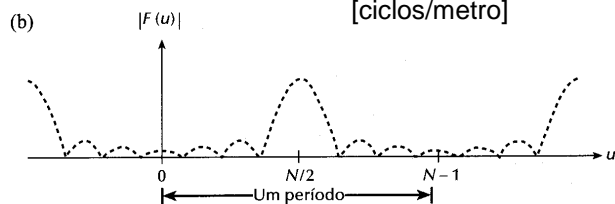
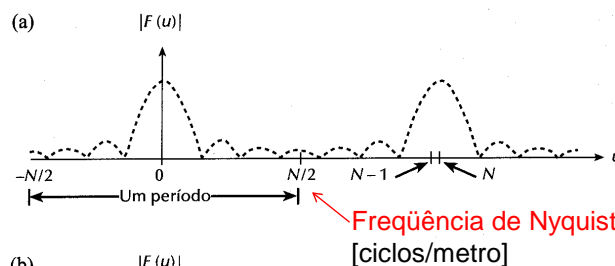
$$F(u_{-n}) = (1/N) \sum_{k=0}^{N-1} f(x_k) e^{-j2\pi(-n)k/N} = (1/N) \sum_{k=0}^{N-1} f(x_k) e^{j2\pi(n)k/N}$$

Se $f(x_k)$ for real, magnitude da transformada discreta é **simétrica** em relação à origem.

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Transformada Discreta de Fourier 1D

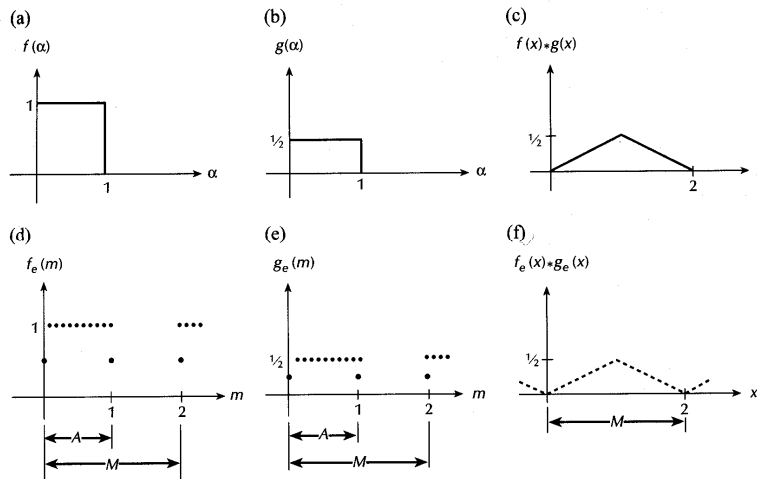
$$F(u_n) = (1/N) \sum_{k=0}^{N-1} f(x_k) e^{-j2\pi nk/N}$$



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Transformada Discreta de Fourier 1D

Convolução



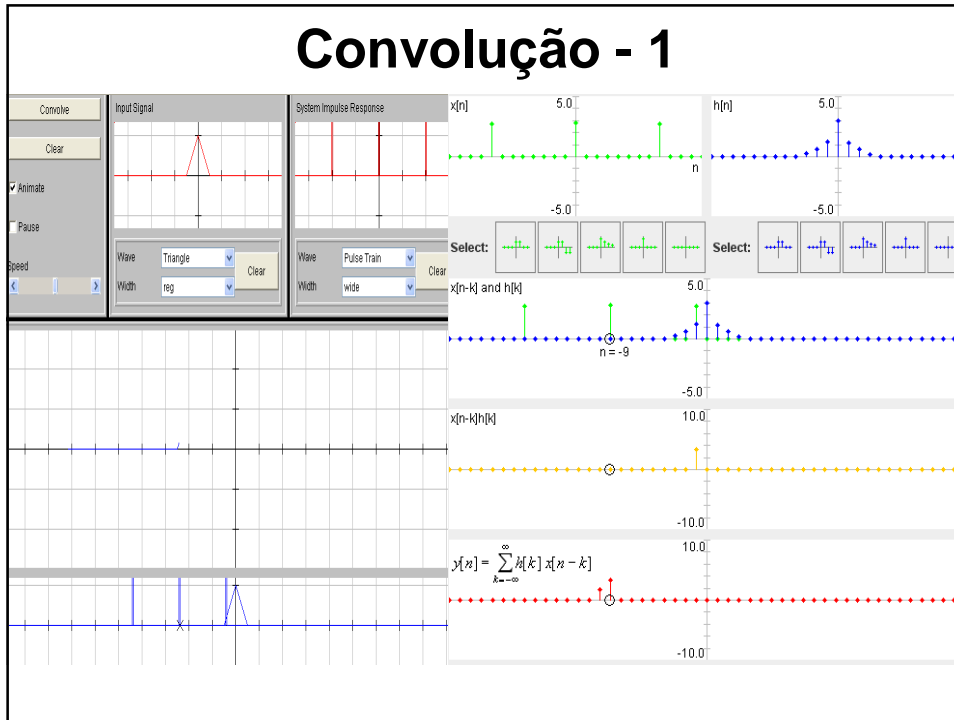
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Convolução - 1

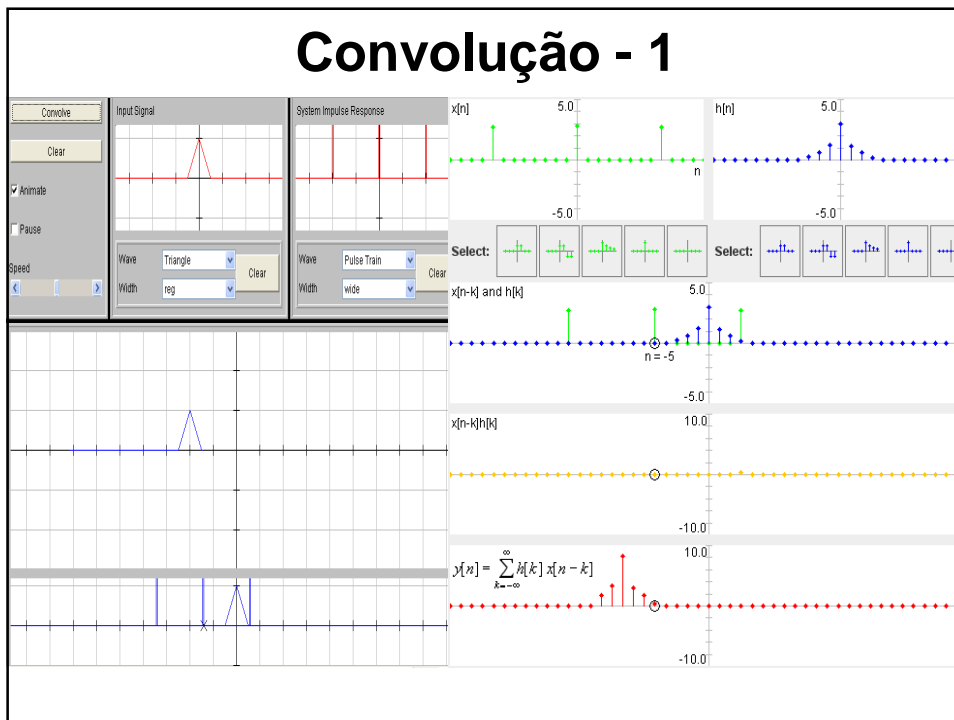
$y(t) = \int f(x) * g(t-x) dx$
 $y(x_n) = \sum f(x_k)g(x_n-x_k)$

<http://maxwell.me.gu.edu.au/sp/Excalibar/Jtg/Conv.html>
<http://www.jhu.edu/signals/discreteconv2/index.html>

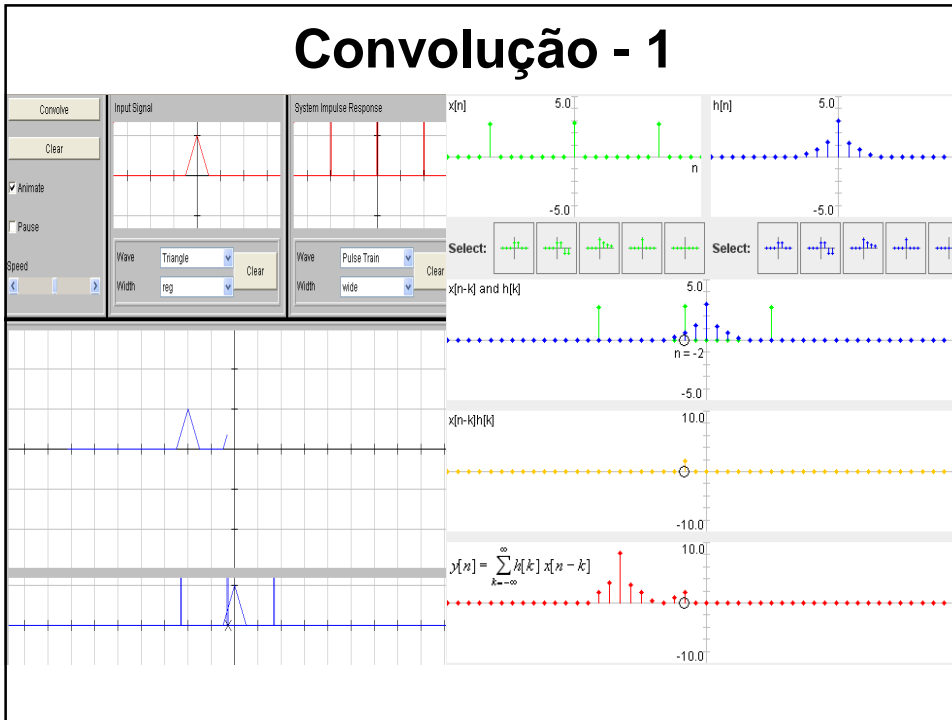
Convolução - 1



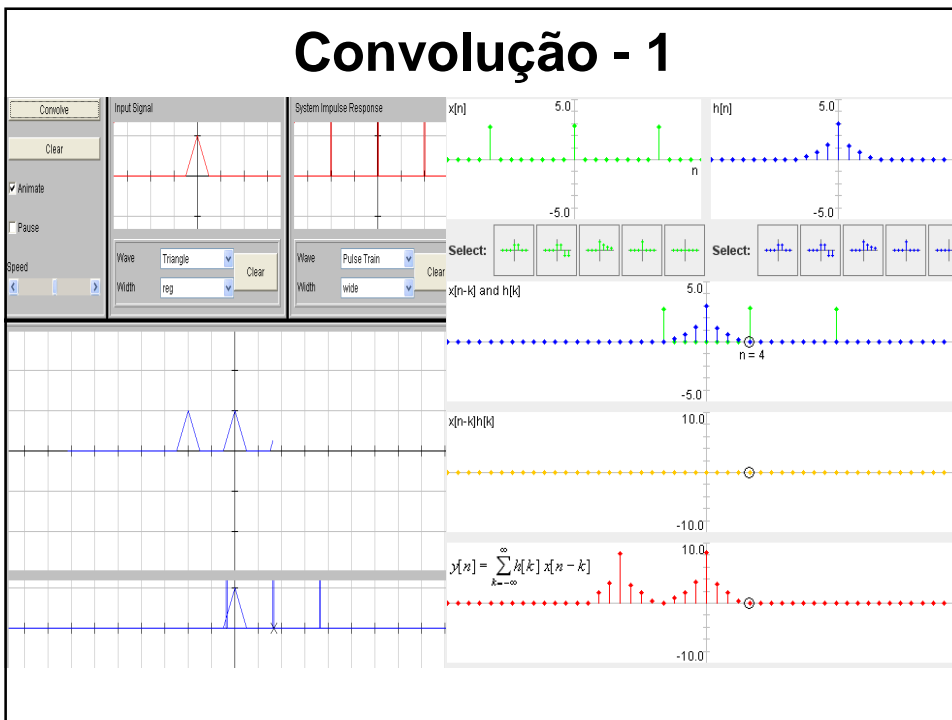
Convolução - 1



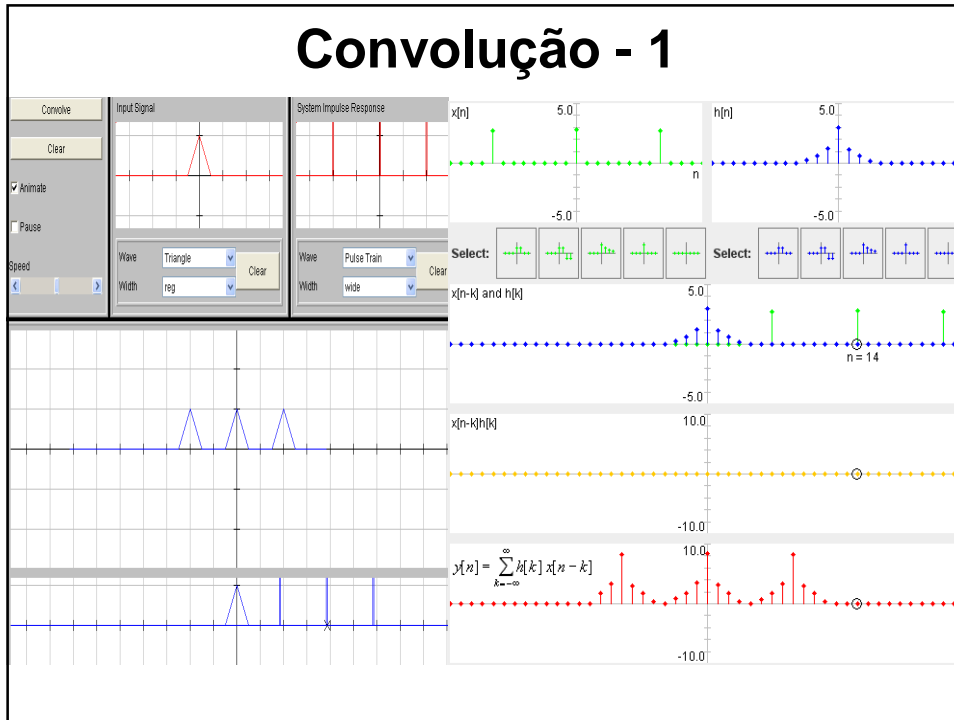
Convolução - 1



Convolução - 1



Convolução - 1

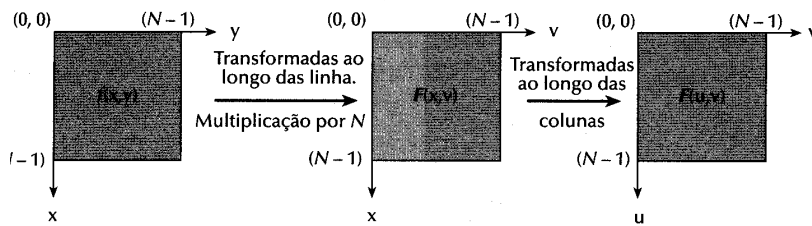


Transformada Discreta de Fourier 2D

$$F(u, v) = \frac{1}{NM} \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} f(x, y) e^{2\pi i (xu/N + yv/M)}$$

Separabilidade

$$F(u, v) = (1/NM) \sum_{x=0}^{N-1} e^{-j2\pi ux/N} \left[\sum_{y=0}^{M-1} f(x, y) e^{-j2\pi vy/N} \right]$$



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Transformada Discreta de Fourier 2D

Outras Propriedades

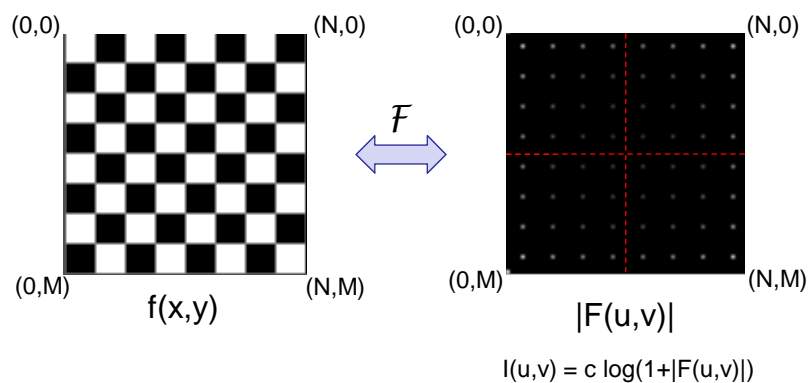
Linearidade
Deslocamento
Periodicidade e Simetria Conjugada
Rotação
Distributividade quanto à adição
Mudança de Escala
Multiplicação
Convolução

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Espectro de Fourier

Análise Visual

Periodicidade → somente um período $N \times M$ é suficiente
Escala muito alta → compressão por função logarítmica



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Espectro de Fourier

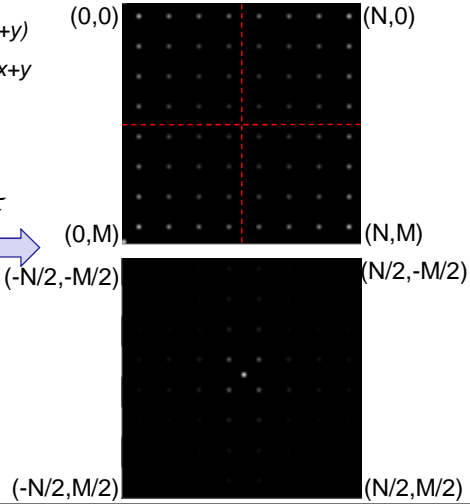
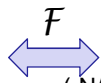
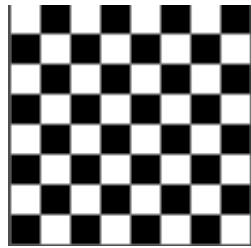
Análise Visual

Simetria Conjugada → imagem simétrica

Deslocamento não afeta amplitude → mover $F(0,0)$ para centro

$$F(u+N/2, v+N/2) \leftrightarrow f(x,y) e^{j\pi(x+y)}$$

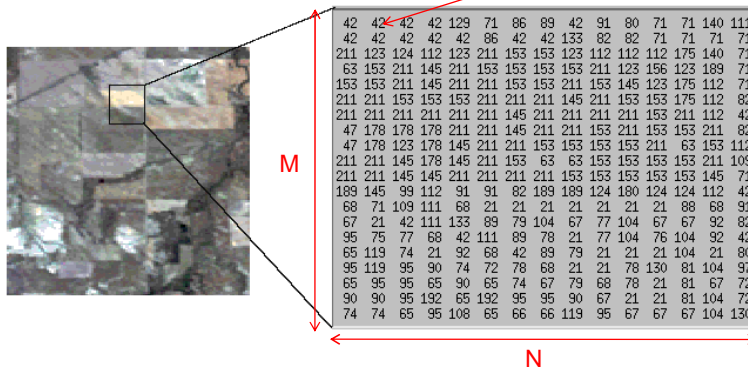
$$= f(x,y)(-1)^{x+y}$$



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Imagem Discreta

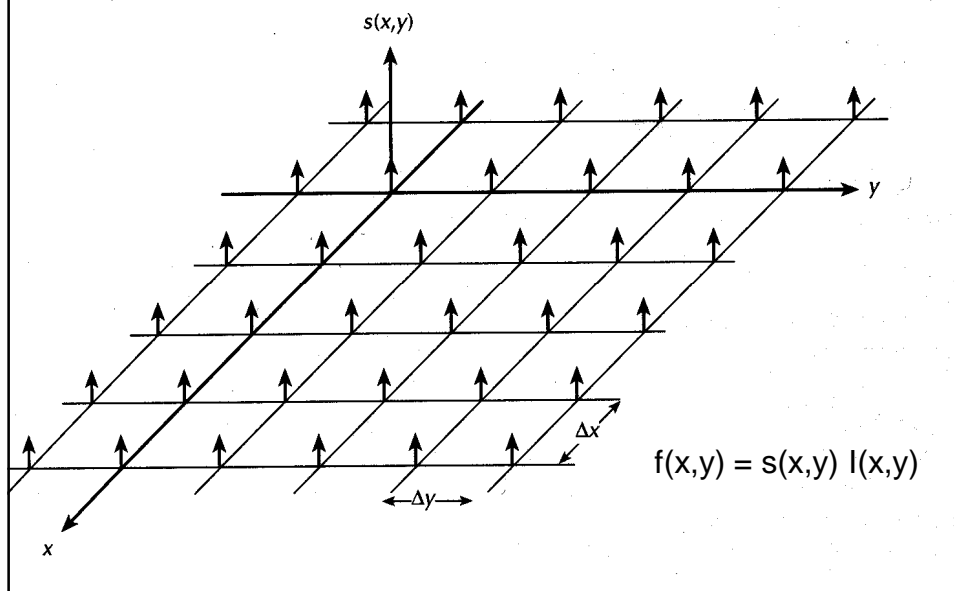
$f(x,y)$: intensidade em cada *pixel*



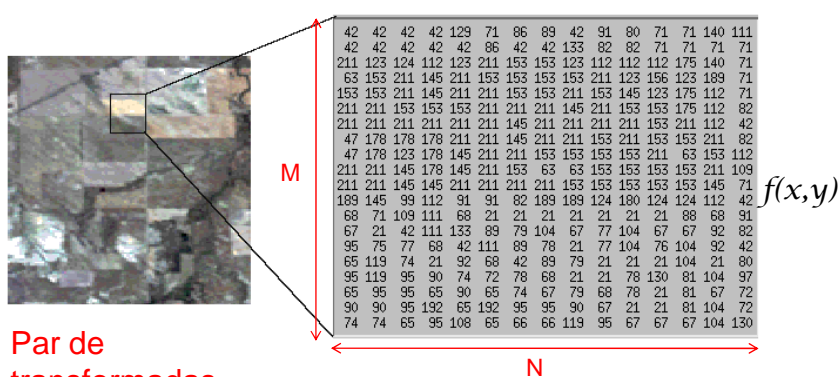
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Quais são as componentes periódicas de intensidade?

Amostragem 2D



Transformada Discreta de Fourier 2D



Par de transformadas

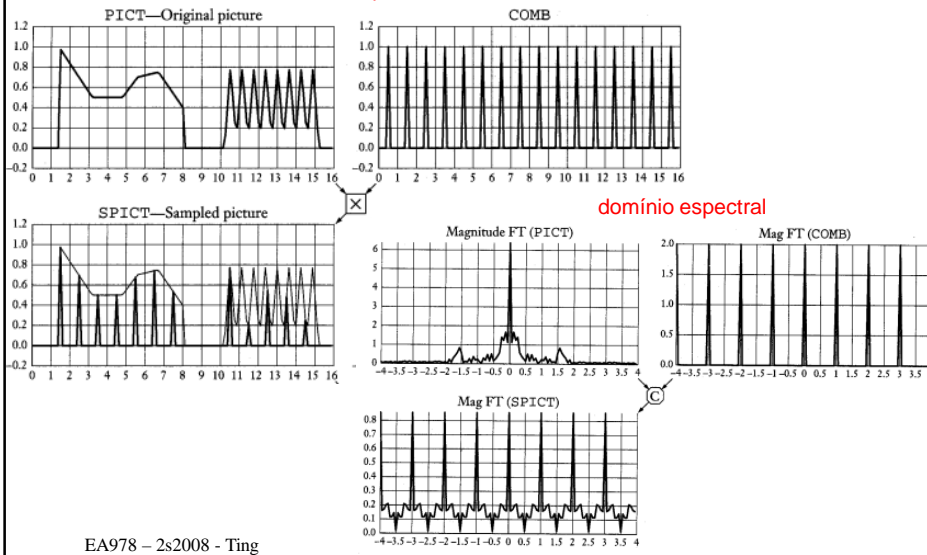
$$F(u, v) = \frac{1}{NM} \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} f(x, y) e^{-2\pi i(xu/N + yv/M)}$$

$$f(x, y) = \sum_{u=0}^{N-1} \sum_{v=0}^{M-1} F(u, v) e^{2\pi i(xu/N + yv/M)}$$

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Amostragem 1D

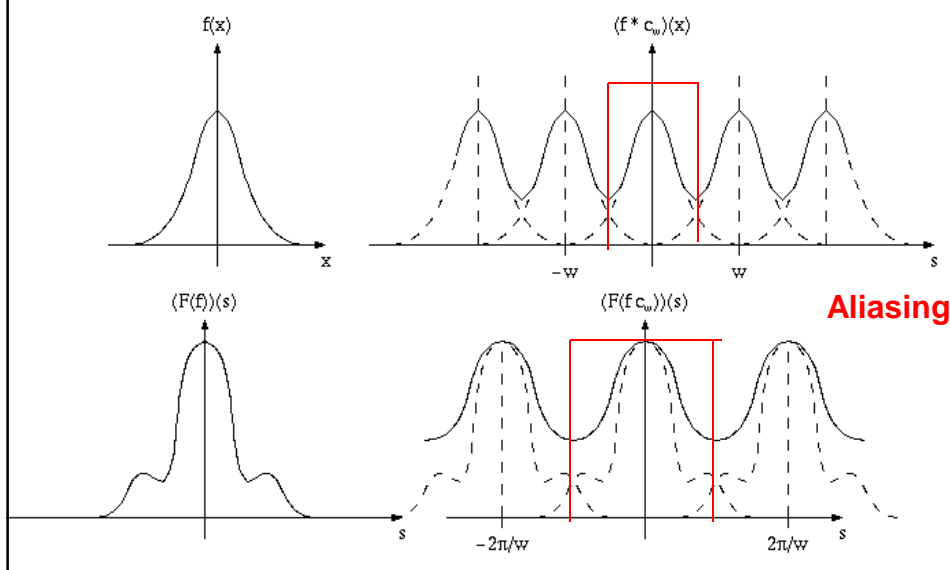
domínio espacial



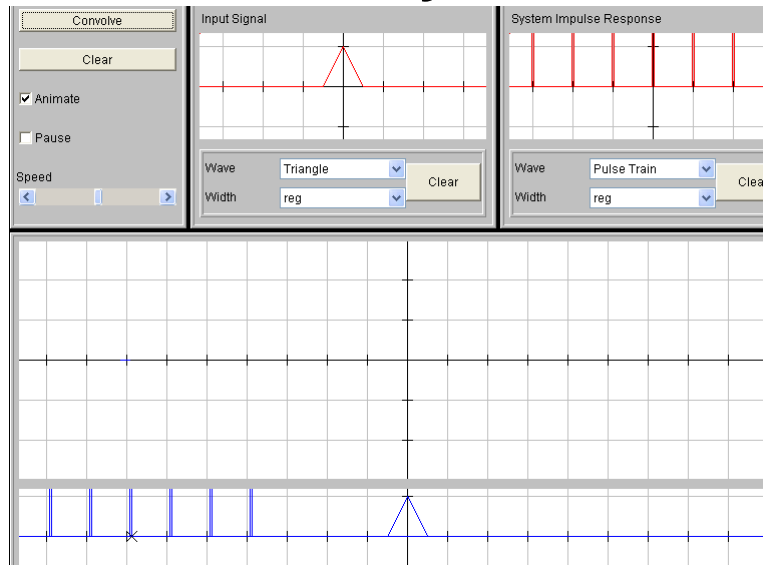
domínio espectral

Amostragem 1D

Subamostragem

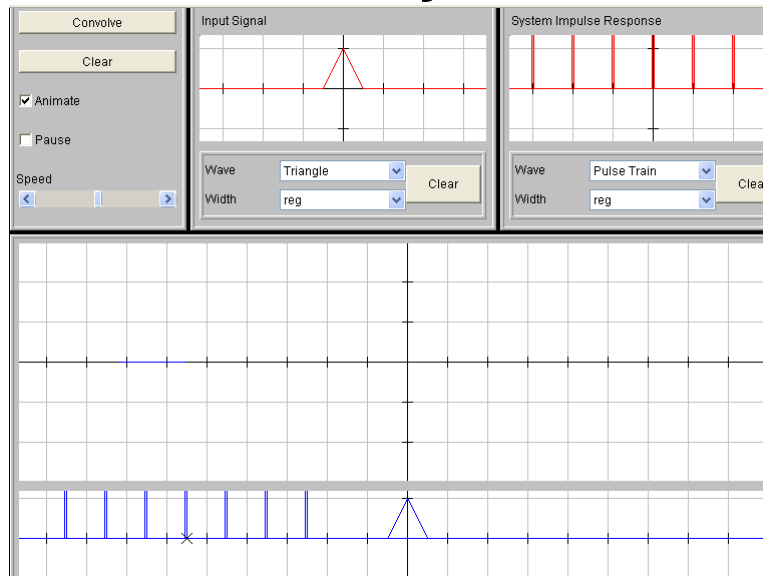


Convolução - 2



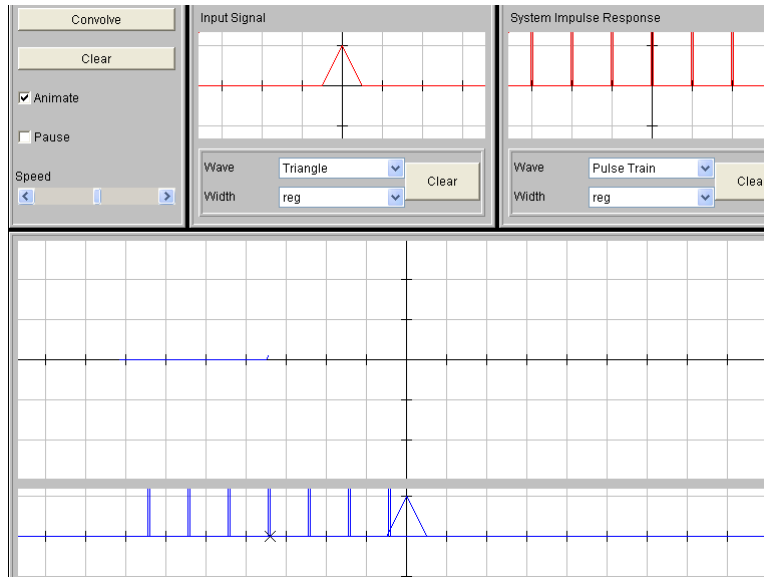
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Convolução - 2



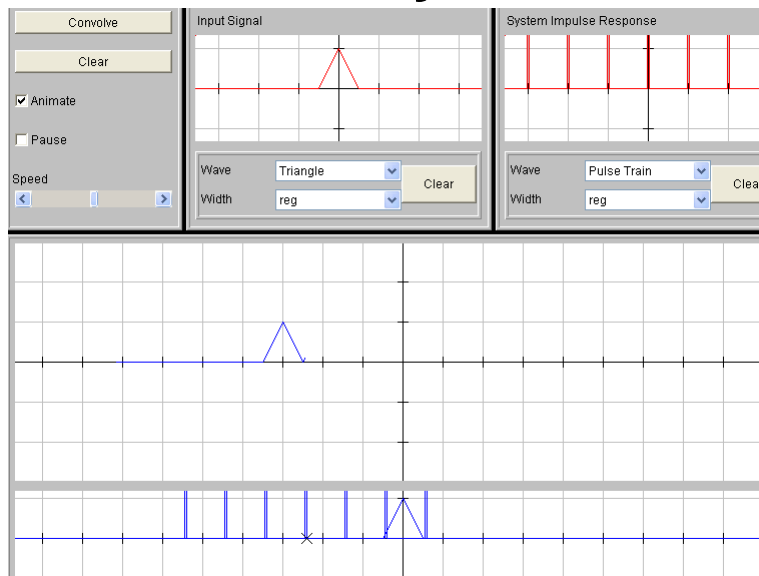
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Convolução - 2



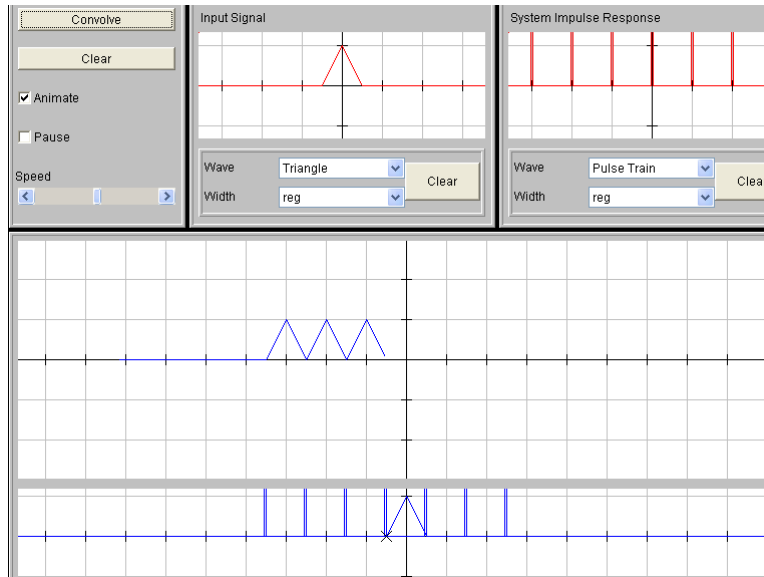
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Convolução - 2



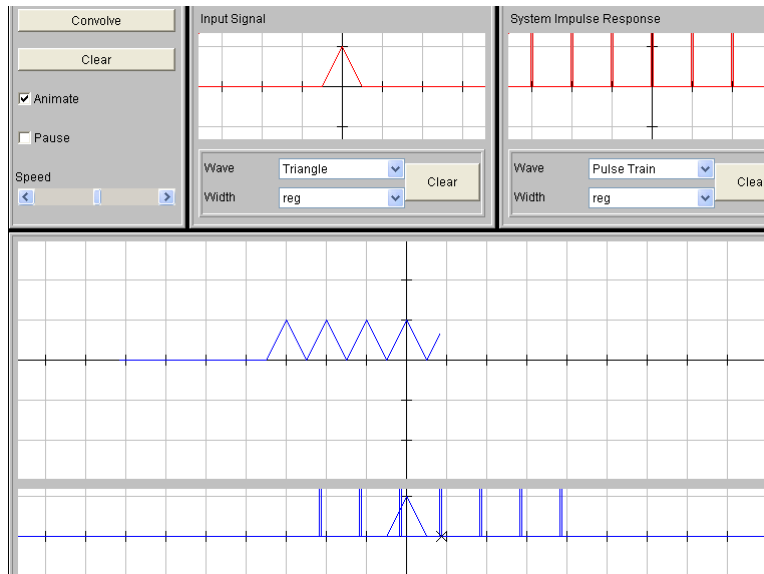
EA978 - 2s2008 - Ting

Convolução - 2



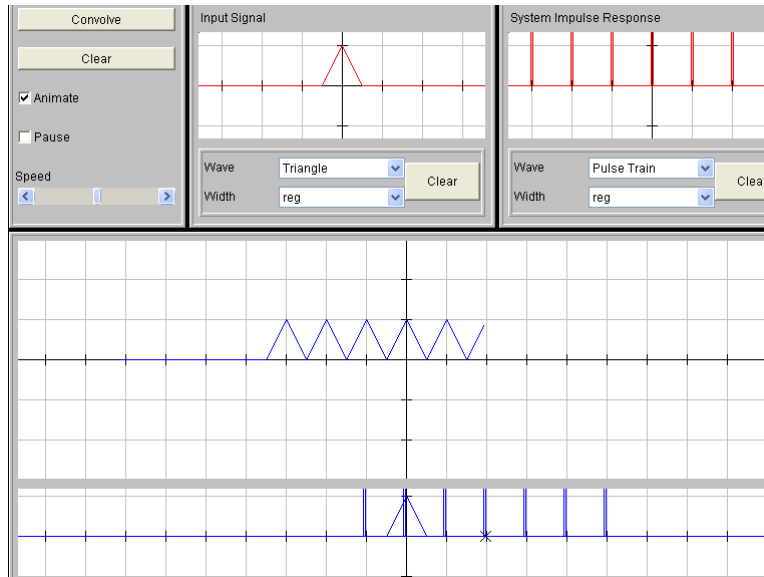
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Convolução - 2



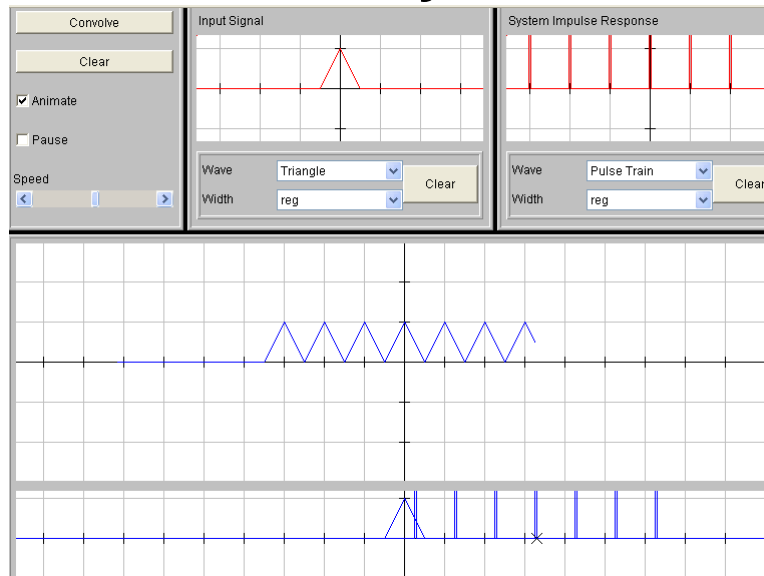
EA978 - 2s2008 - Ting

Convolução - 2



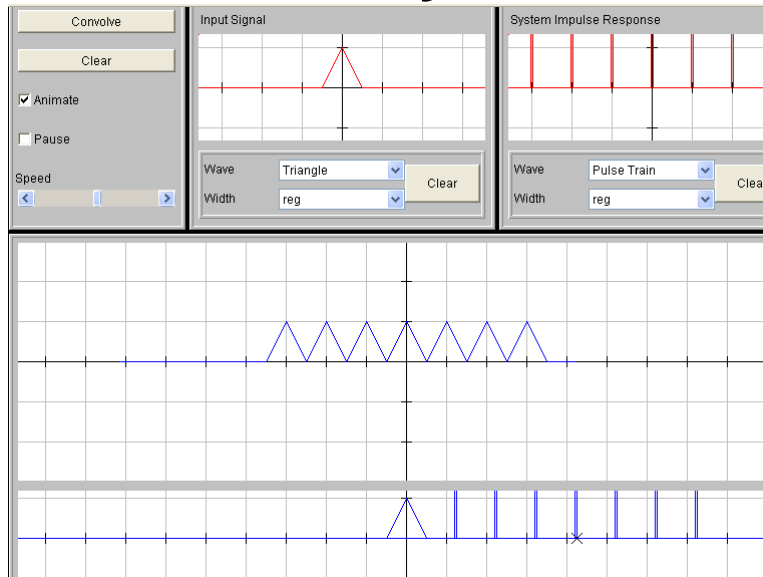
EA978 - 2s2008 - Ting

Convolução - 2



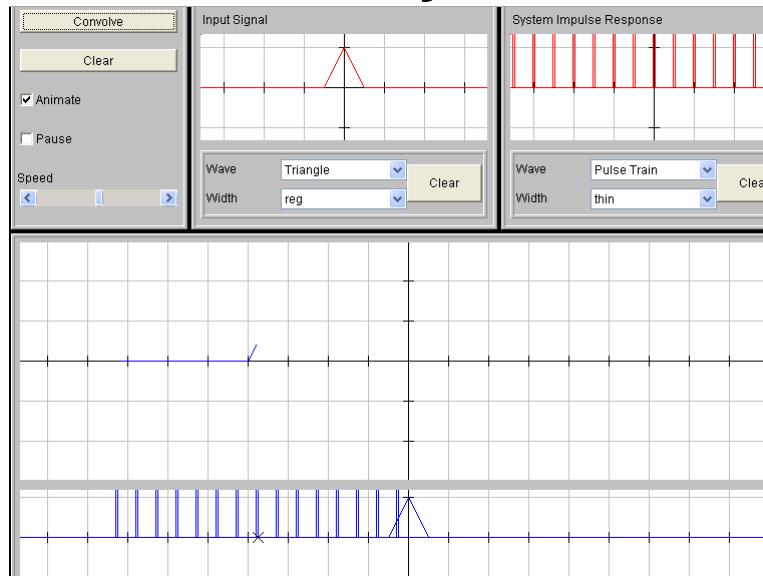
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Convolução - 2



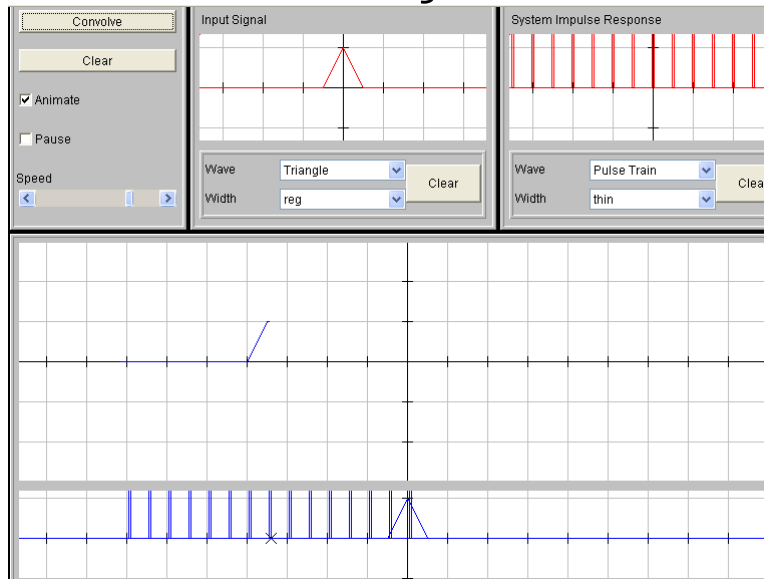
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Convolução - 3



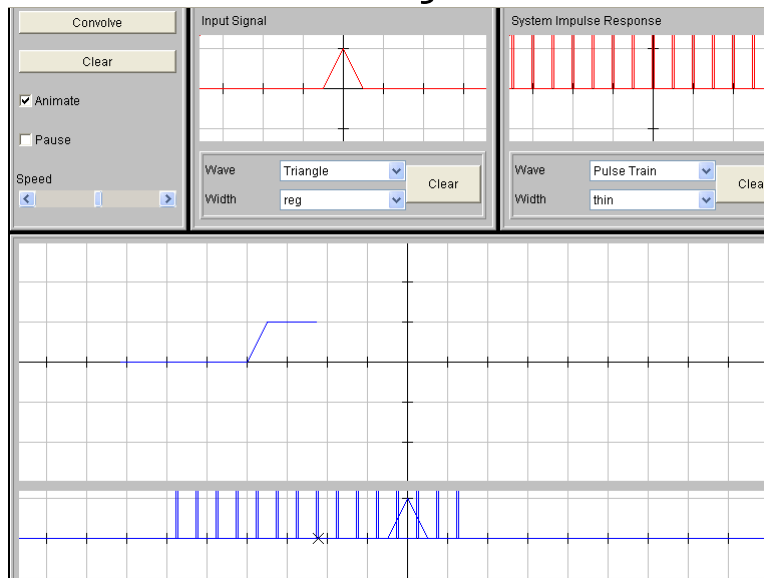
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Convolução - 3



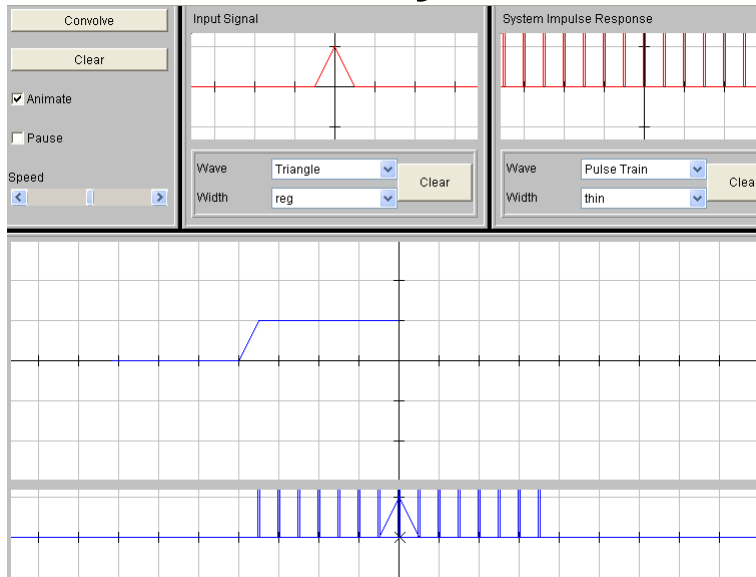
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Convolução - 3



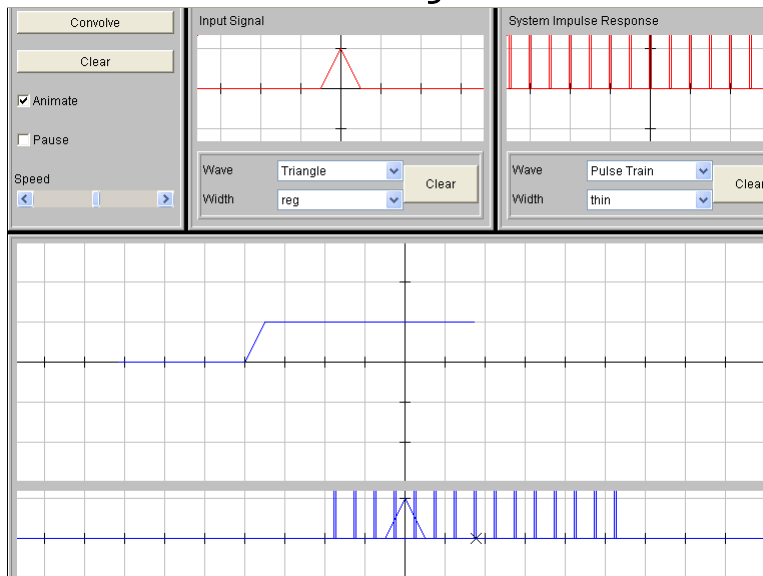
EA978 - 2s2008 - Ting

Convolução - 3



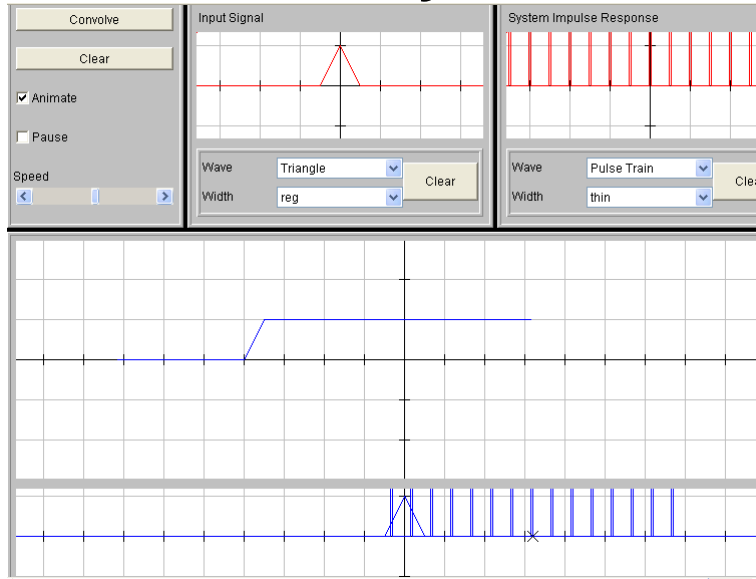
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Convolução - 3



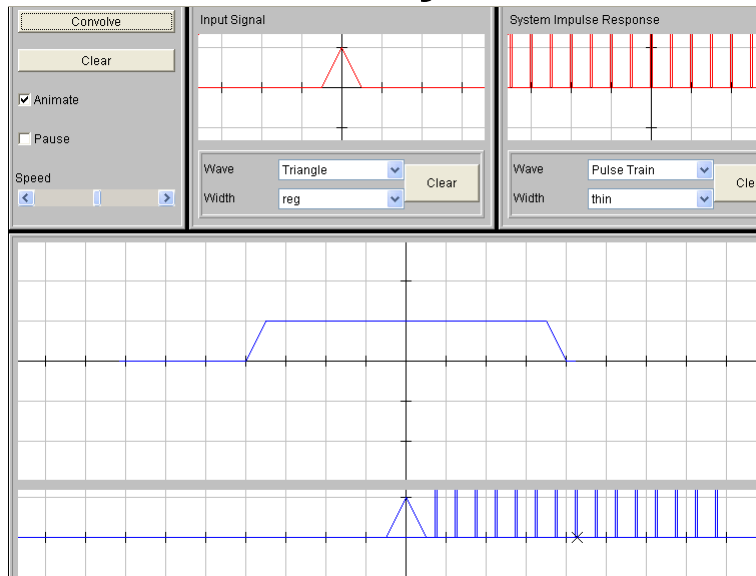
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Convolução - 3



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Convolução - 3



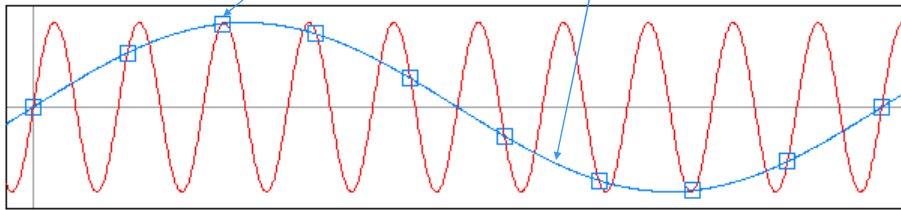
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Amostragem



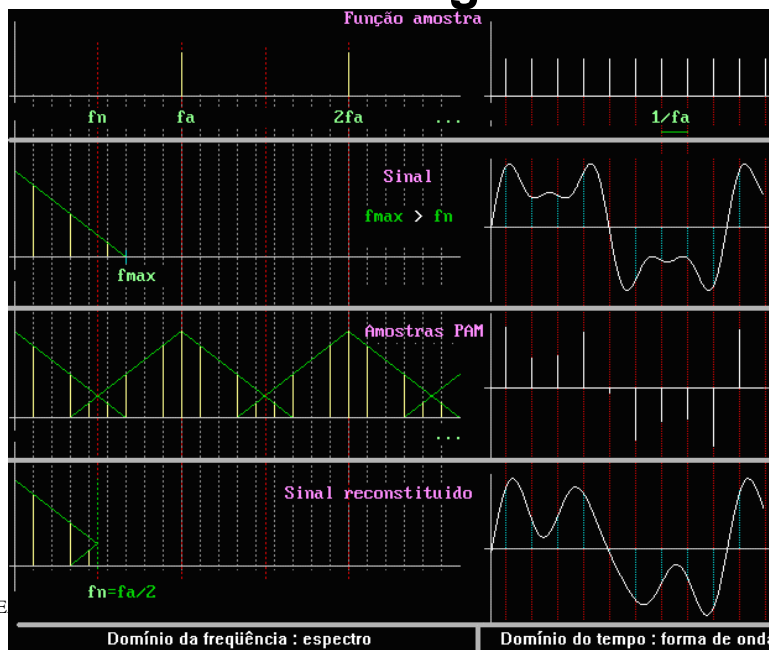
Amostragem

Reconstrução

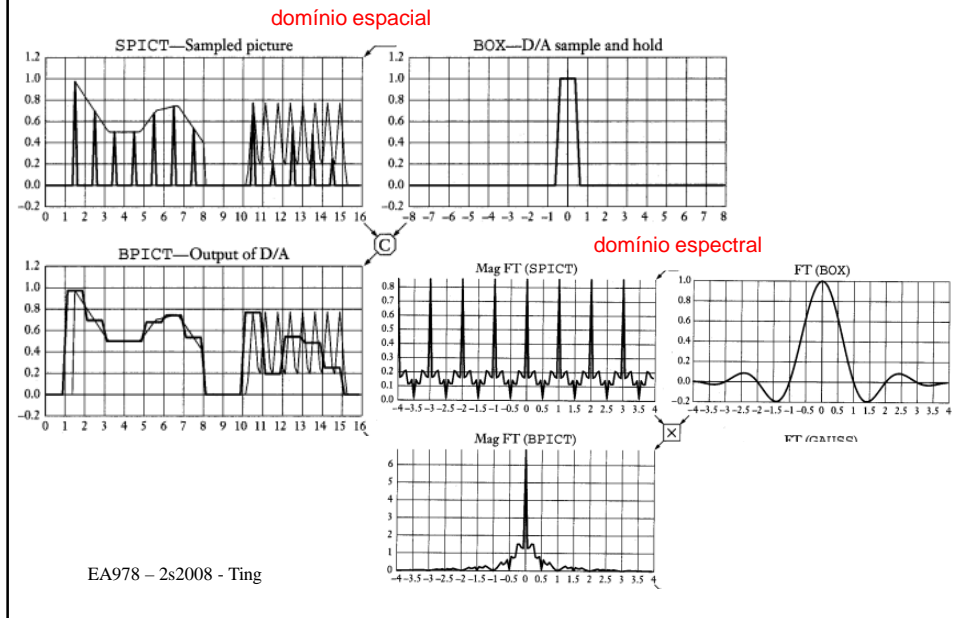


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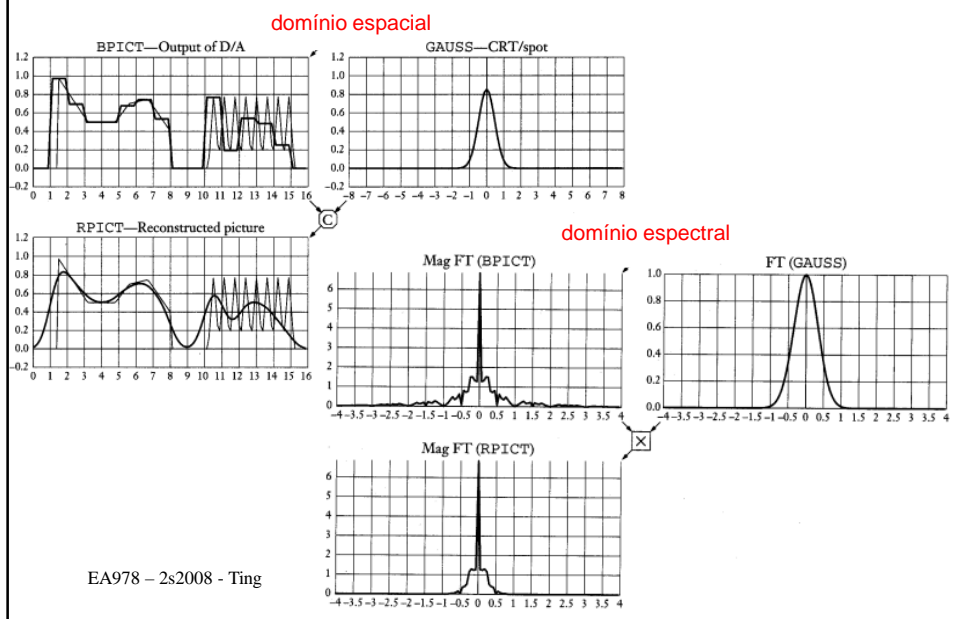
Amostragem



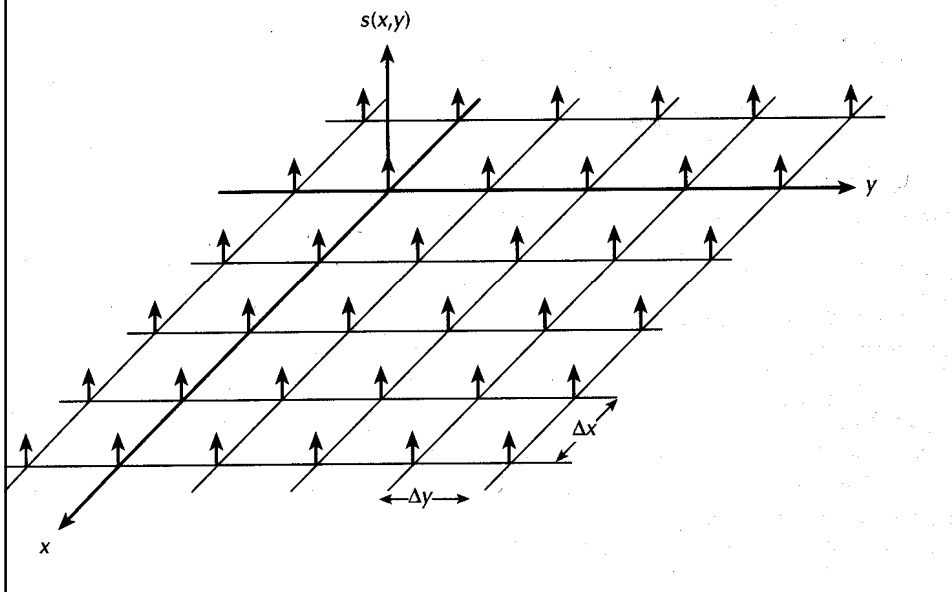
Reconstrução



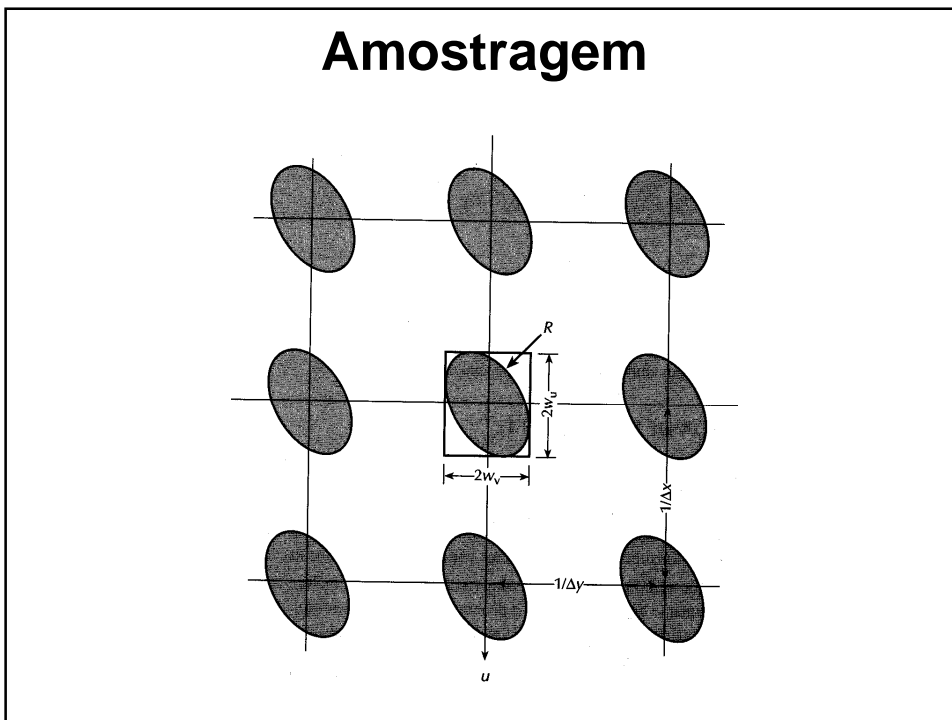
Exibição



Amostragem 2D



Amostragem



Anti-aliasing

Aliased



Anti-Aliased

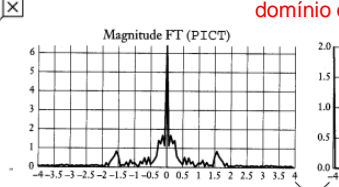
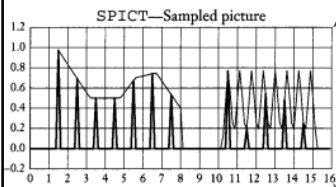
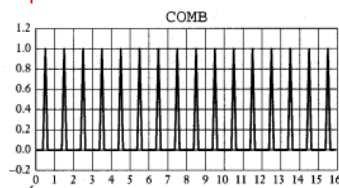
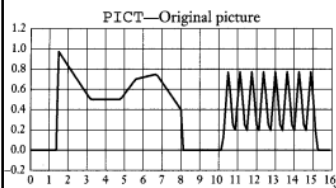
Aumentar a taxa de amostragem

Eliminar as altas frequências antes da amostragem → **Filtragem**

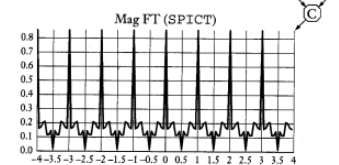
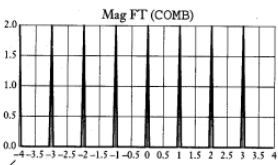
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Amostragem 1D

domínio espacial



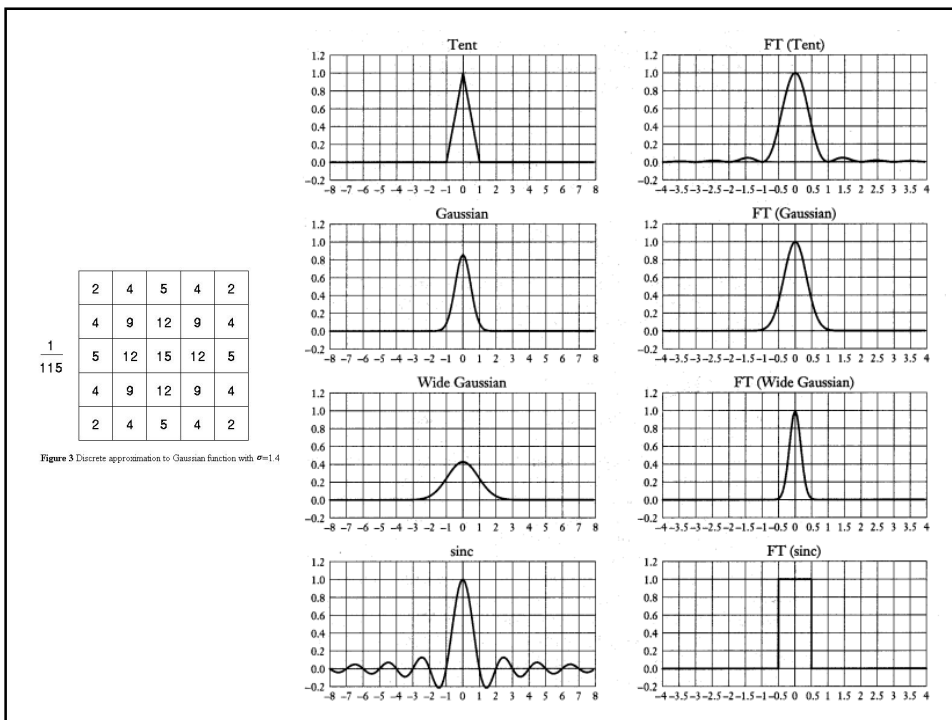
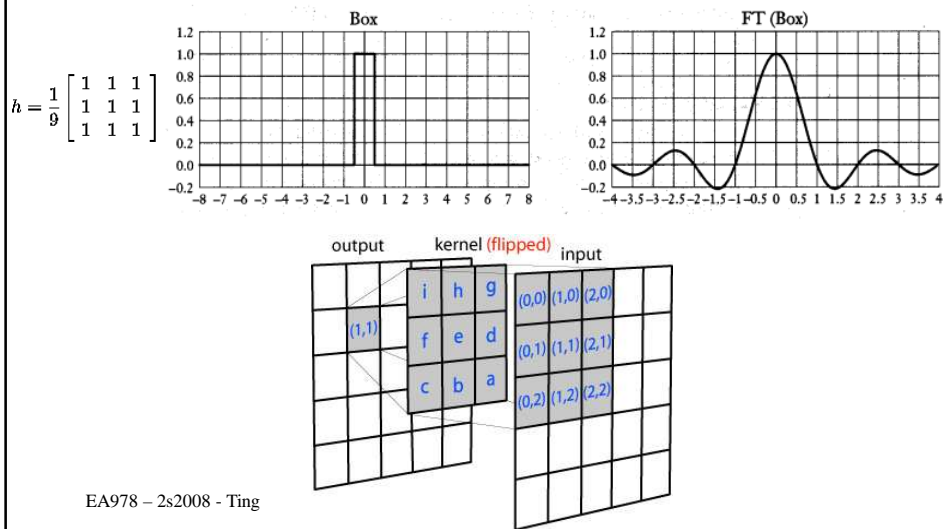
domínio espectral



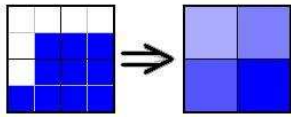
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Filtragem

Função de Espalhamento Pontual



Super amostragem



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